King Abdullah University of Science and Technology KAUST

CS 380 - GPU and GPGPU Programming
Lecture 22: GPU Texturing, Pt. 4

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Reading Assignment #12 (until Nov 24)

Read (required):

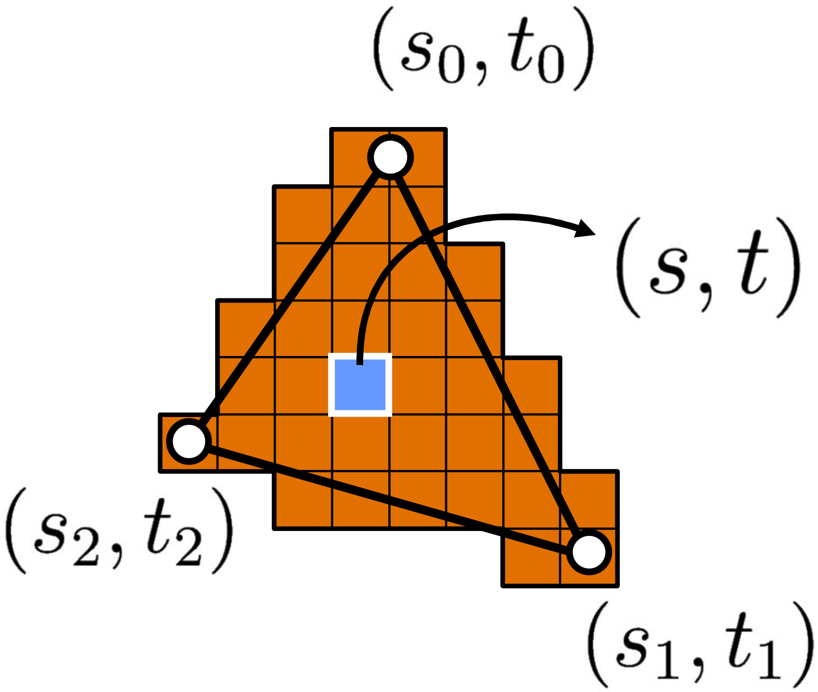
- Look at Vulkan *sparse resources*, especially *sparse partially-resident images*
  - `https://docs.vulkan.org/spec/latest/chapters/sparsemem.html`
- Read about shadow mapping
  - `https://en.wikipedia.org/wiki/Shadow_mapping`
- Look at Unreal Engine 5 virtual texturing
  - `https://dev.epicgames.com/documentation/en-us/unreal-engine/virtual-texturing-in-unreal-engine/`
- Look at Unreal Engine 5 MegaLights
  - `https://dev.epicgames.com/documentation/en-us/unreal-engine/megalights-in-unreal-engine/`

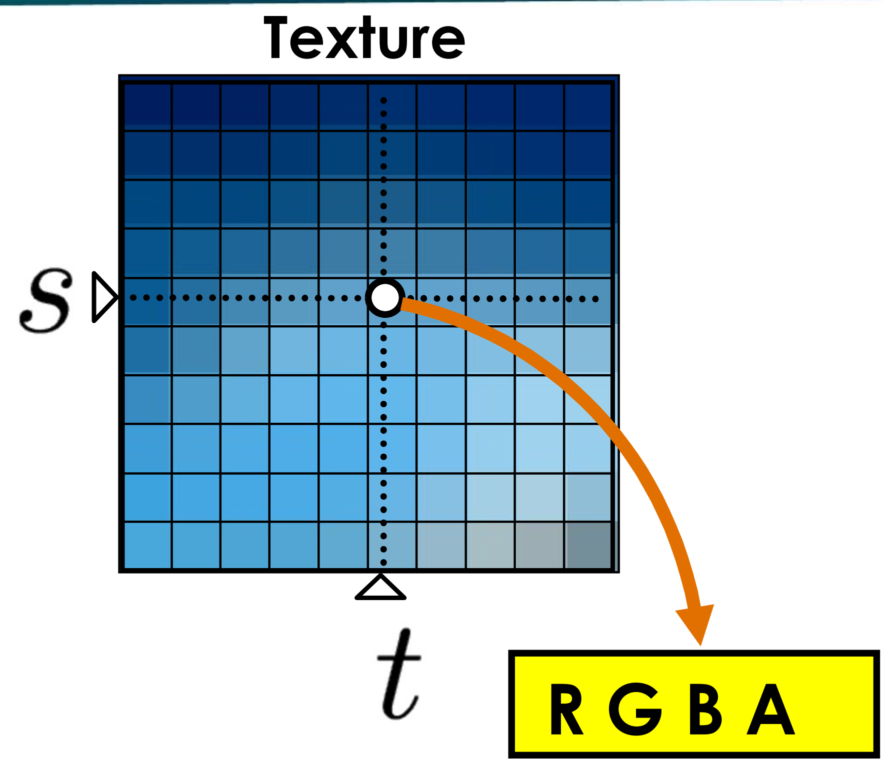
Read (optional):

- CUDA Warp-Level Primitives
  - `https://developer.nvidia.com/blog/using-cuda-warp-level-primitives/`
- Warp-aggregated atomics
  - `https://developer.nvidia.com/blog/cuda-pro-tip-optimized-filtering-warp-aggregated-atomics/`

GPU Texturing

2D Texture Mapping





For each fragment:
interpolate the
texture coordinates
(barycentric)
Or:
Use arbitrary, computed coordinates

Texture-Lookup:
interpolate the
texture data
(bi-linear)
Or:
Nearest-neighbor for "array lookup"

Interpolation #1

Interpolation Type + Purpose #1:

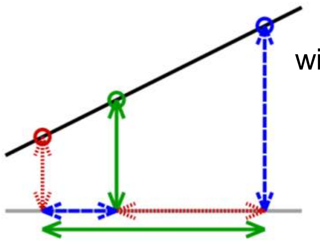
Interpolation of Texture Coordinates

(Linear / Rational-Linear Interpolation)

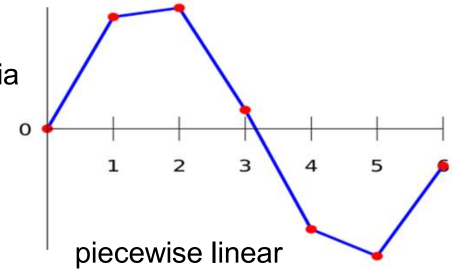
Linear Interpolation / Convex Combinations

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$



wikipedia



piecewise linear

Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad \alpha_1 + \alpha_2 = 1$$

$$f(\alpha) = v_1 + \alpha(v_2 - v_1) \qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

Compare to line parameterization with parameter t: $v(t) = v_1 + t(v_2 - v_1)$

Linear Interpolation / Convex Combinations

Linear combination (n-dim. space):

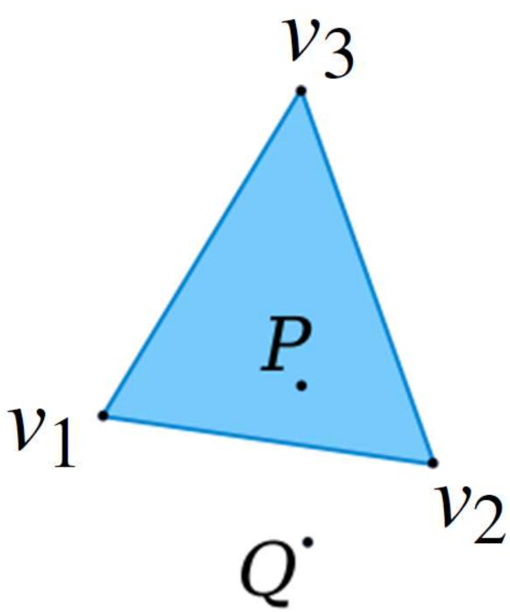
$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

Affine combination: Restrict to $(n-1)$-dim. subspace:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

Convex combination: $\alpha_i \geq 0$

(restrict to simplex in subspace)



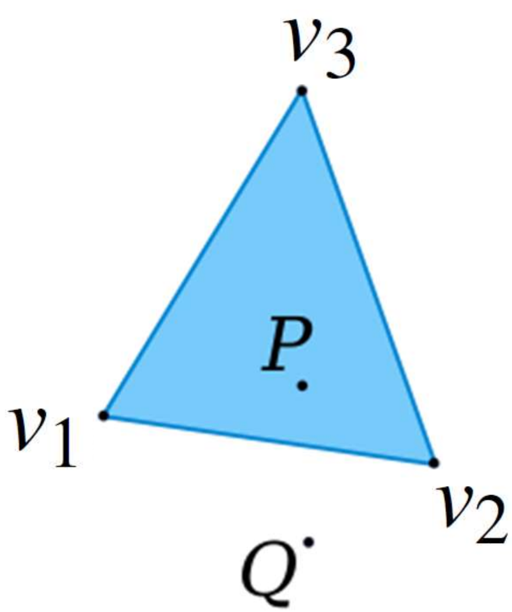
Linear Interpolation / Convex Combinations

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

Re-parameterize to get affine coordinates:

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 =$$
$$\tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1$$
$$\tilde{\alpha}_1 = \alpha_2$$
$$\tilde{\alpha}_2 = \alpha_3$$



Linear Interpolation / Convex Combinations

The weights α_i are the (normalized) barycentric coordinates

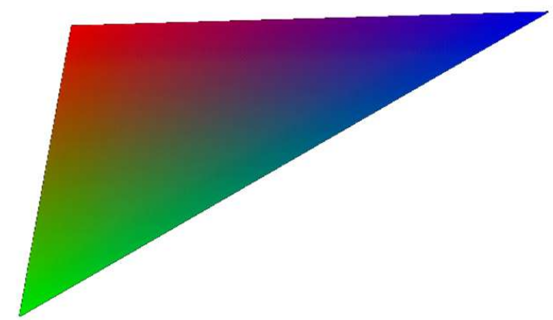
→ linear attribute interpolation in simplex

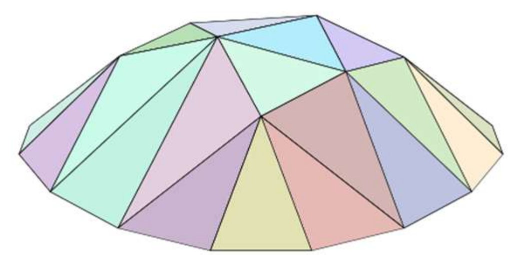
$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

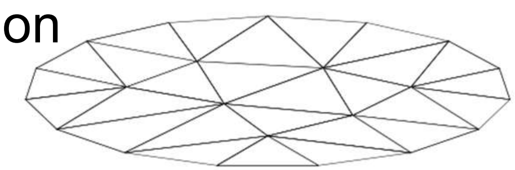
$$\alpha_i \geq 0$$

attribute interpolation





spatial position interpolation



wikipedia

Homogeneous Coordinates (1)

Projective geometry

- (Real) projective spaces RP^n:

  Real projective line RP^1, real projective plane RP^2, ...
- A point in RP^n is a line through the origin (i.e., all the scalar multiples of the same vector) in an (n+1)-dimensional (real) vector space

Homogeneous coordinates of 2D projective point in RP^2

- Coordinates differing only by a non-zero factor λ map to the same point

  $(\lambda x, \lambda y, \lambda)$ dividing out the λ gives $(x, y, 1)$, corresponding to (x,y) in R^2
- Coordinates with last component = 0 map to “points at infinity”

  $(\lambda x, \lambda y, 0)$ division by last component not allowed; but again this is the same point if it only differs by a scalar factor, e.g., this is the same point as $(x, y, 0)$

Homogeneous Coordinates (2)

Examples of usage

- Translation (with translation vector $\vec{b}$)
- Affine transformations (linear transformation + translation)

$$\vec{y} = A\vec{x} + \vec{b}.$$

- With homogeneous coordinates:

$$\begin{bmatrix} \vec{y} \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & A & & \vec{b} \\ 0 & \ldots & 0 & 1 \end{array} \right] \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix}$$

- Setting the last coordinate = 1 and the last row of the matrix to [0, ..., 0, 1] results in translation of the point $\vec{x}$ (via addition of translation vector $\vec{b}$)
- The matrix above is a linear map, but because it is one dimension higher, it does not have to move the origin in the (n+1)-dimensional space for translation

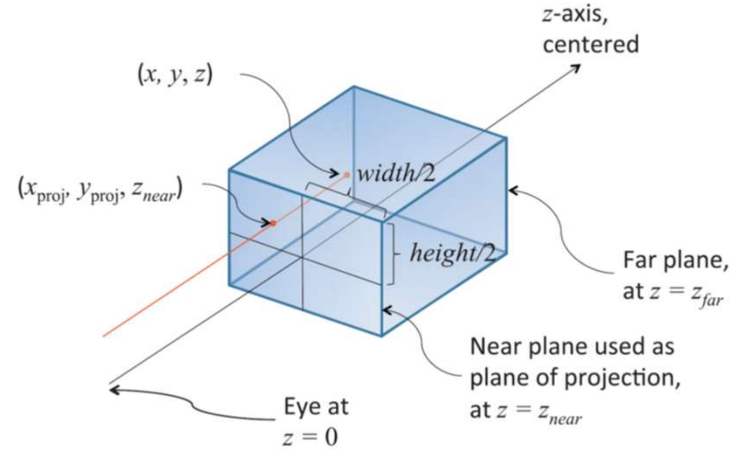
Homogeneous Coordinates (3)

Examples of usage

- Projection (e.g., OpenGL projection matrices)

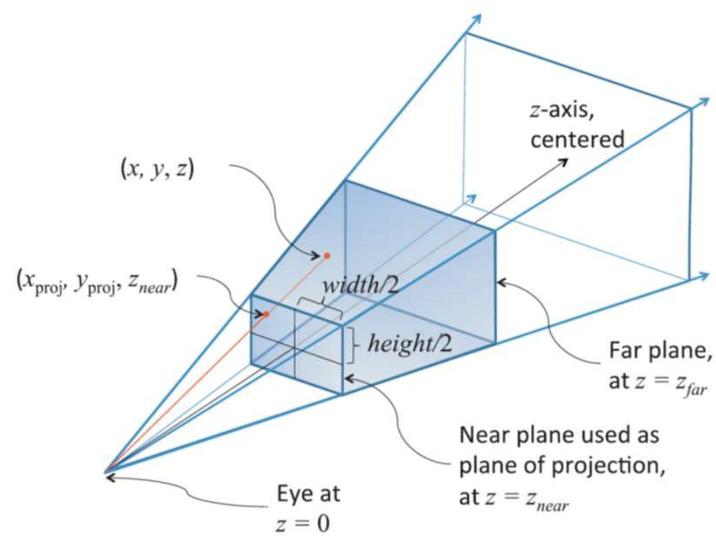
$$\begin{bmatrix} \frac{2}{\text{right}-\text{left}} & 0 & 0 & -\frac{\text{right}+\text{left}}{\text{right}-\text{left}} \\ 0 & \frac{2}{\text{top}-\text{bottom}} & 0 & -\frac{\text{top}+\text{bottom}}{\text{top}-\text{bottom}} \\ 0 & 0 & \frac{-2}{\text{far}-\text{near}} & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

orthographic



$$\begin{bmatrix} \frac{z_{near}}{width/2} & 0.0 & \frac{left+right}{width/2} & 0.0 \\ 0.0 & \frac{z_{near}}{height/2} & \frac{top+bottom}{height/2} & 0.0 \\ 0.0 & 0.0 & -\frac{z_{far}+z_{near}}{z_{far}-z_{near}} & \frac{2z_{far}z_{near}}{z_{far}-z_{near}} \\ 0.0 & 0.0 & -1.0 & 0.0 \end{bmatrix}$$

perspective



Texture Mapping

2D (3D) Texture Space

Texture Transformation

2D Object Parameters

Parameterization

3D Object Space

Model Transformation

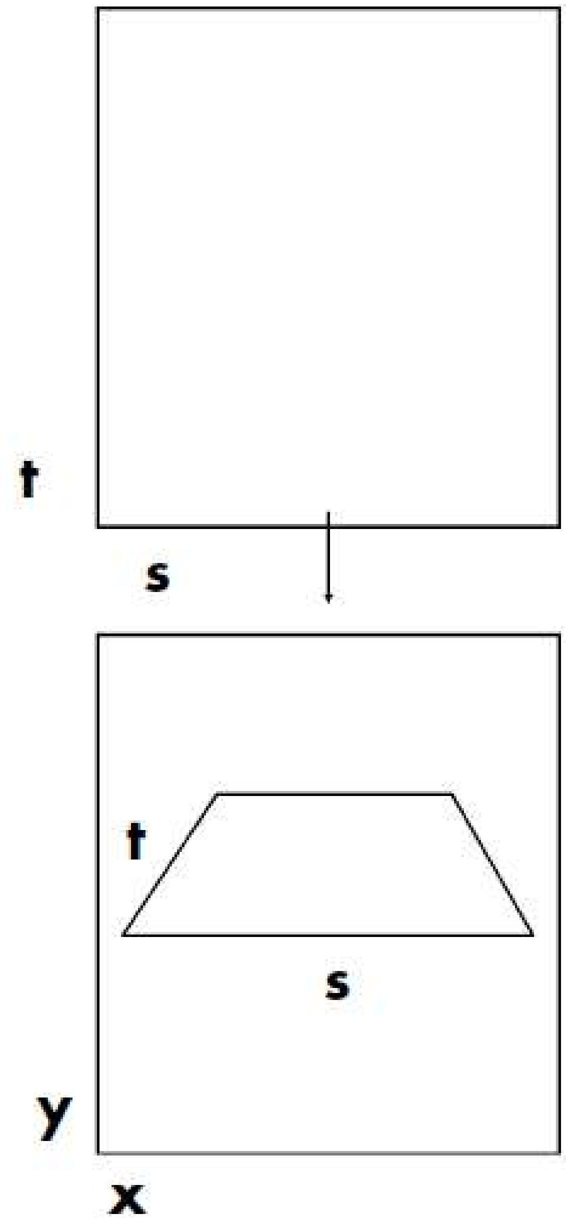
3D World Space

Viewing Transformation

3D Camera Space

Projection

2D Image Space



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Texture Mapping Polygons

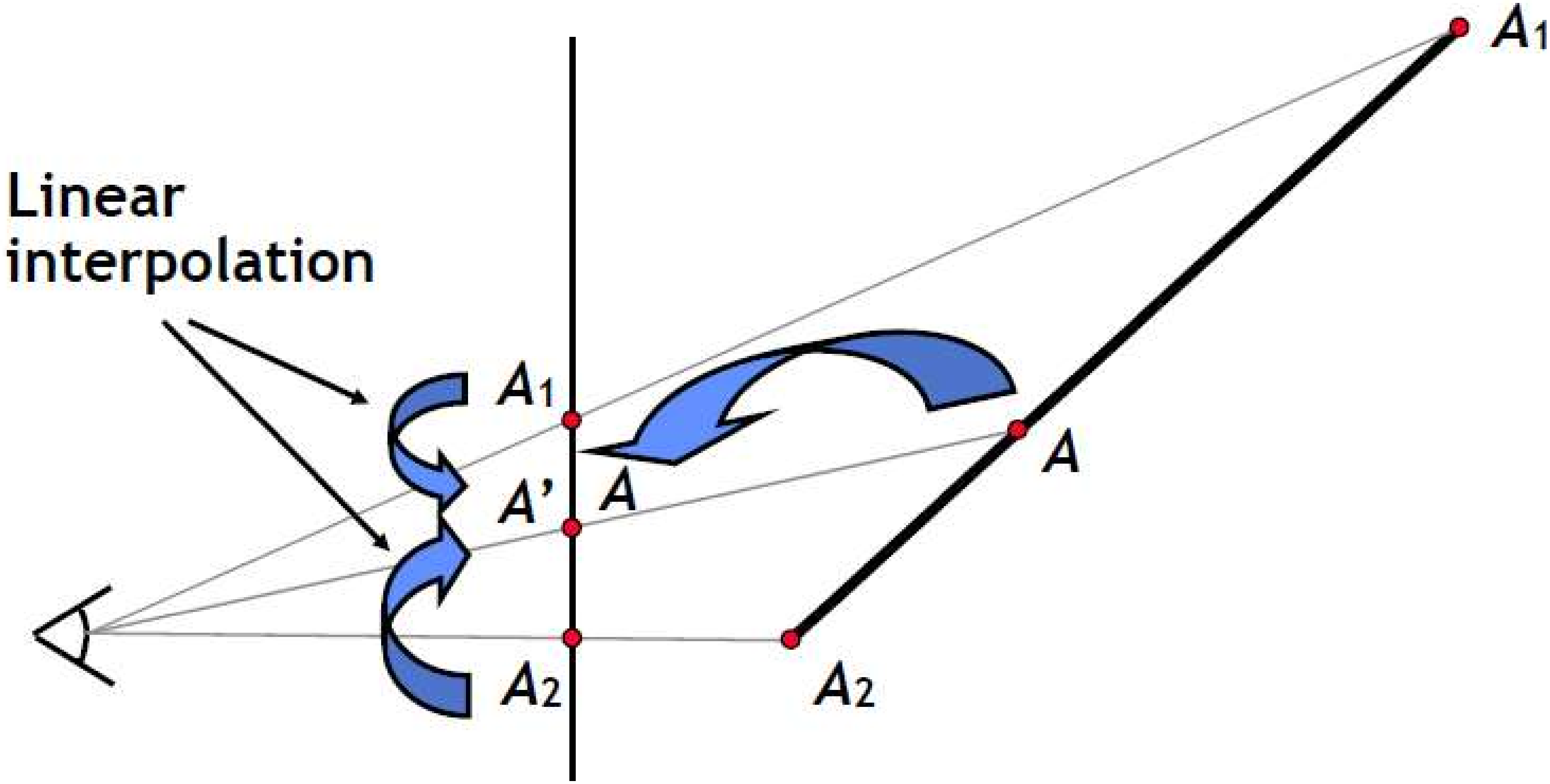
Forward transformation: linear projective map

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} s \\ t \\ r \end{bmatrix}$$

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Incorrect attribute interpolation



$\boxed{A' \neq A\,!}$

Kurt Akeley, Pat Hanrahan

Linear interpolation

Compute intermediate attribute value

- Along a line: $A = aA_1 + bA_2$, $a+b=1$
- On a plane: $A = aA_1 + bA_2 + cA_3$, $a+b+c=1$

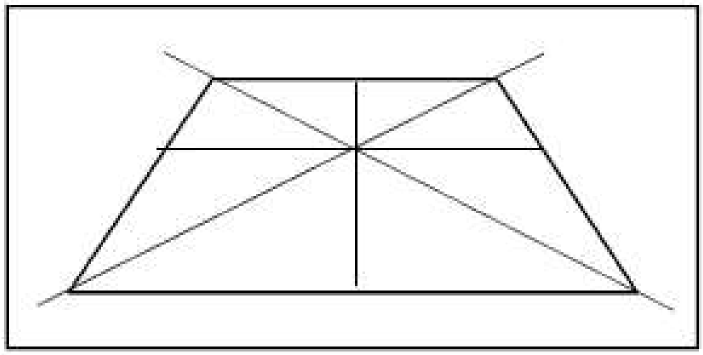
Only projected values interpolate linearly in screen space (straight lines project to straight lines)

- x and y are projected (divided by w)
- Attribute values are not naturally projected

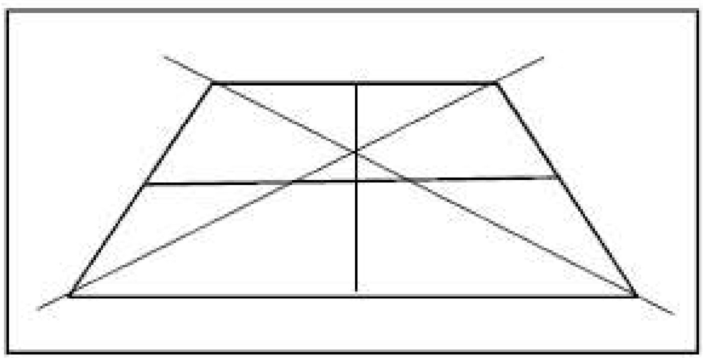
Choice for attribute interpolation in screen space

- Interpolate unprojected values
  - Cheap and easy to do, but gives wrong values
  - Sometimes OK for color, but
  - Never acceptable for texture coordinates
- Do it right

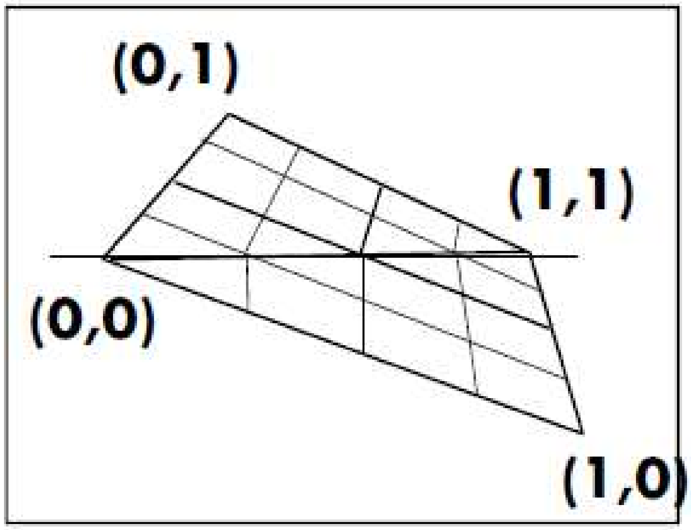
Linear Perspective



Correct Linear Perspective



Incorrect Perspective

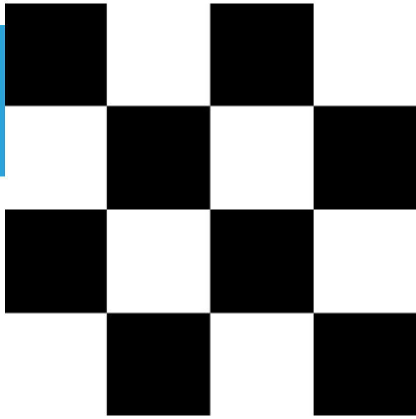


Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

Kurt Akeley, Pat Hanrahan

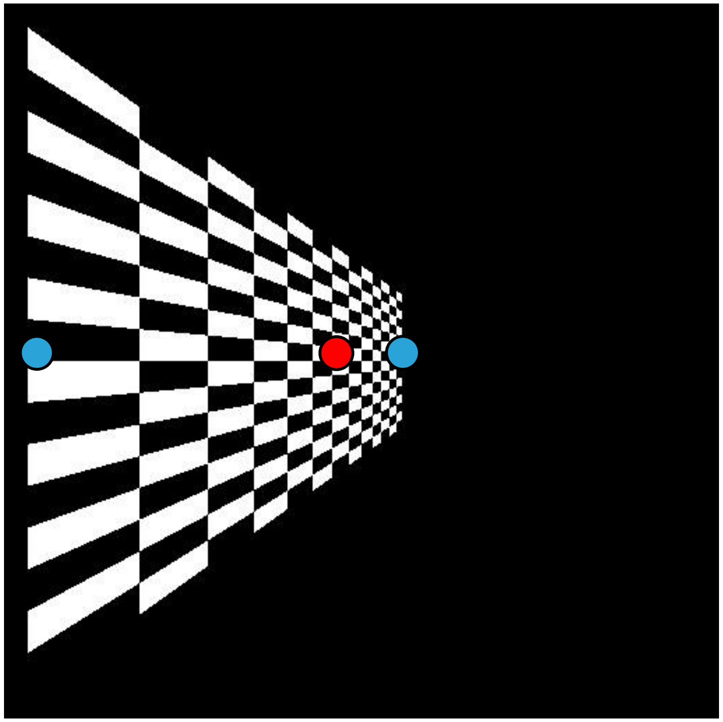
Perspective Texture Mapping

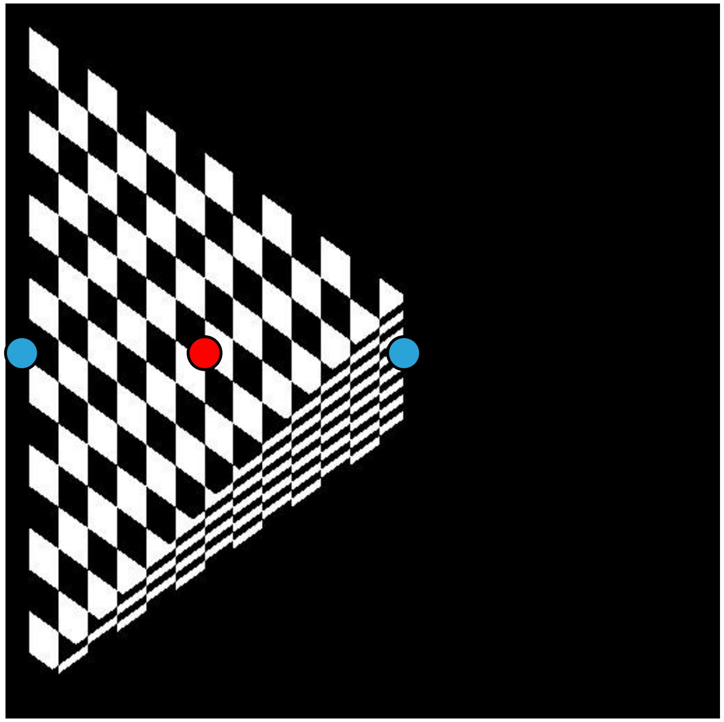


linear interpolation in object space

$$\frac{a x_1 + b x_2}{a w_1 + b w_2} \neq a\frac{x_1}{w_1} + b\frac{x_2}{w_2}$$

linear interpolation in screen space





$a = b = 0.5$

Perspective-correct linear interpolation

Only projected values interpolate correctly, so project A

- Linearly interpolate A_1/w_1 and A_2/w_2

Also interpolate $1/w_1$ and $1/w_2$

- These also interpolate linearly in screen space

Divide interpolants at each sample point to recover A

- $(A/w) \,/\, (1/w) = A$
- Division is expensive (more than add or multiply), so
  - Recover w for the sample point (reciprocate), and
  - Multiply each projected attribute by w

Barycentric triangle parameterization:

$$A = \frac{aA_1/w_1 + bA_2/w_2 + cA_3/w_3}{a/w_1 + b/w_2 + c/w_3} \qquad a + b + c = 1$$

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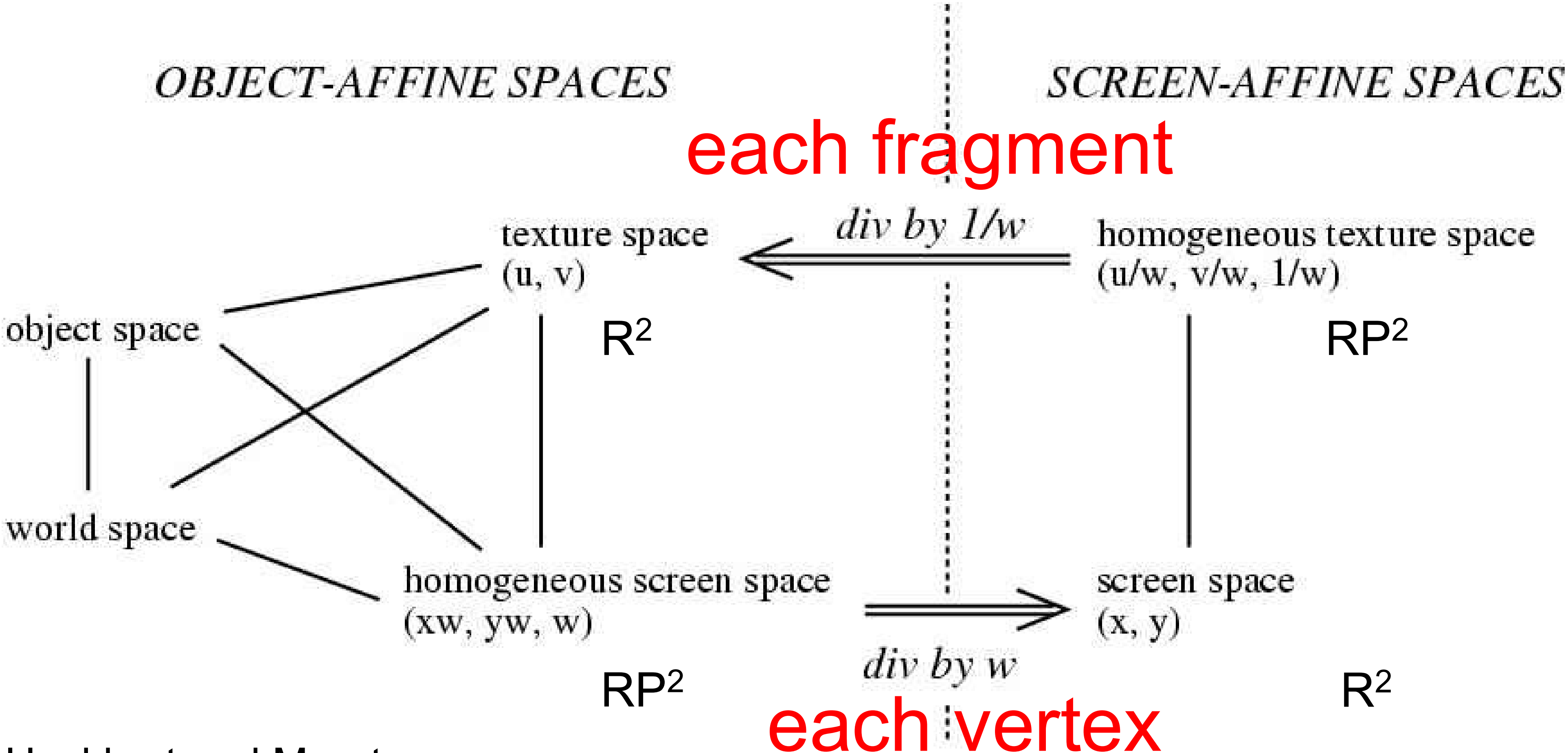
Perspective Texture Mapping

- Solution: interpolate (s/w, t/w, 1/w)
- (s/w) / (1/w) = s etc. at every fragment



Heckbert and Moreton

Perspective-Correct Interpolation Recipe

$$r_i(x,y) = \frac{r_i(x,y)/w(x,y)}{1/w(x,y)}$$

(1) Associate a record containing the n parameters of interest $(r_1, r_2, \cdots, r_n)$ with each vertex of the polygon.

(2) For each vertex, transform object space coordinates to homogeneous screen space using 4×4 object to screen matrix, yielding the values (xw, yw, zw, w).

(3) Clip the polygon against plane equations for each of the six sides of the viewing frustum, linearly interpolating all the parameters when new vertices are created.

(4) At each vertex, divide the homogeneous screen coordinates, the parameters r_i, and the number 1 by w to construct the variable list $(x, y, z, s_1, s_2, \cdots, s_{n+1})$, where $s_i = r_i/w$ for $i \le n$, $s_{n+1} = 1/w$.

(5) Scan convert in screen space by linear interpolation of all parameters, at each pixel computing $r_i = s_i/s_{n+1}$ for each of the n parameters; use these values for shading.

Heckbert and Moreton

Projective Map vs. Interpolation Recipe (1)

In general (see previous slides), we had the projective map:

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Let's rename and rewrite this as:

$$\begin{bmatrix} s \\ t \\ q \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_{world} \\ y_{world} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \cdot w \\ y \cdot w \\ w \end{bmatrix}$$

For homogeneous points we can also divide by w:

$$\begin{bmatrix} s \\ t \\ q \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \cdot w \\ y \cdot w \\ w \end{bmatrix},$$

$$\begin{bmatrix} s/w \\ t/w \\ q/w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.$$

Coordinates on the right become screen space coordinates!

Projective Map vs. Interpolation Recipe (2)

In general (see previous slides), we had the projective map:

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Let's rename and rewrite this as:

$$\begin{bmatrix} s \\ t \\ q \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_{world} \\ y_{world} \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \cdot w \\ y \cdot w \\ w \end{bmatrix}$$

For homogeneous points we can also divide by w:

$$\begin{bmatrix} s \\ t \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \cdot w \\ y \cdot w \\ w \end{bmatrix},$$

(special case $q = 1$)

Coordinates on the right become screen space coordinates!

$$\boxed{\begin{bmatrix} s/w \\ t/w \\ 1/w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}.}$$

Projective Map vs. Interpolation Recipe (3)

In general (see previous slides), we had the projective map:

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Now consider scanline interpolation:

(barycentric interpolation is linear along any line: here, horizontal line)

$$\begin{bmatrix} s/w \\ t/w \\ 1/w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x + \Delta x \\ y \\ 1 \end{bmatrix},$$

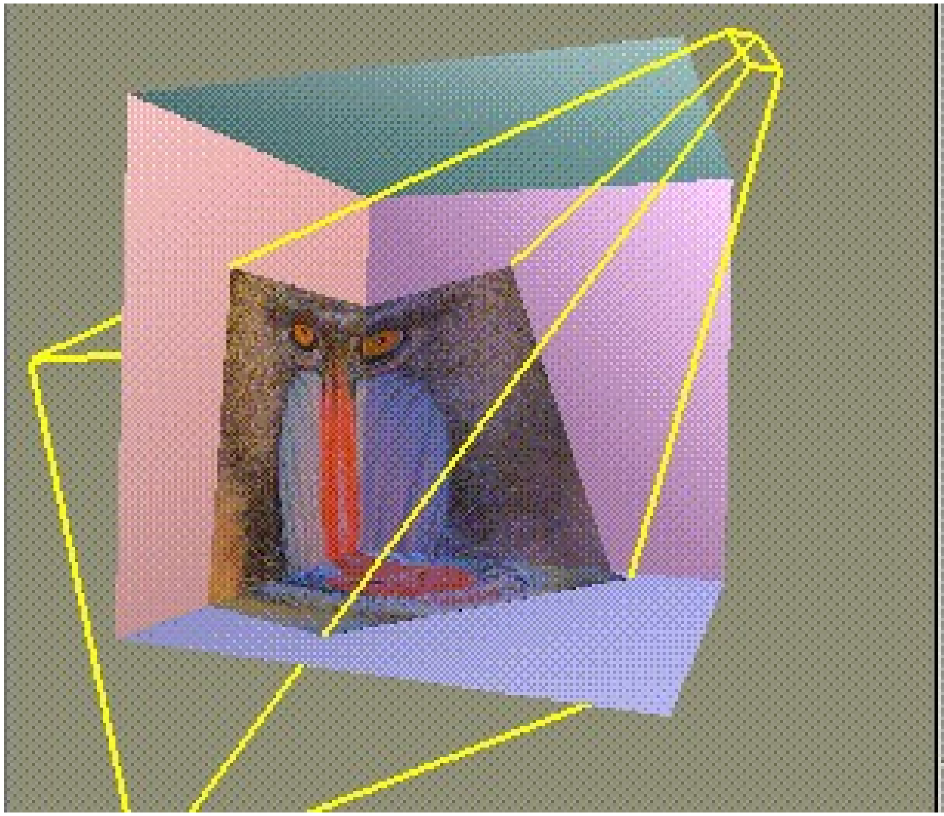
$$\begin{bmatrix} s/w \\ t/w \\ 1/w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} + \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \Delta x \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta_x \begin{bmatrix} s/w \\ t/w \\ 1/w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} \Delta x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a \cdot \Delta x \\ d \cdot \Delta x \\ g \cdot \Delta x \end{bmatrix} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$$

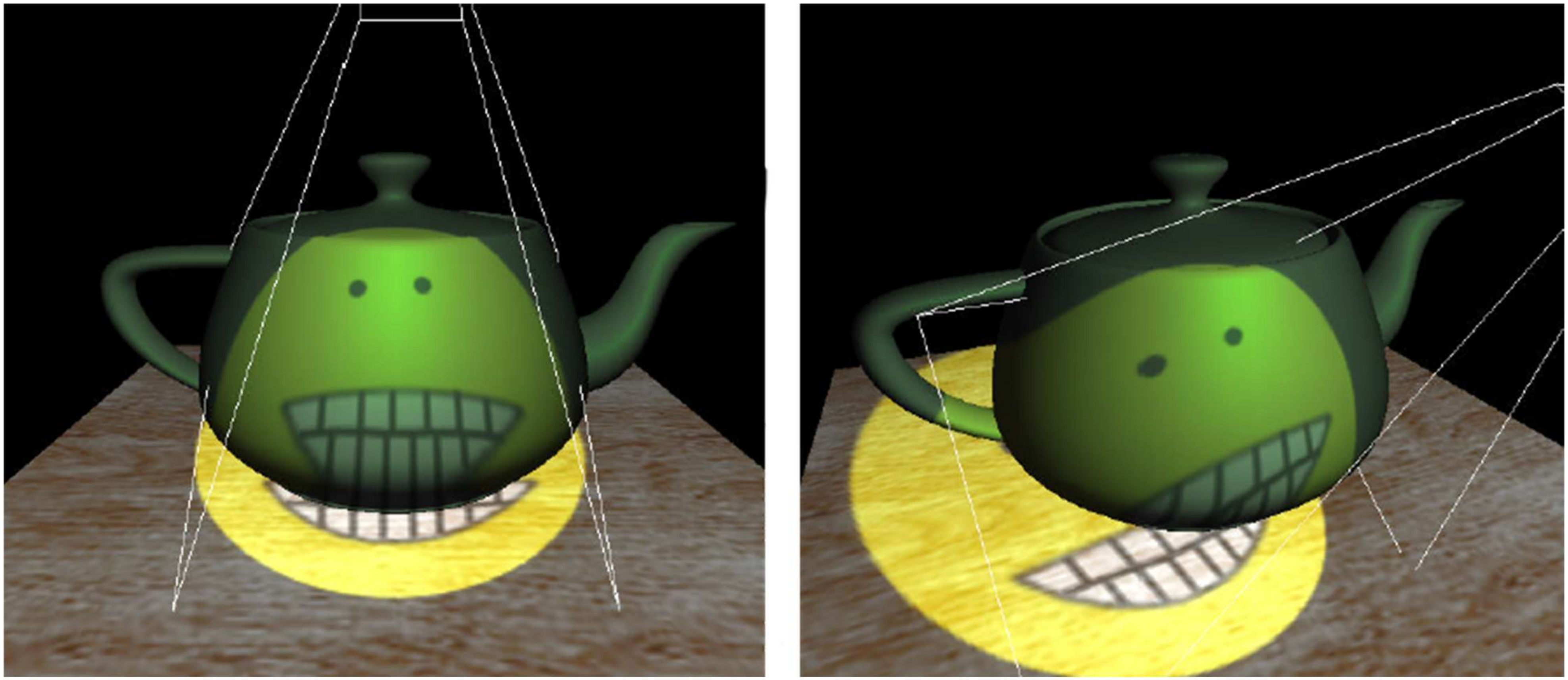
$$(\Delta x = 1)$$

Projective Texture Mapping

- Want to simulate a beamer
  - … or a flashlight, or a slide projector
- Precursor to shadows
- Interesting mathematics: 2 perspective projections involved!
- Easy to program!



Projective Texture Mapping



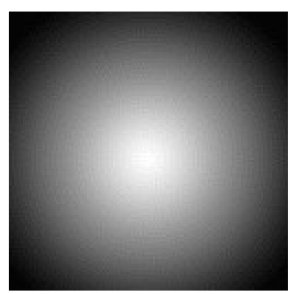
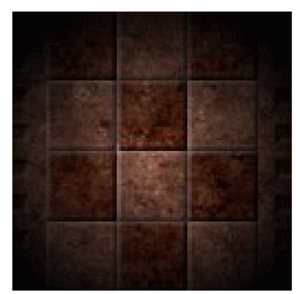
Projective Shadows in Doom 3

Projective Texturing

- What about homogeneous texture coords?
- Need to do perspective divide also for projector!
  - (s, t, q) → (s/q, t/q) for every fragment
- How does OpenGL do that?
  - Needs to be perspective correct as well!
  - Trick: interpolate (s/w, t/w, r/w, q/w)
  - (s/w) / (q/w) = s/q etc. at every fragment
- Remember: s,t,r,q are equivalent to x,y,z,w in projector space! → r/q = projector depth!

Multitexturing

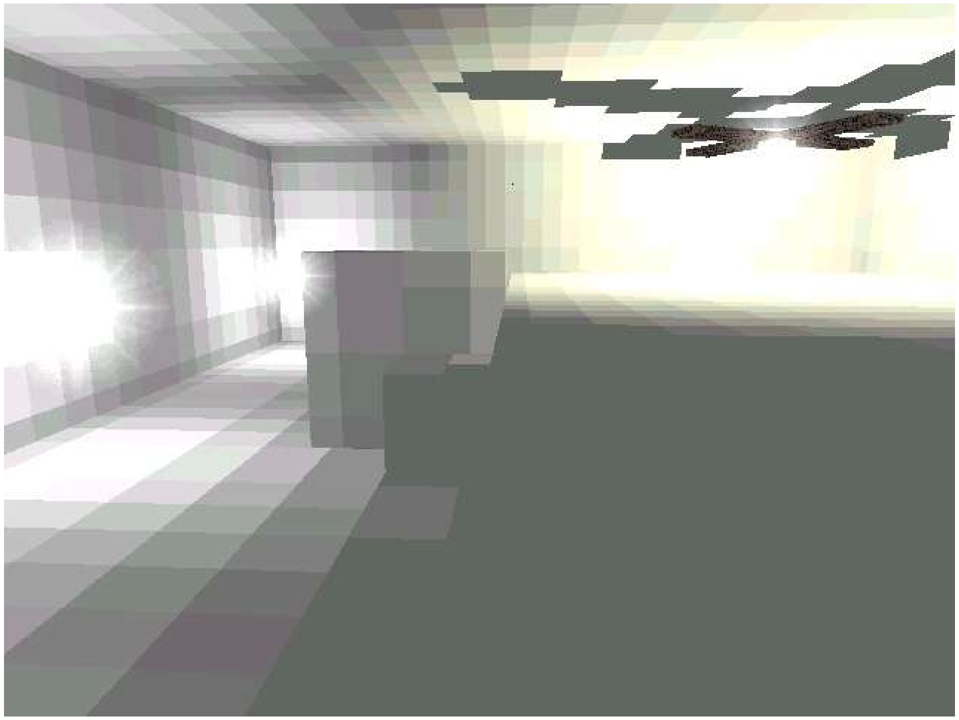
- Apply multiple textures in one pass
- *Integral* part of programmable shading
  - e.g. diffuse texture map + gloss map
  - e.g. diffuse texture map + light map
- Performance issues
  - How many textures are free?
  - How many are available

Example: Light Mapping

- Used in virtually every commercial game
- Precalculate diffuse lighting on static objects
  - Only low resolution necessary
  - Diffuse lighting is view independent!
- Advantages:
  - No runtime lighting necessary
    - VERY fast!
  - Can take global effects (shadows, color bleeds) into account

Light Mapping

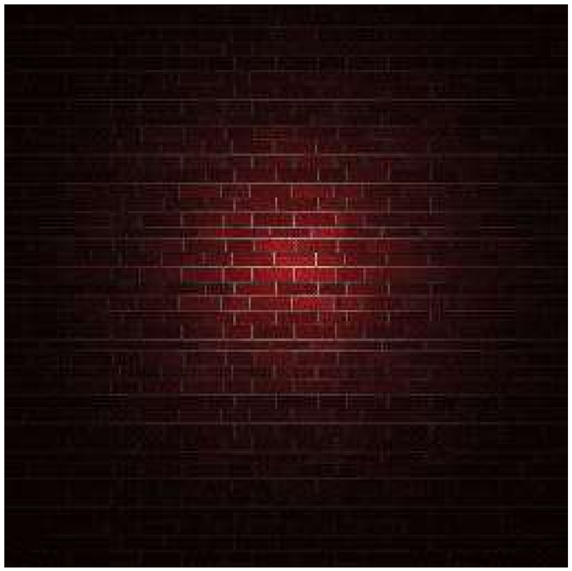


Original LM texels

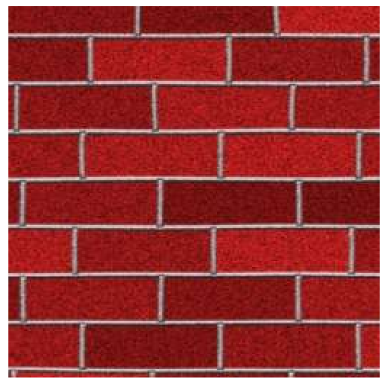
Bilinear Filtering

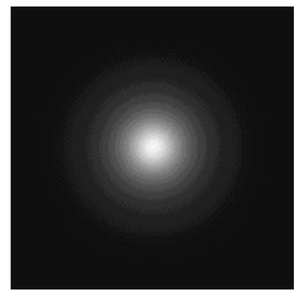
Light Mapping Issues

- Why premultiplication is bad…



Full Size Texture
(with Lightmap)





Tiled Surface Texture
plus Lightmap

→ use tileable surface textures and low resolution lightmaps

Light Mapping

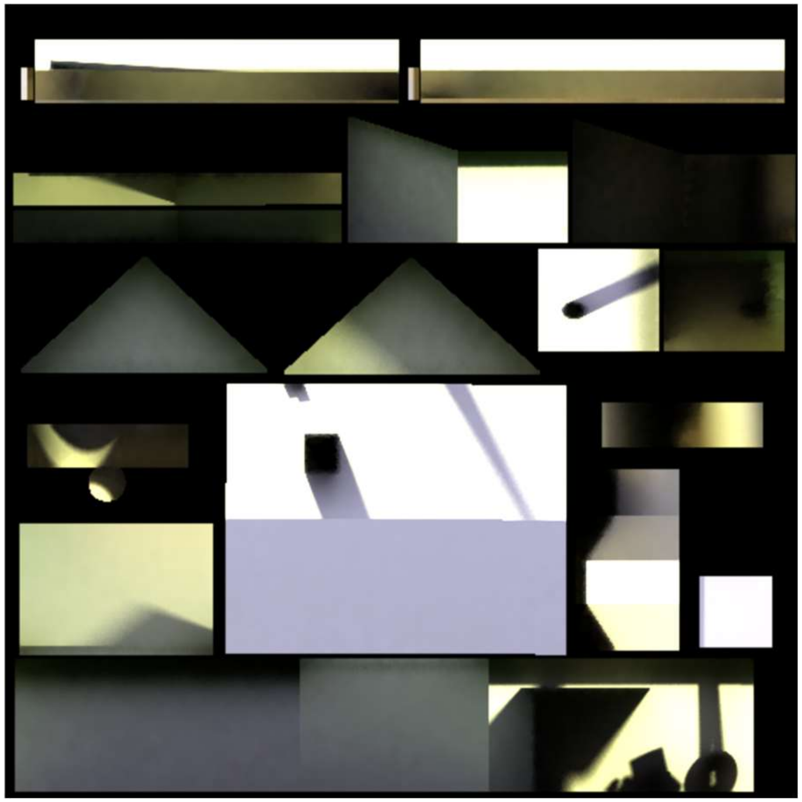
Original scene

Light-mapped

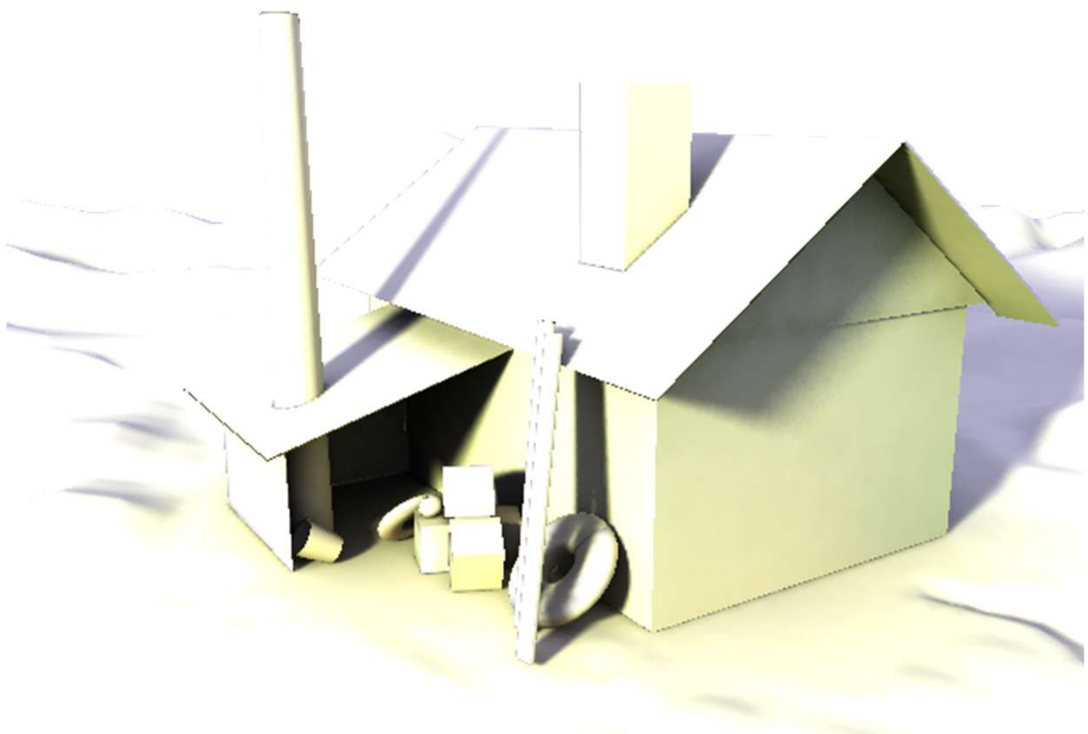
Example: Light Mapping

- Precomputation based on non-realtime methods
  - Radiosity
  - Ray tracing
    - Monte Carlo Integration
    - Path tracing
    - Photon mapping

Light Mapping

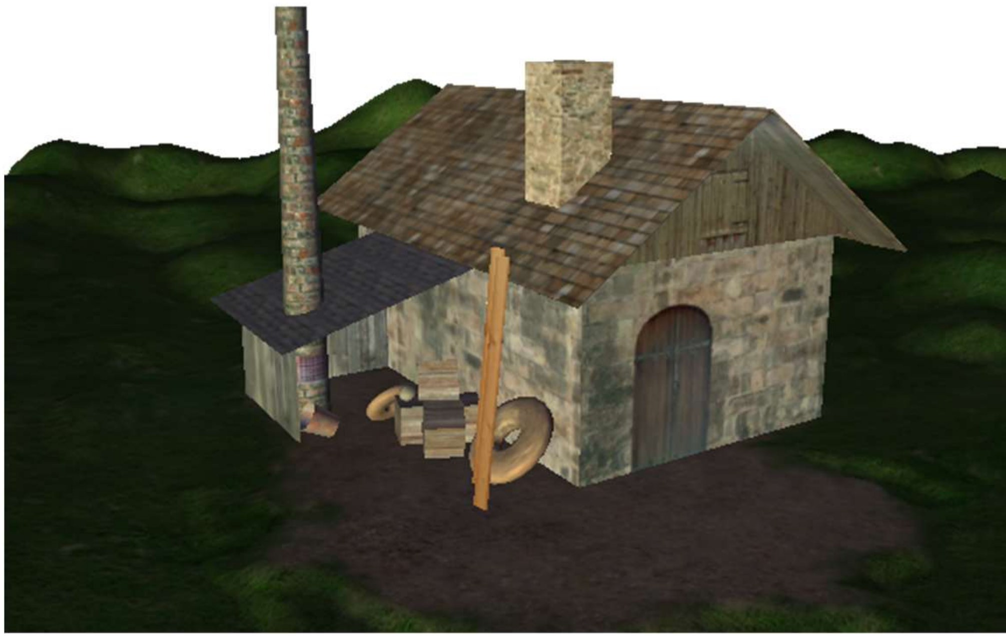


Lightmap



mapped

Light Mapping



Original scene

Light-mapped

Thank you.