King Abdullah University of Science and Technology KAUST

CS 380 - GPU and GPGPU Programming
Lecture 18: GPU Texturing 5

Markus Hadwiger, KAUST

Reading Assignment #10 (until Nov 9)

Read (required):

- **Brook for GPUs: Stream Computing on Graphics Hardware**
  Ian Buck et al., SIGGRAPH 2004

http://graphics.stanford.edu/papers/brookgpu/

Read (optional):

- **The Imagine Stream Processor**
  Ujval Kapasi et al.; IEEE ICCD 2002

http://cva.stanford.edu/publications/2002/imagine-overview-iccd/

- **Merrimac: Supercomputing with Streams**
  Bill Dally et al.; SC 2003

https://dl.acm.org/citation.cfm?doid=1048935.1050187

Texture Magnification

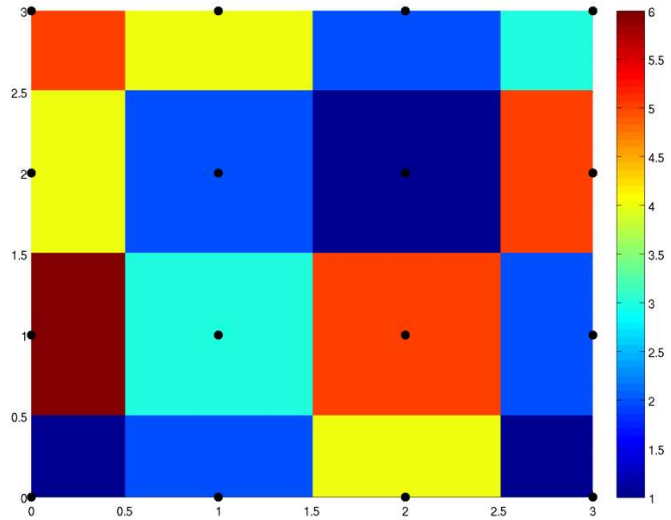
Magnification (Bi-linear Filtering Example)

Original image

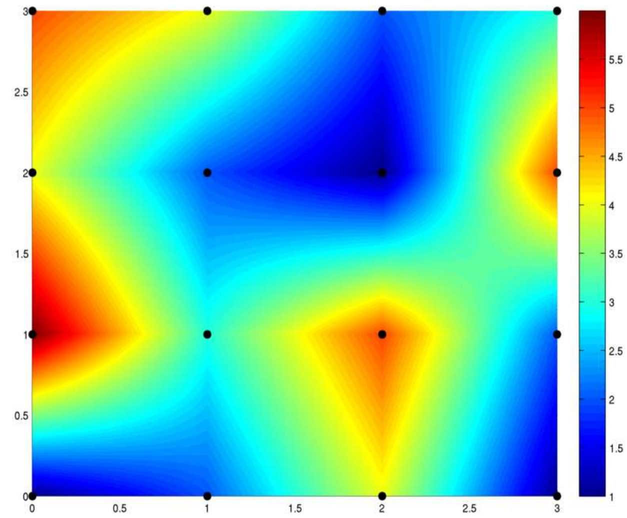
Nearest neighbor

Bi-linear filtering

Nearest-Neighbor vs. Bi-Linear Interpolation

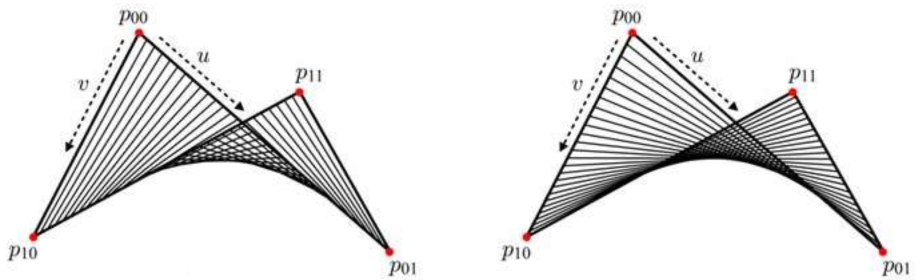


nearest-neighbor



bi-linear

wikipedia



Bilinear patch (courtesy J. Han)

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples (e.g., pixel centers):

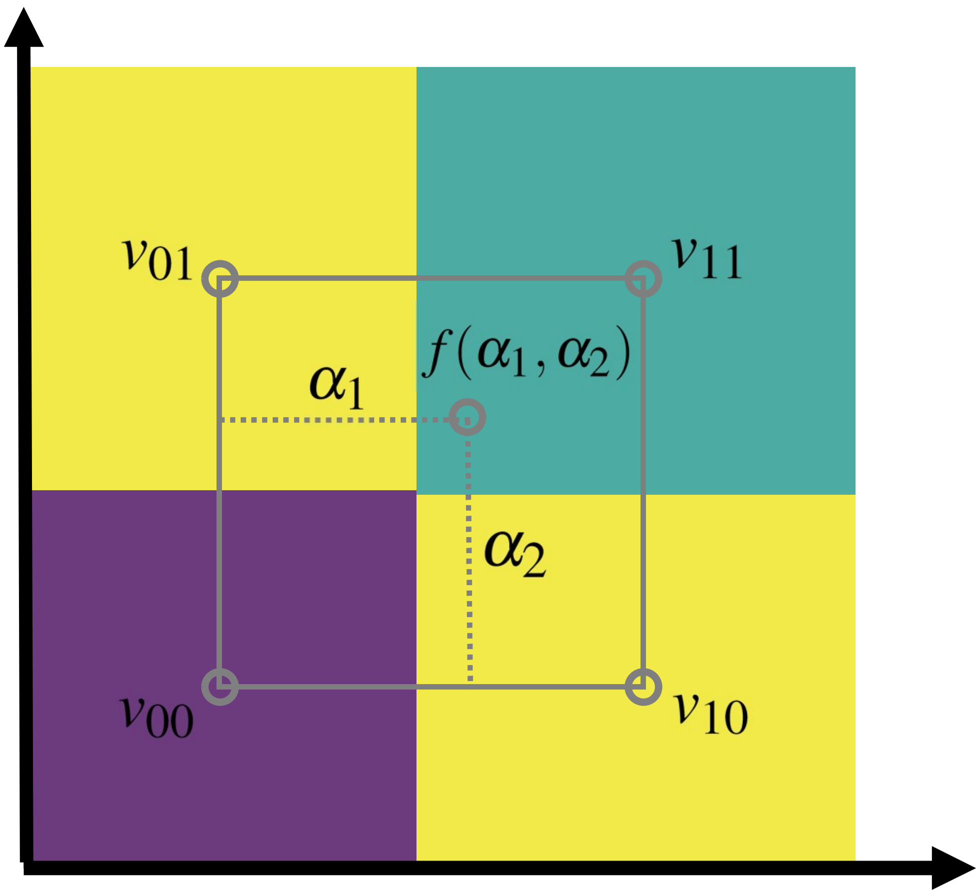
Given any (fractional) position

$$\alpha_1 := x_1 - \lfloor x_1 \rfloor \qquad \alpha_1 \in [0.0, 1.0)$$
$$\alpha_2 := x_2 - \lfloor x_2 \rfloor \qquad \alpha_2 \in [0.0, 1.0)$$

and 2x2 sample values

$$\begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix}$$

Compute: $f(\alpha_1, \alpha_2)$



Bi-Linear Interpolation

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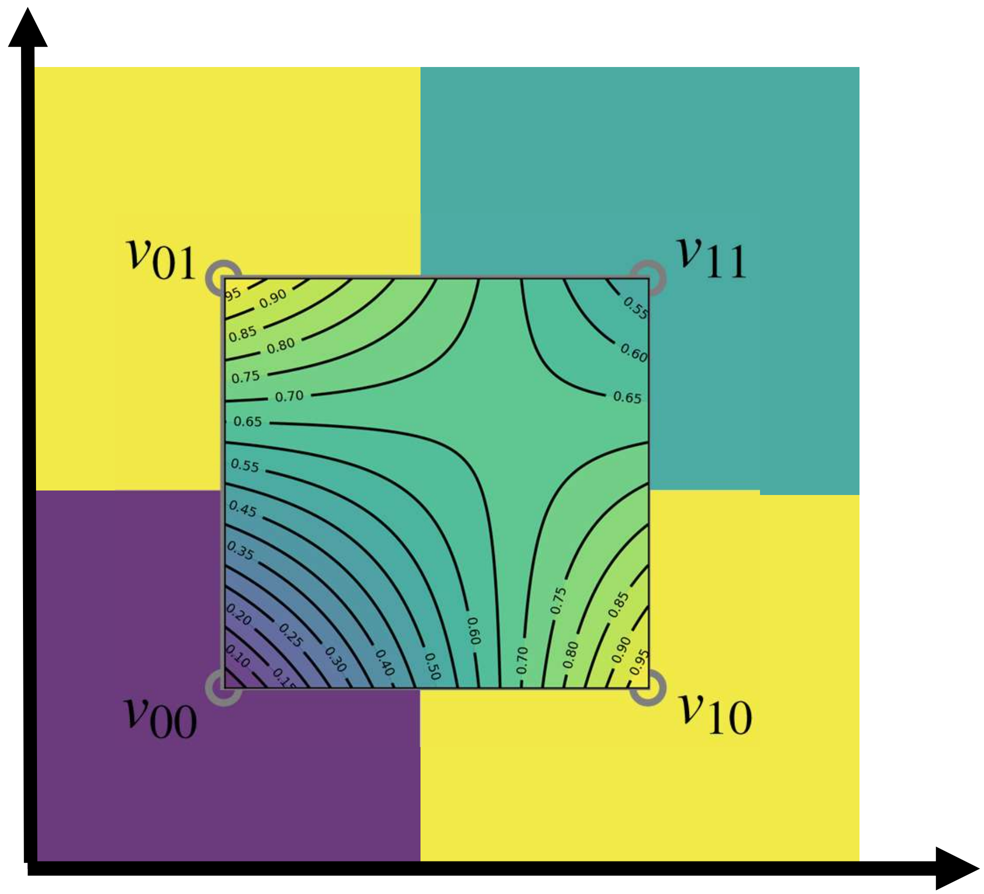


Bi-Linear Interpolation

Weights in 2x2 format:

$$\begin{bmatrix} \alpha_2 \\ (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1) & \alpha_1 \end{bmatrix} = \begin{bmatrix} (1-\alpha_1)\alpha_2 & \alpha_1\alpha_2 \\ (1-\alpha_1)(1-\alpha_2) & \alpha_1(1-\alpha_2) \end{bmatrix}$$

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1)v_{01} + \alpha_1 v_{11} \\ (1-\alpha_1)v_{00} + \alpha_1 v_{10} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 v_{01} + (1-\alpha_2)v_{00} & \alpha_2 v_{11} + (1-\alpha_2)v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

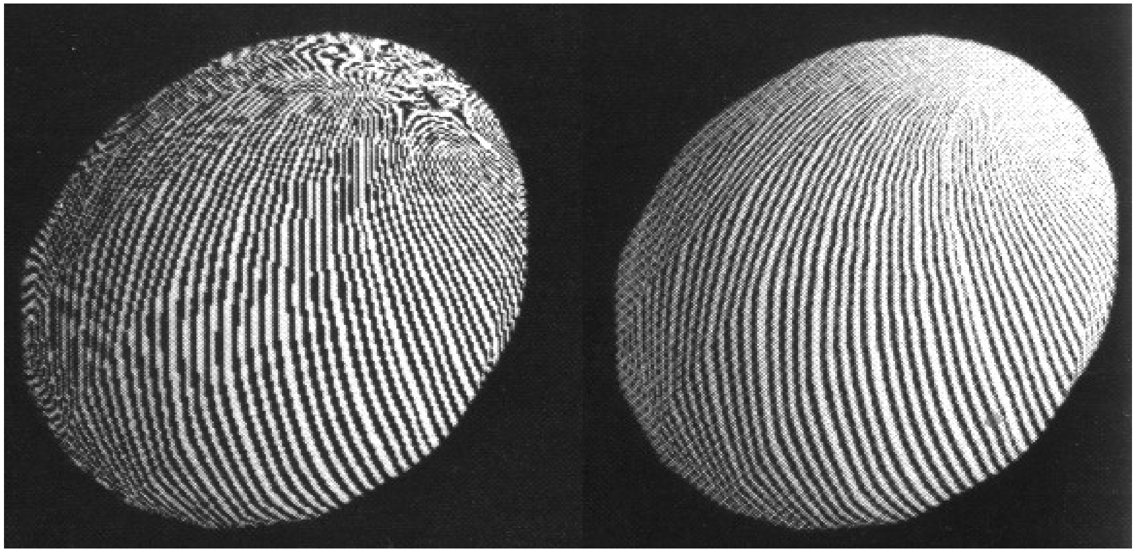
$$= (1-\alpha_1)(1-\alpha_2)v_{00} + \alpha_1(1-\alpha_2)v_{10} + (1-\alpha_1)\alpha_2 v_{01} + \alpha_1\alpha_2 v_{11}$$

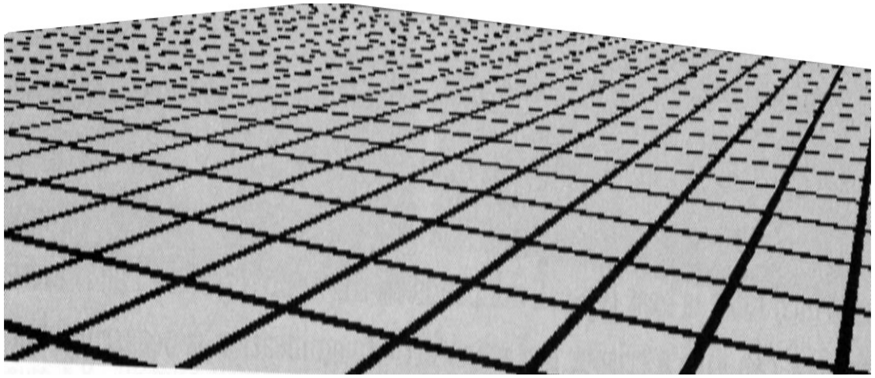
$$= v_{00} + \alpha_1(v_{10} - v_{00}) + \alpha_2(v_{01} - v_{00}) + \alpha_1\alpha_2(v_{00} + v_{11} - v_{10} - v_{01})$$

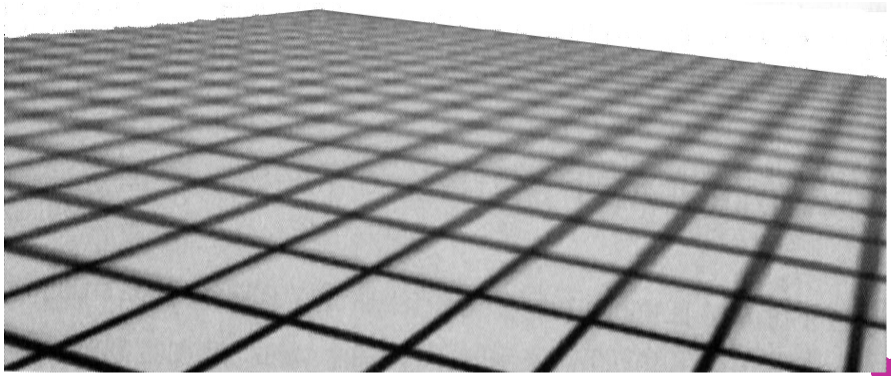
Texture Minification

Texture Aliasing: Minification

- Problem: One pixel in image space covers many texels

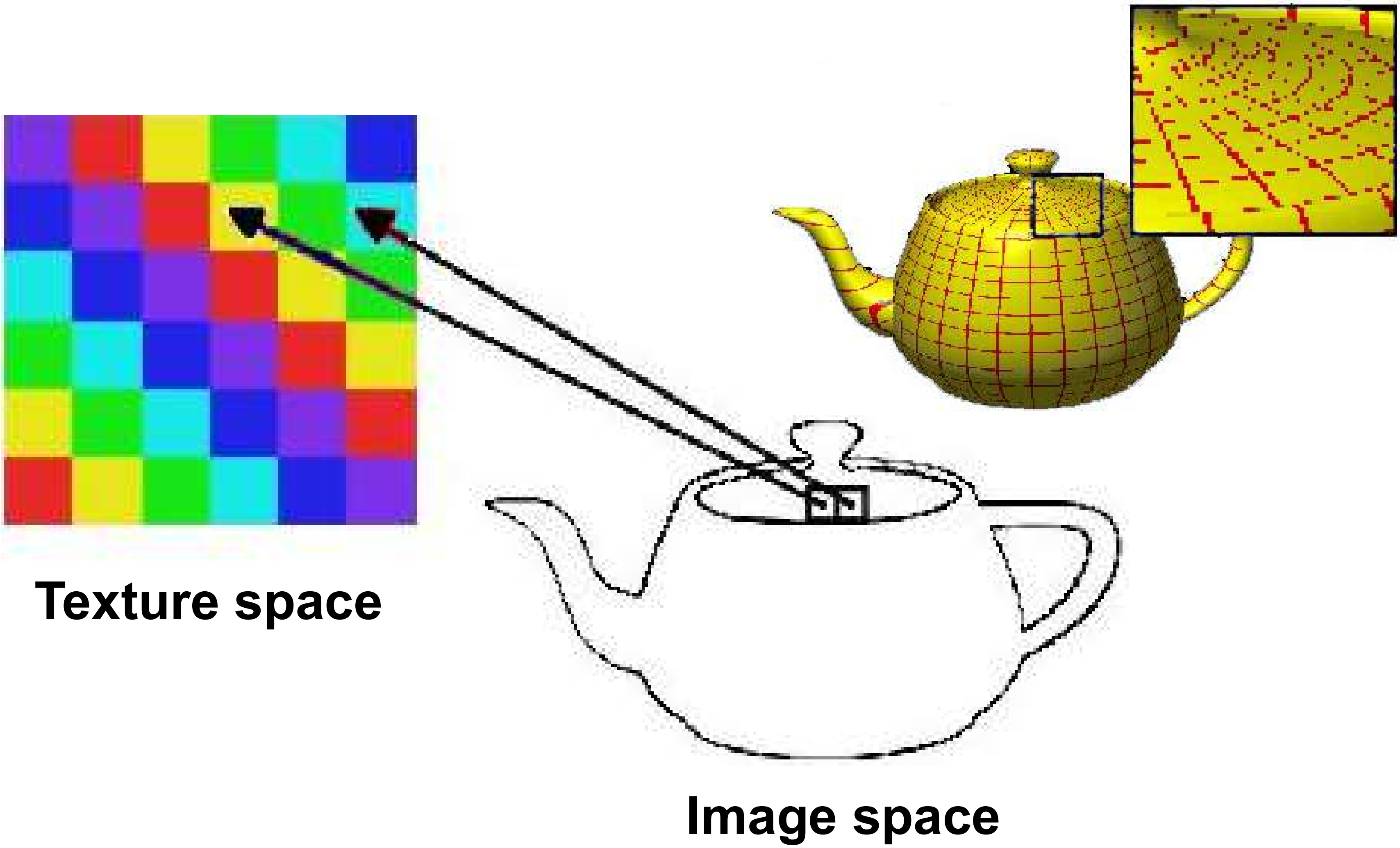






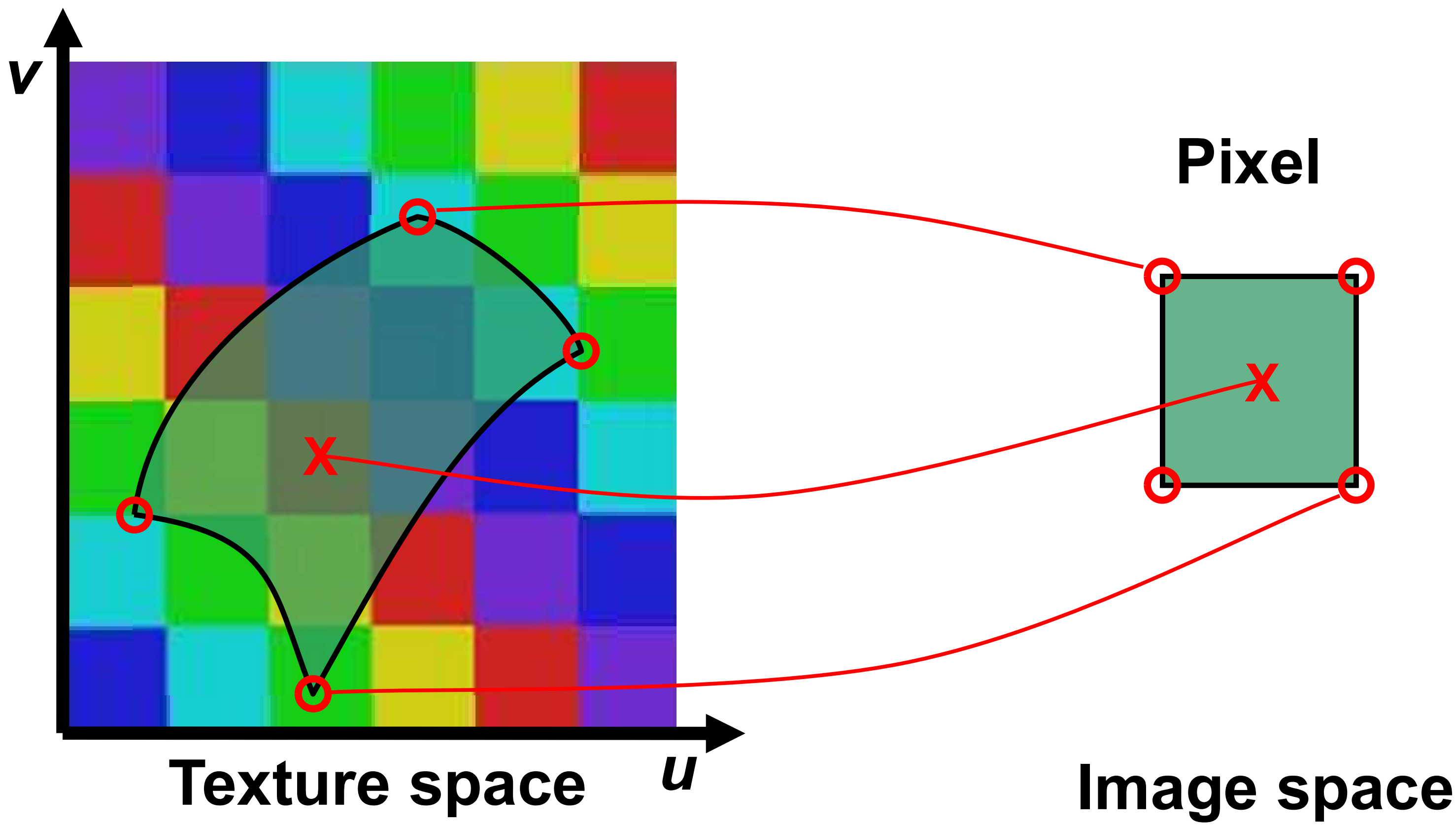
Texture Aliasing: Minification

- Caused by *undersampling*: texture information is lost



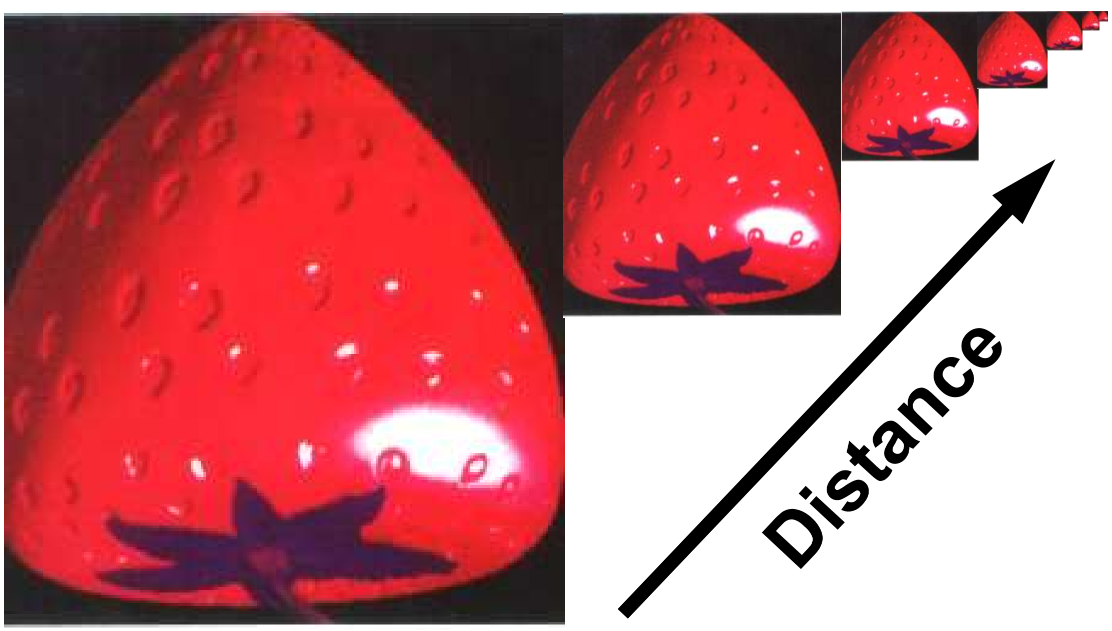
Texture Anti-Aliasing: Minification

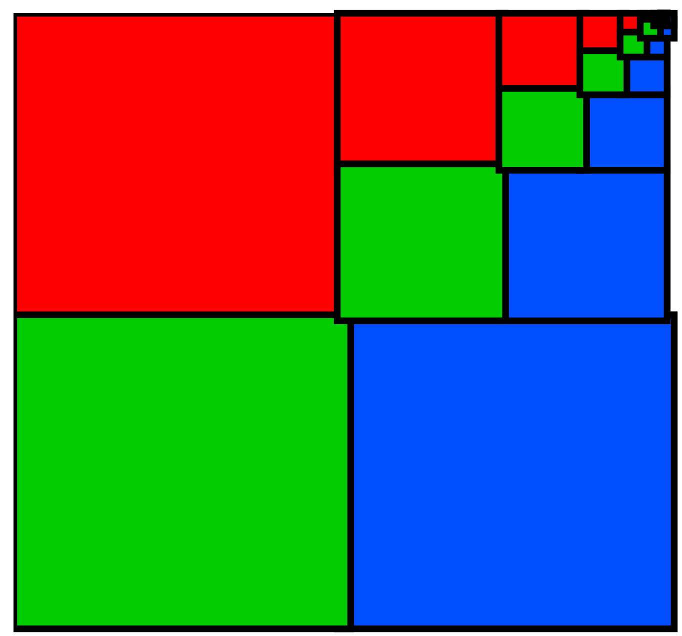
- A good pixel value is the weighted mean of the pixel area projected into texture space



Texture Anti-Aliasing: MIP Mapping

- MIP Mapping ("Multum In Parvo")
  - Texture size is reduced by factors of 2 (*downsampling* = "many things in a small place")
  - Simple (4 pixel average) and memory efficient
  - Last image is only ONE texel





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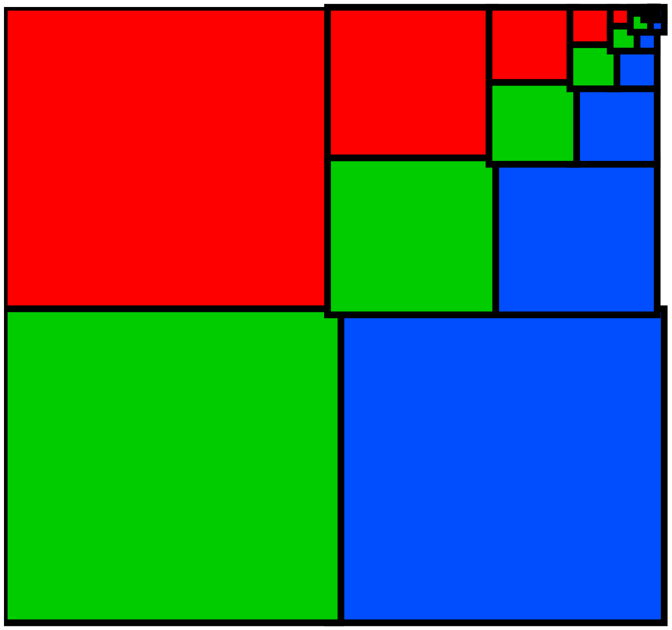
geometric series:

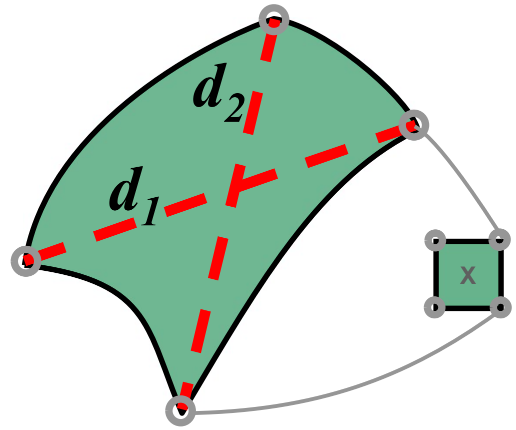
$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} =$$

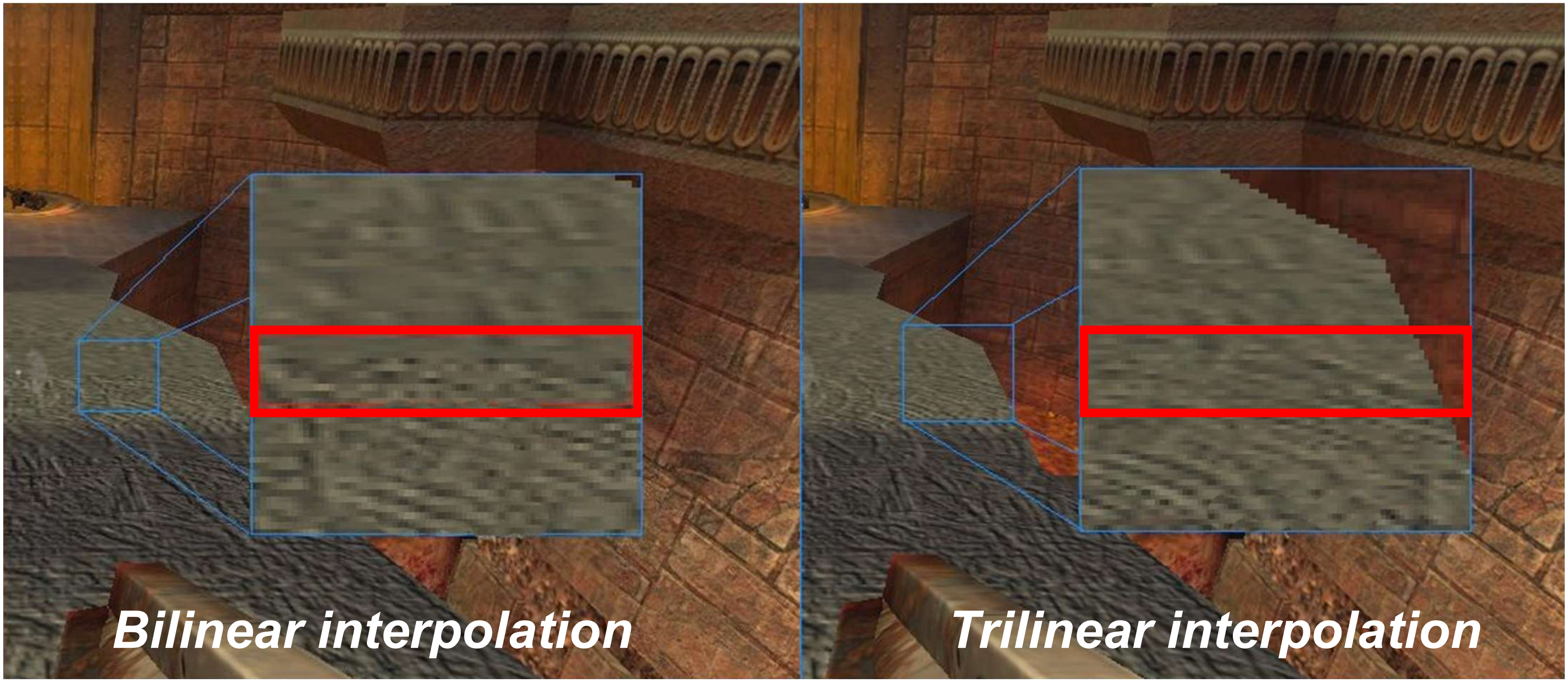
$$= \sum_{k=0}^{n-1} ar^k = a\left(\frac{1-r^n}{1-r}\right)$$



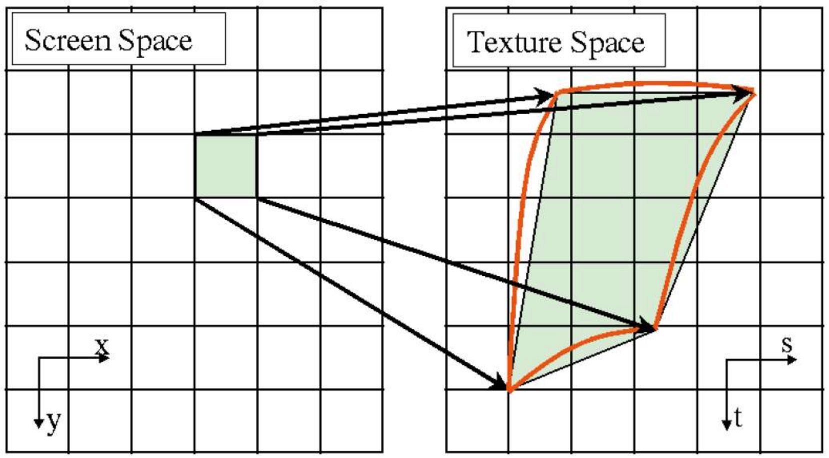
Texture Anti-Aliasing: MIP Mapping

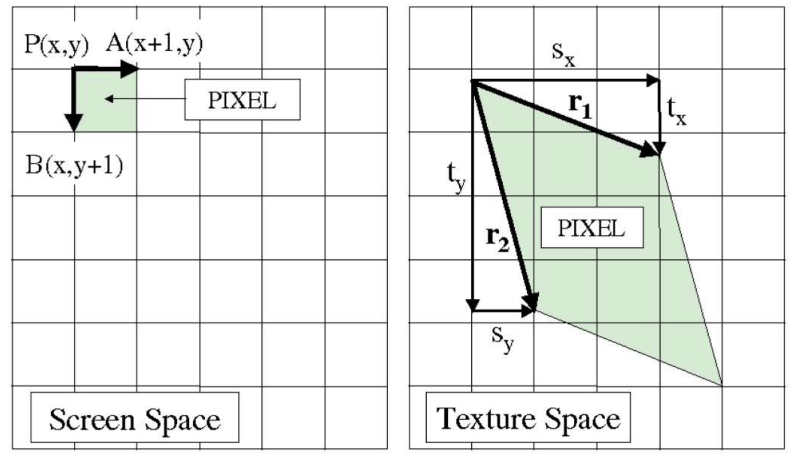
- MIP Mapping Algorithm
- $D := ld(max(d_1,d_2))$
- T_0 := value from texture $D_0 = trunc\ (D)$ ("Mip Map level")
  - Use *bilinear interpolation*





MIP-Map Level Computation





- Use the partial derivatives of texture coordinates with respect to screen space coordinates
- This is the Jacobian matrix

$$\begin{pmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{pmatrix} = \begin{pmatrix} s_x & s_y \\ t_x & t_y \end{pmatrix}$$

- Area of parallelogram is the absolute value of the Jacobian determinant (the *Jacobian*)

MIP-Map Level Computation (OpenGL)

- OpenGL 4.6 core specification, pp. 251-264

(3D tex coords!)

$$\lambda_{base}(x,y) = \log_2[\rho(x,y)]$$

$$\rho = \max\left\{\sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2}, \sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2}\right\}$$

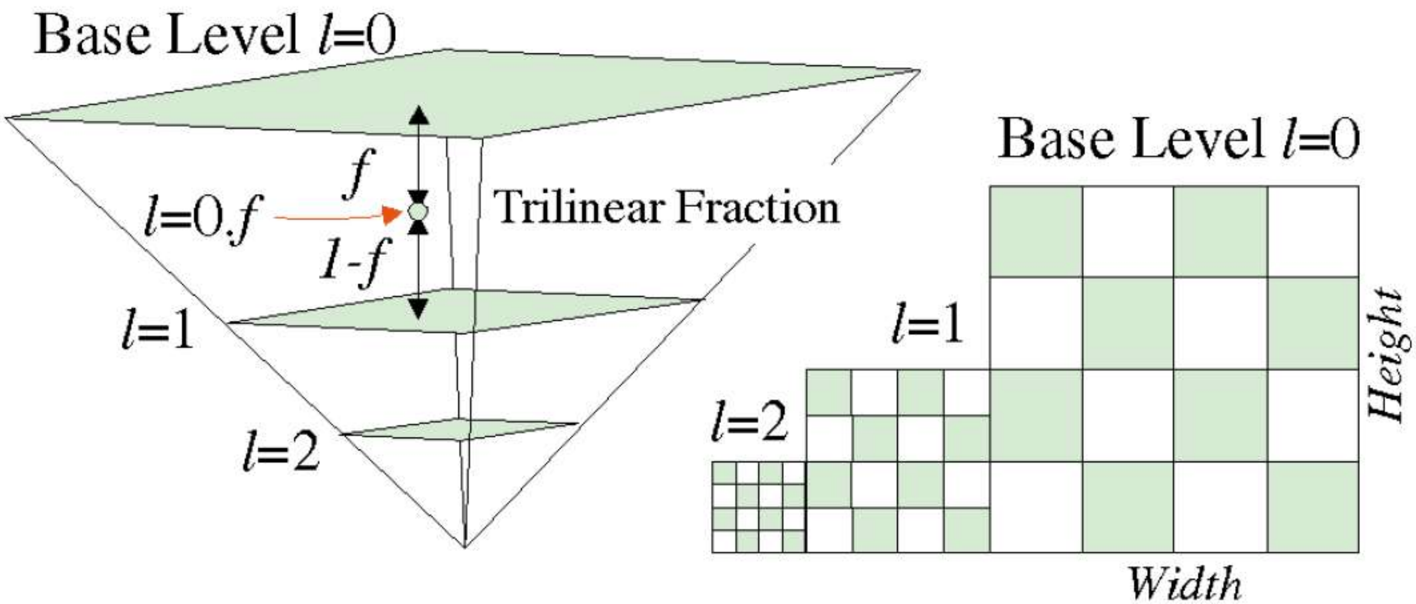
Does not use area of parallelogram but greater hypotenuse [Heckbert, 1983]

- Approximation without square-roots

$$m_u = \max\left\{\left|\frac{\partial u}{\partial x}\right|, \left|\frac{\partial u}{\partial y}\right|\right\} \quad m_v = \max\left\{\left|\frac{\partial v}{\partial x}\right|, \left|\frac{\partial v}{\partial y}\right|\right\} \quad m_w = \max\left\{\left|\frac{\partial w}{\partial x}\right|, \left|\frac{\partial w}{\partial y}\right|\right\}$$

$$\max\{m_u, m_v, m_w\} \le f(x,y) \le m_u + m_v + m_w$$

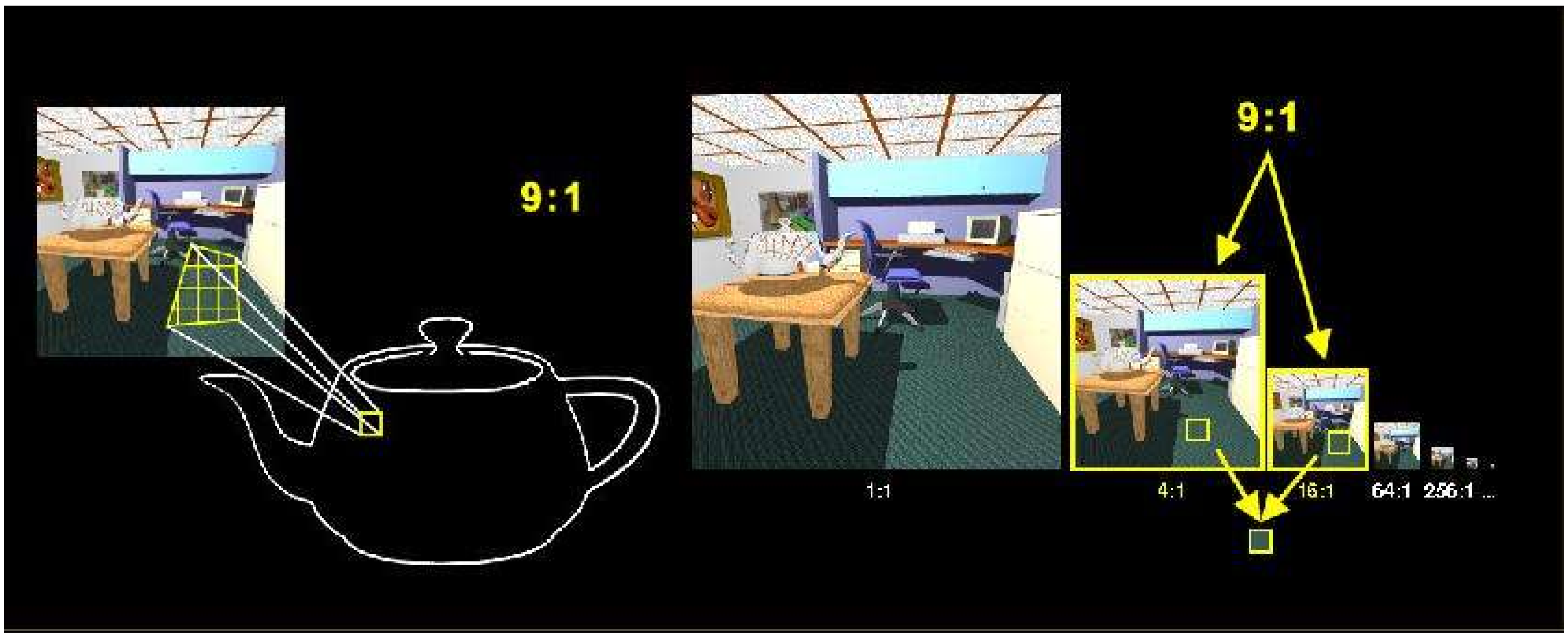
MIP-Map Level Interpolation



- Level of detail value is fractional!
- Use fractional part to blend (lin.) between two adjacent mipmap levels

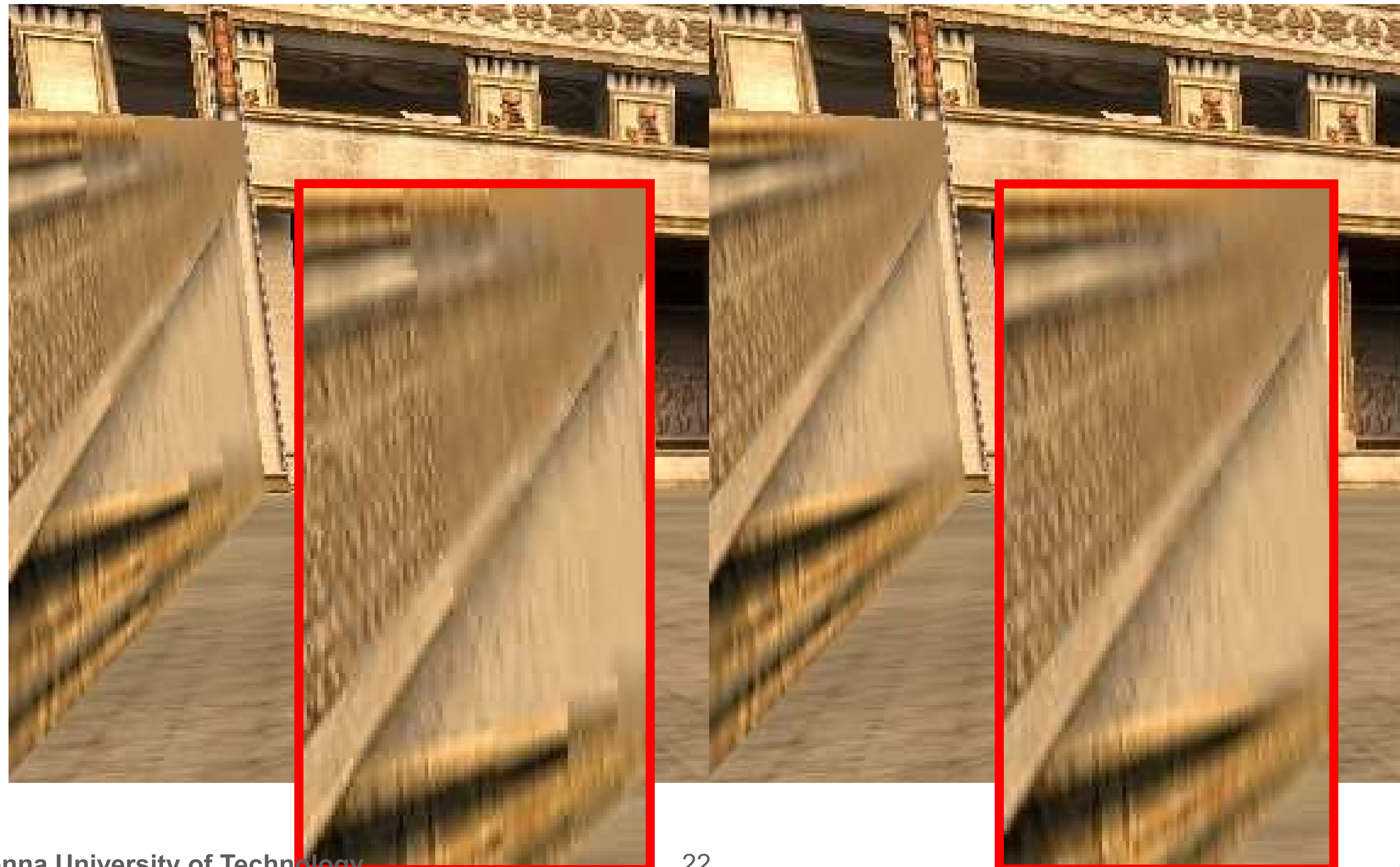
Texture Anti-Aliasing: MIP Mapping

- Trilinear interpolation:
  - T_1 := value from texture $D_1 = D_0+1$ (bilin.interpolation)
  - Pixel value := $(D_1 – D) \cdot T_0 + (D – D_0) \cdot T_1$
    - Linear interpolation between successive MIP Maps
  - Avoids "Mip banding" (but doubles texture lookups)



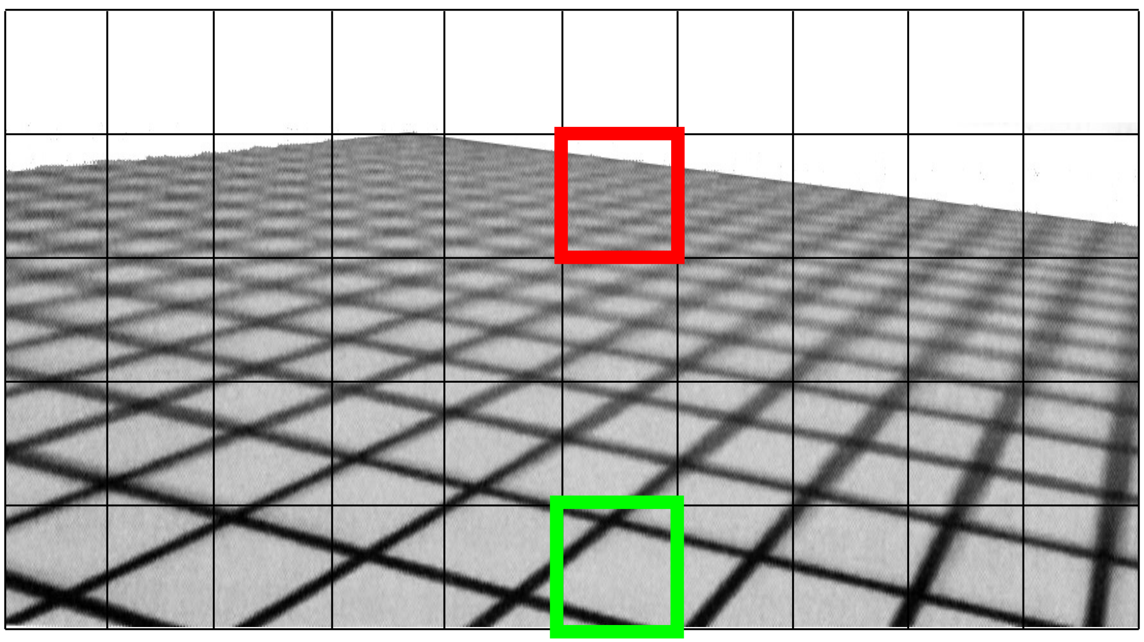
Texture Anti-Aliasing: MIP Mapping

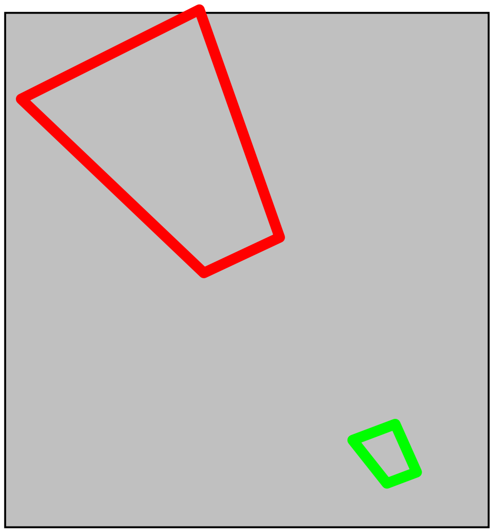
- Other example for bilinear vs. trilinear filtering



Anti-Aliasing: Anisotropic Filtering

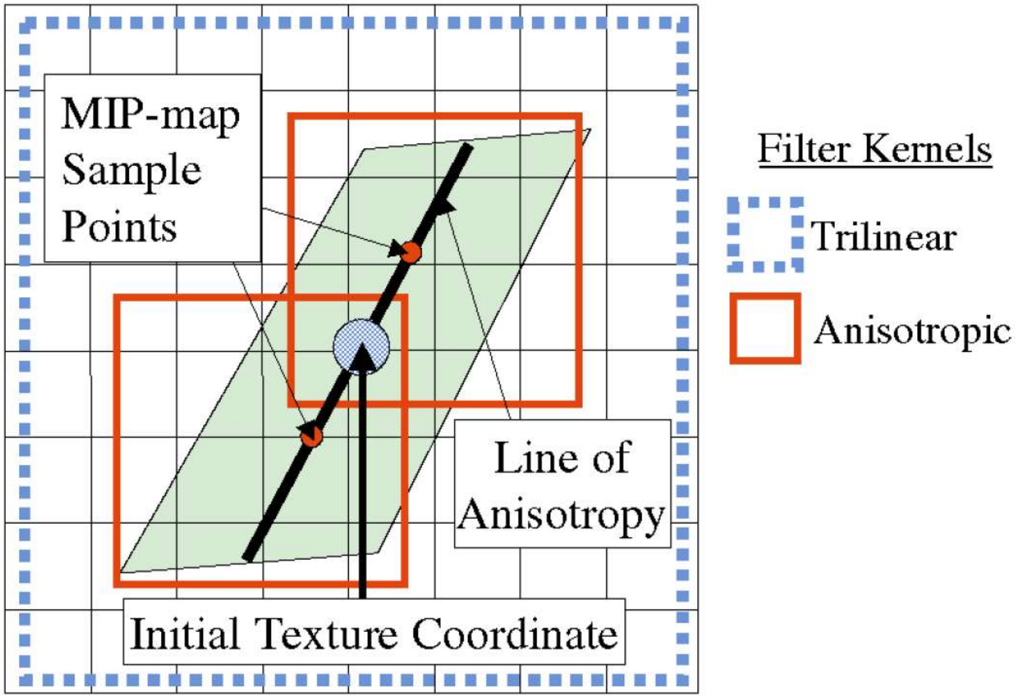
- Anisotropic filtering
  - View-dependent filter kernel
  - Implementation: *summed area table*, "*RIP Mapping*", *footprint assembly*, *elliptical weighted average* (EWA)





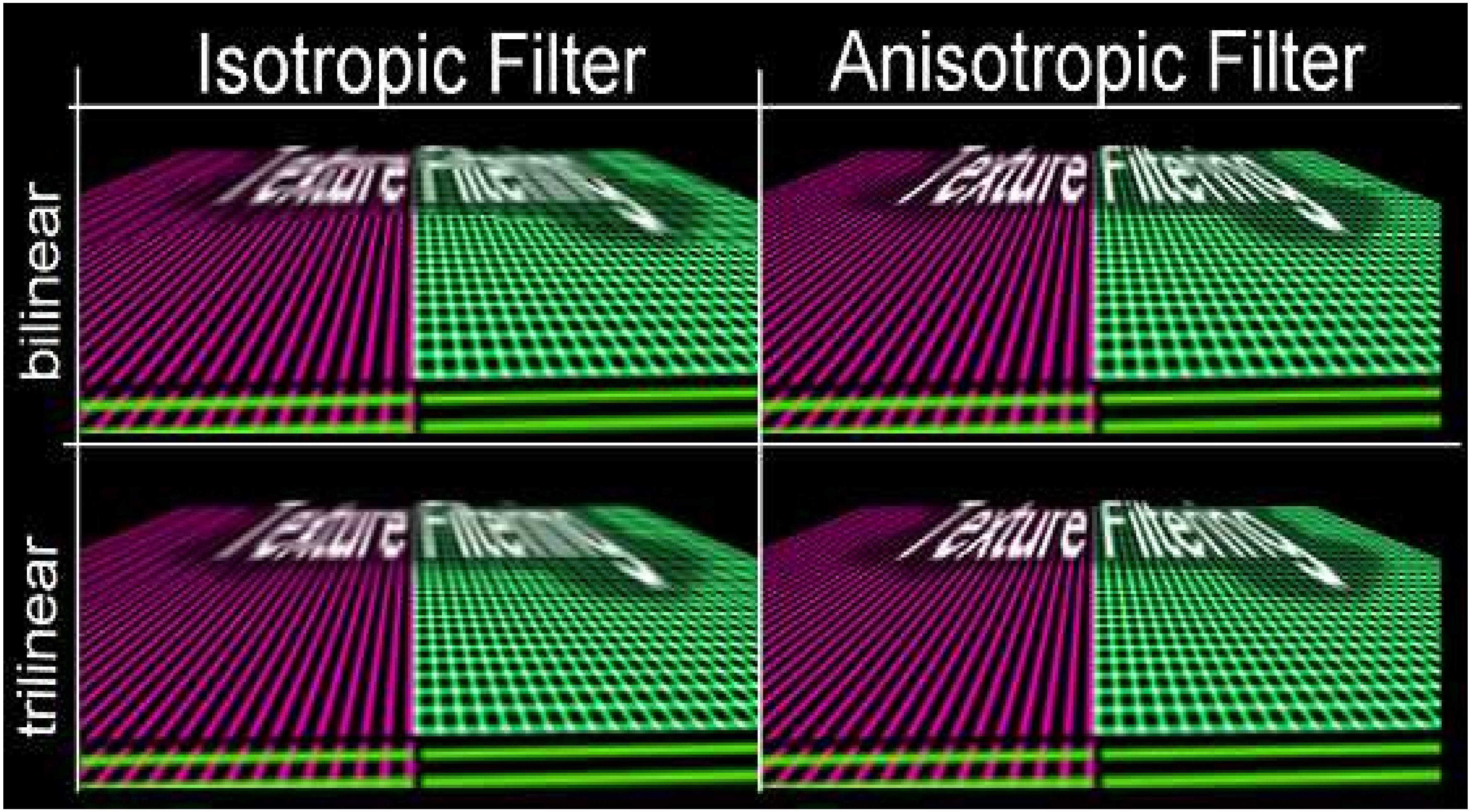
Texture space

Anisotropic Filtering: Footprint Assembly



Anti-Aliasing: Anisotropic Filtering

- Example



Texture Anti-aliasing

- Basically, everything done in hardware
- `gluBuild2DMipmaps()` generates MIPmaps
- Set parameters in `glTexParameter()`
  - `GL_TEXTURE_MAG_FILTER: GL_NEAREST, GL_LINEAR, …`
  - `GL_TEXTURE_MIN_FILTER: GL_LINEAR_MIPMAP_NEAREST`
- Anisotropic filtering is an extension:
  - `GL_EXT_texture_filter_anisotropic`
  - Number of samples can be varied (4x,8x,16x)
    - Vendor specific support and extensions

Thank you.