CS 380 - GPU and GPGPU Programming
Lecture 17: GPU Texturing 4

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Reading Assignment #10 (until Nov 9)

Read (required):

- **Brook for GPUs: Stream Computing on Graphics Hardware**
  Ian Buck et al., SIGGRAPH 2004

`http://graphics.stanford.edu/papers/brookgpu/`

Read (optional):

- **The Imagine Stream Processor**
  Ujval Kapasi et al.; IEEE ICCD 2002

`http://cva.stanford.edu/publications/2002/imagine-overview-iccd/`

- **Merrimac: Supercomputing with Streams**
  Bill Dally et al.; SC 2003

`https://dl.acm.org/citation.cfm?doid=1048935.1050187`

Interpolation Type + Purpose #1:

Interpolation of Texture Coordinates

(Linear / Rational-Linear Interpolation)

Texture Mapping

2D (3D) Texture Space

↓ Texture Transformation

2D Object Parameters

↓ Parameterization

3D Object Space

↓ Model Transformation

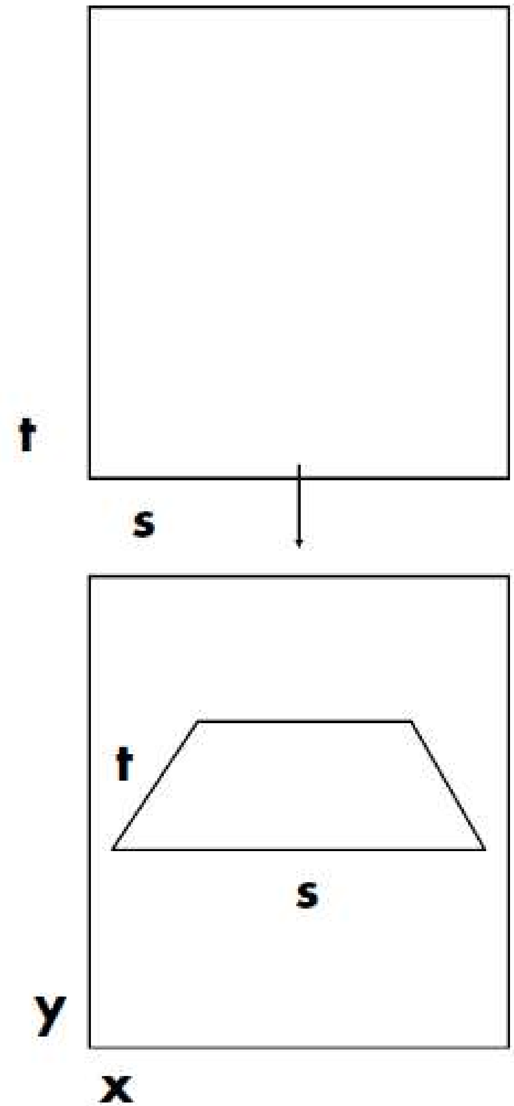
3D World Space

↓ Viewing Transformation

3D Camera Space

↓ Projection

2D Image Space



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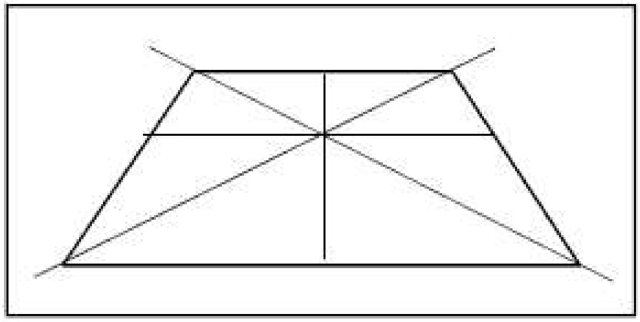
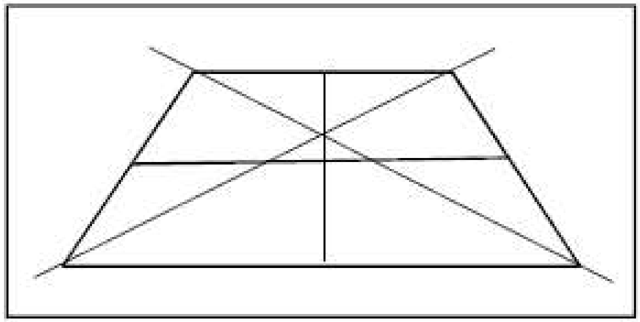
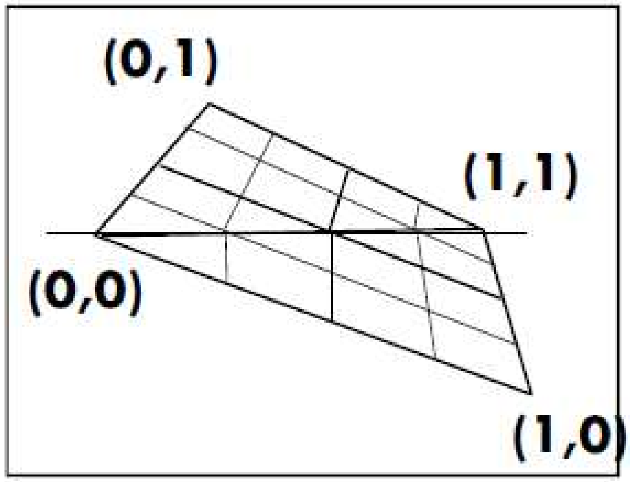
Linear Perspective



Correct Linear Perspective



Incorrect Perspective



Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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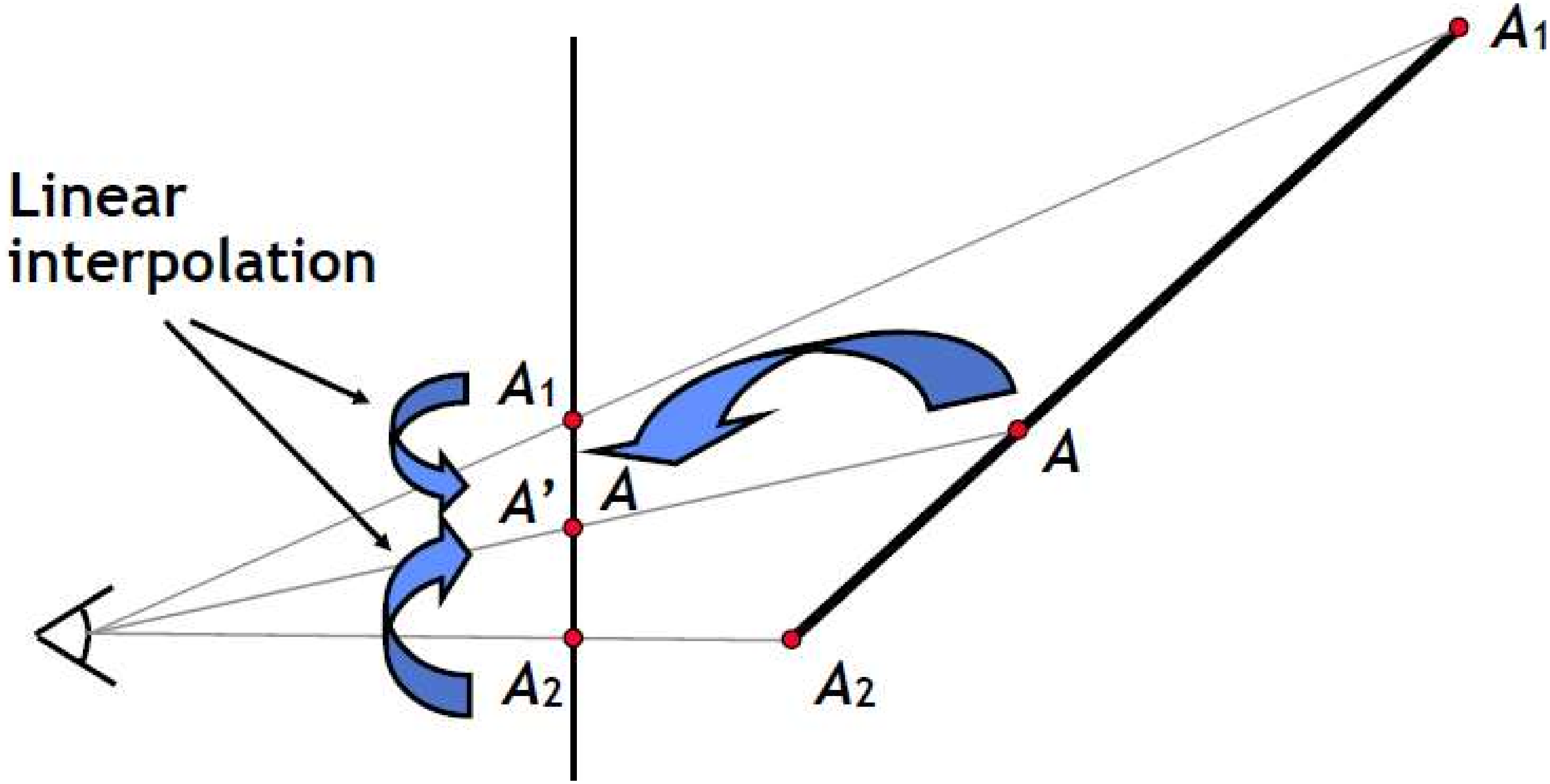
Incorrect attribute interpolation



$A' \neq A$!

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Linear interpolation

Compute intermediate attribute value

- Along a line: $A = aA_1 + bA_2$, $a+b=1$
- On a plane: $A = aA_1 + bA_2 + cA_3$, $a+b+c=1$

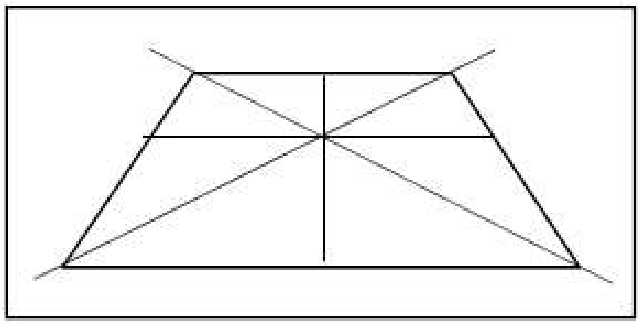
Only projected values interpolate linearly in screen space (straight lines project to straight lines)

- x and y are projected (divided by w)
- Attribute values are not naturally projected

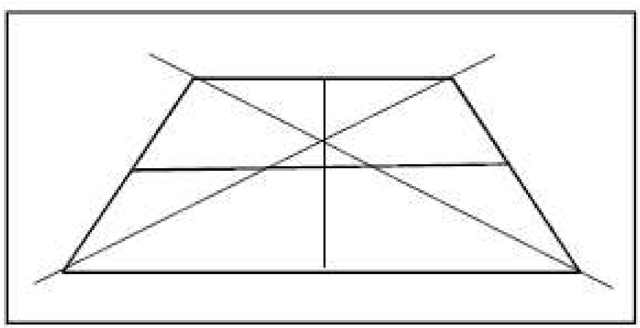
Choice for attribute interpolation in screen space

- Interpolate unprojected values
  - Cheap and easy to do, but gives wrong values
  - Sometimes OK for color, but
  - Never acceptable for texture coordinates
- Do it right

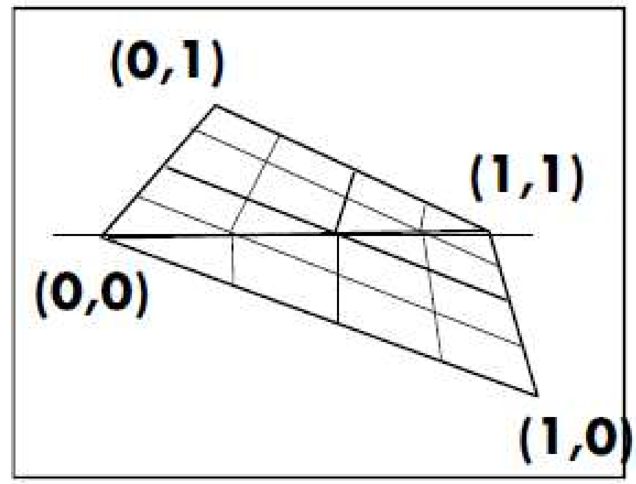
Linear Perspective



Correct Linear Perspective



Incorrect Perspective



Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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Perspective-correct linear interpolation

Only projected values interpolate correctly, so project *A*

- Linearly interpolate A_1/w_1 and A_2/w_2

Also interpolate $1/w_1$ and $1/w_2$

- These also interpolate linearly in screen space

Divide interpolants at each sample point to recover *A*

- $(A/w)\ /\ (1/w) = A$
- Division is expensive (more than add or multiply), so
  - Recover *w* for the sample point (reciprocate), and
  - Multiply each projected attribute by *w*

Barycentric triangle parameterization:

$$A = \frac{aA_1/w_1 + bA_2/w_2 + cA_3/w_3}{a/w_1 + b/w_2 + c/w_3} \qquad a + b + c = 1$$

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Perspective-Correct Interpolation Recipe

$$r_i(x,y) = \frac{r_i(x,y)/w(x,y)}{1/w(x,y)}$$

(1) Associate a record containing the n parameters of interest $(r_1, r_2, \cdots, r_n)$ with each vertex of the polygon.

(2) For each vertex, transform object space coordinates to homogeneous screen space using 4×4 object to screen matrix, yielding the values (xw, yw, zw, w).

(3) Clip the polygon against plane equations for each of the six sides of the viewing frustum, linearly interpolating all the parameters when new vertices are created.

(4) At each vertex, divide the homogeneous screen coordinates, the parameters r_i, and the number 1 by w to construct the variable list $(x, y, z, s_1, s_2, \cdots, s_{n+1})$, where $s_i = r_i/w$ for $i \leq n$, $s_{n+1} = 1/w$.

(5) Scan convert in screen space by linear interpolation of all parameters, at each pixel computing $r_i = s_i/s_{n+1}$ for each of the n parameters; use these values for shading.

Heckbert and Moreton

Interpolation Type + Purpose #2:

Interpolation of Samples in Texture Space

(Multi-Linear Interpolation)

Types of Textures

- Spatial layout
  - Cartesian grids: 1D, 2D, 3D, 2D_ARRAY, …
  - Cube maps, …
- Formats (too many), e.g. OpenGL
  - GL_LUMINANCE16_ALPHA16
  - GL_RGB8, GL_RGBA8, …: integer texture formats
  - GL_RGB16F, GL_RGBA32F, …: float texture formats
  - compressed formats, high dynamic range formats, …
- External (CPU) format vs. internal (GPU) format
  - OpenGL driver converts from external to internal

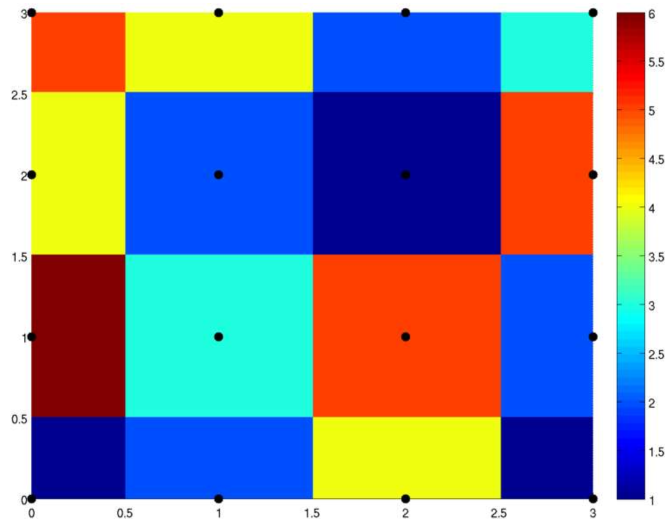
Magnification (Bi-linear Filtering Example)

Original image

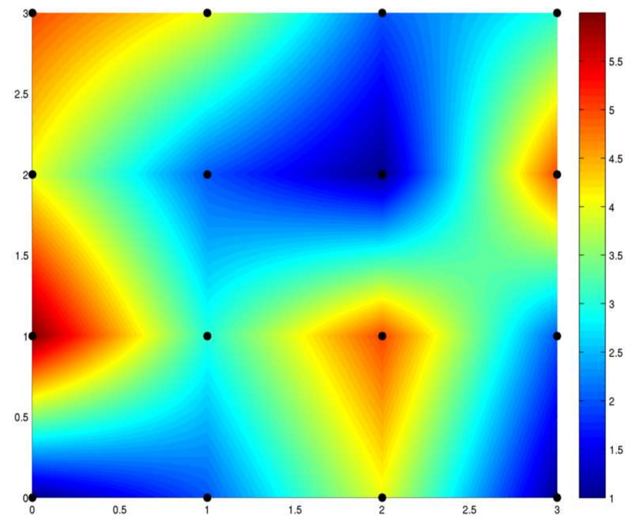
Nearest neighbor

Bi-linear filtering

Nearest-Neighbor vs. Bi-Linear Interpolation

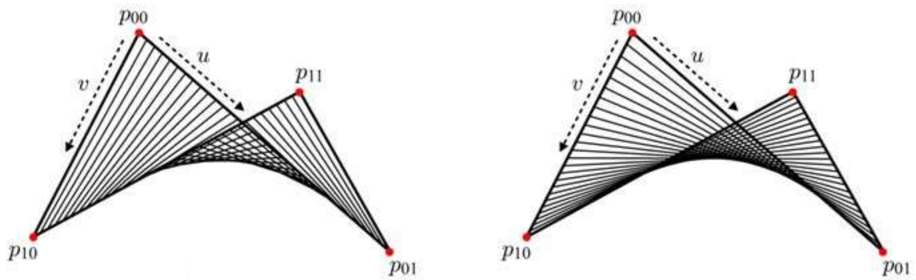


nearest-neighbor



bi-linear

wikipedia



Bilinear patch (courtesy J. Han)

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples (e.g., pixel centers):

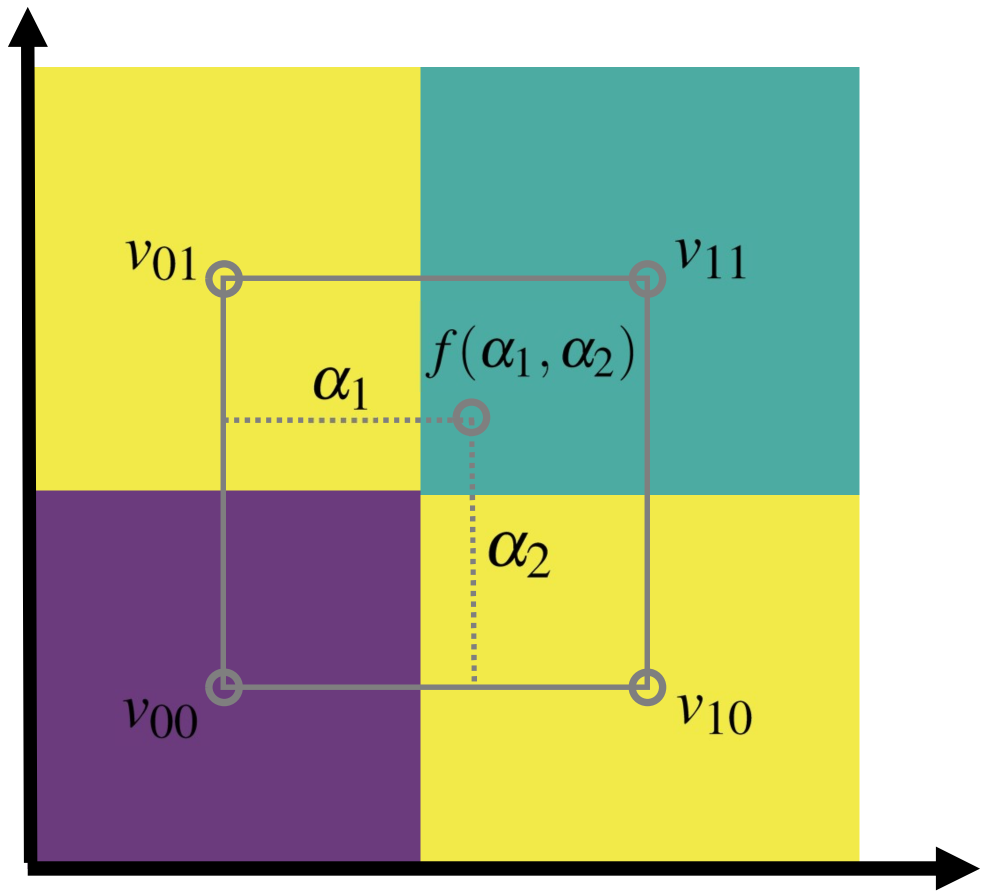
Given any (fractional) position

$$\alpha_1 := x_1 - \lfloor x_1 \rfloor \qquad \alpha_1 \in [0.0, 1.0)$$
$$\alpha_2 := x_2 - \lfloor x_2 \rfloor \qquad \alpha_2 \in [0.0, 1.0)$$

and 2x2 sample values

$$\begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix}$$

Compute: $f(\alpha_1, \alpha_2)$



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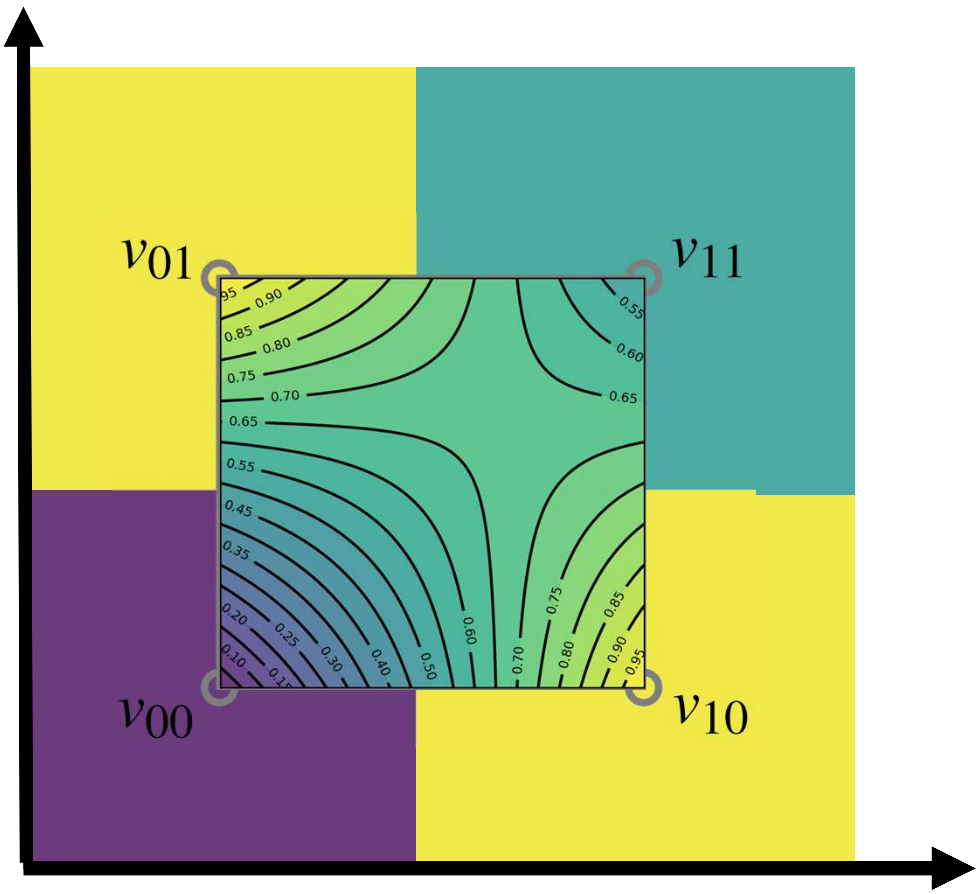

Bi-Linear Interpolation

Weights in 2x2 format:

$$\begin{bmatrix} \alpha_2 \\ (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1) & \alpha_1 \end{bmatrix} = \begin{bmatrix} (1-\alpha_1)\alpha_2 & \alpha_1\alpha_2 \\ (1-\alpha_1)(1-\alpha_2) & \alpha_1(1-\alpha_2) \end{bmatrix}$$

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} (1-\alpha_1)v_{01} + \alpha_1 v_{11} \\ (1-\alpha_1)v_{00} + \alpha_1 v_{10} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_2 v_{01} + (1-\alpha_2)v_{00} & \alpha_2 v_{11} + (1-\alpha_2)v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

Bi-Linear Interpolation

Interpolate function at (fractional) position (α_1, α_2) :

$$f(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_2 & (1-\alpha_2) \end{bmatrix} \begin{bmatrix} v_{01} & v_{11} \\ v_{00} & v_{10} \end{bmatrix} \begin{bmatrix} (1-\alpha_1) \\ \alpha_1 \end{bmatrix}$$

$$= (1-\alpha_1)(1-\alpha_2)v_{00} + \alpha_1(1-\alpha_2)v_{10} + (1-\alpha_1)\alpha_2 v_{01} + \alpha_1\alpha_2 v_{11}$$

$$= v_{00} + \alpha_1(v_{10} - v_{00}) + \alpha_2(v_{01} - v_{00}) + \alpha_1\alpha_2(v_{00} + v_{11} - v_{10} - v_{01})$$

REALLY IMPORTANT:

this is a different thing (for a different purpose) than the linear (or, in perspective, rational-linear) interpolation of texture coordinates!!

Thank you.