King Abdullah University of Science and Technology

KAUST

CS 380 - GPU and GPGPU Programming
Lecture 16: GPU Texturing 3

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Reading Assignment #9 (until Nov 2)

Read (required):

- MIP-Map Level Selection for Texture Mapping

  `https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=765326`

- Homogeneous Coordinates

  `https://en.wikipedia.org/wiki/Homogeneous_coordinates`

- *Don‘t forget:*

  Interpolation for Polygon Texture Mapping and Shading,
  Paul Heckbert and Henry Moreton

  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.48.7886`

Read (optional):

- Vulkan Tutorial

  `https://vulkan-tutorial.com`

Interpolation Type + Purpose #1:

Interpolation of Texture Coordinates

(Linear / Rational-Linear Interpolation)

Homogeneous Coordinates (1)

Projective geometry

- (Real) projective spaces RP^n:

  Real projective line RP^1, real projective plane RP^2, ...

- A point in RP^n is a line through the origin (i.e., all the scalar multiples of the same vector) in an (n+1)-dimensional (real) vector space

Homogeneous coordinates of 2D projective point in RP^2

- Coordinates differing only by a non-zero factor λ map to the same point

  $(\lambda x, \lambda y, \lambda)$ dividing out the λ gives $(x, y, 1)$, corresponding to (x,y) in R^2

- Coordinates with last component = 0 map to "points at infinity"

  $(\lambda x, \lambda y, 0)$ division by last component not allowed; but again this is the same point if it only differs by a scalar factor, e.g., this is the same point as $(x, y, 0)$

Homogeneous Coordinates (2)

Examples of usage

- Translation (with translation vector $\vec{b}$)
- Affine transformations (linear transformation + translation)

$$\vec{y} = A\vec{x} + \vec{b}.$$

- With homogeneous coordinates:

$$\begin{bmatrix} \vec{y} \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & A & & \vec{b} \\ 0 & \ldots & 0 & 1 \end{array} \right] \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix}$$

- Setting the last coordinate = 1 and the last row of the matrix to [0, ..., 0, 1] results in translation of the point $\vec{x}$ (via addition of translation vector $\vec{b}$)
- The matrix above is a linear map, but because it is one dimension higher, it does not have to move the origin in the (n+1)-dimensional space for translation

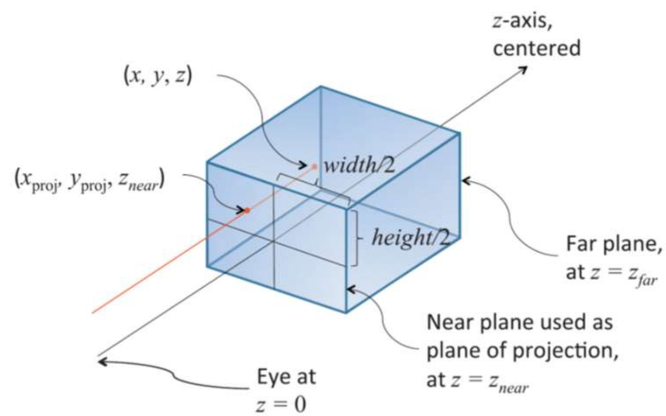
Homogeneous Coordinates (3)

Examples of usage

- Projection (e.g., OpenGL projection matrices)

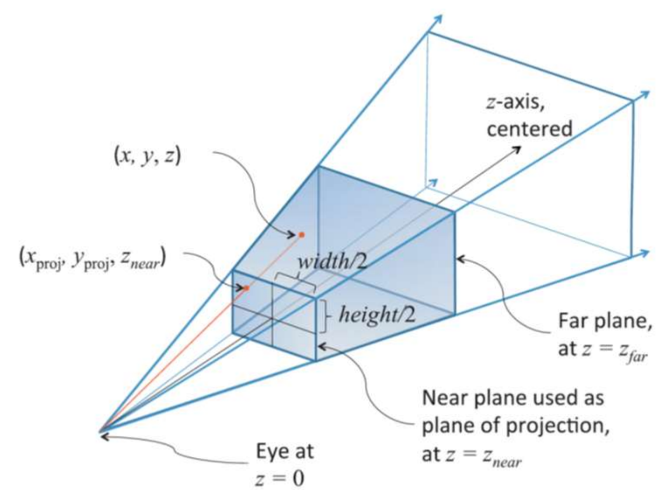
$$
\begin{bmatrix}
\frac{2}{\text{right}-\text{left}} & 0 & 0 & -\frac{\text{right}+\text{left}}{\text{right}-\text{left}} \\
0 & \frac{2}{\text{top}-\text{bottom}} & 0 & -\frac{\text{top}+\text{bottom}}{\text{top}-\text{bottom}} \\
0 & 0 & \frac{-2}{\text{far}-\text{near}} & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

orthographic



$$
\begin{bmatrix}
\frac{z_{near}}{width/2} & 0.0 & \frac{left+right}{width/2} & 0.0 \\
0.0 & \frac{z_{near}}{height/2} & \frac{top+bottom}{height/2} & 0.0 \\
0.0 & 0.0 & -\frac{z_{far}+z_{near}}{z_{far}-z_{near}} & \frac{2z_{far}z_{near}}{z_{far}-z_{near}} \\
0.0 & 0.0 & -1.0 & 0.0
\end{bmatrix}
$$

perspective



Texture Mapping

2D (3D) Texture Space

Texture Transformation

2D Object Parameters

Parameterization

3D Object Space

Model Transformation

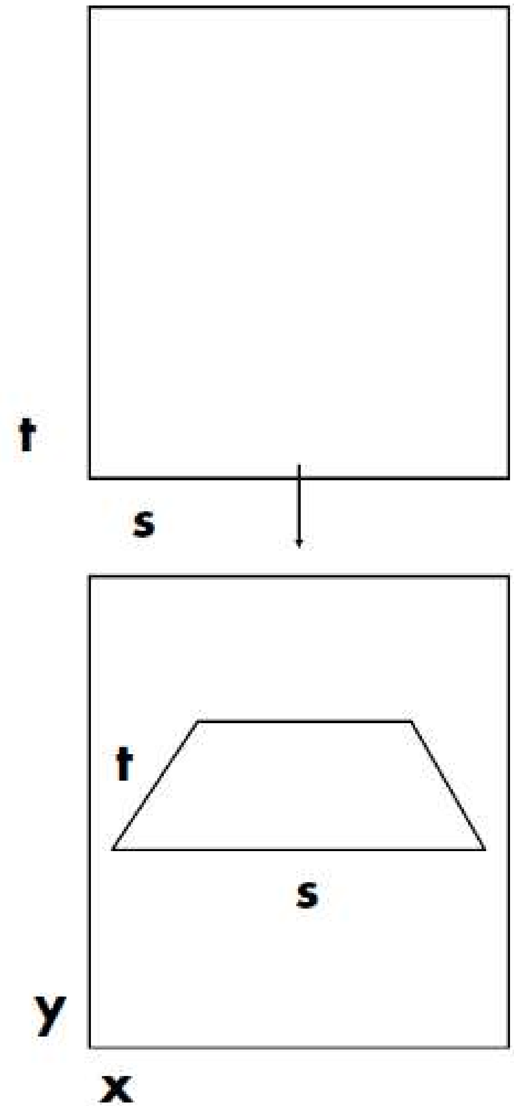
3D World Space

Viewing Transformation

3D Camera Space

Projection

2D Image Space



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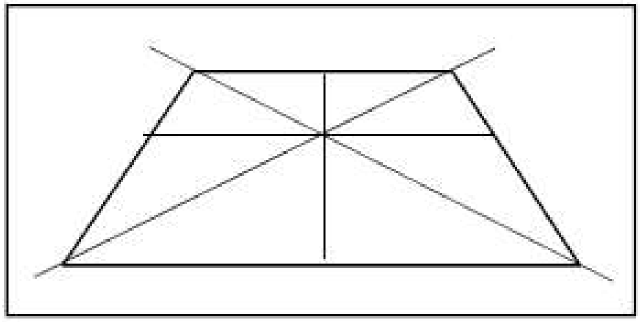
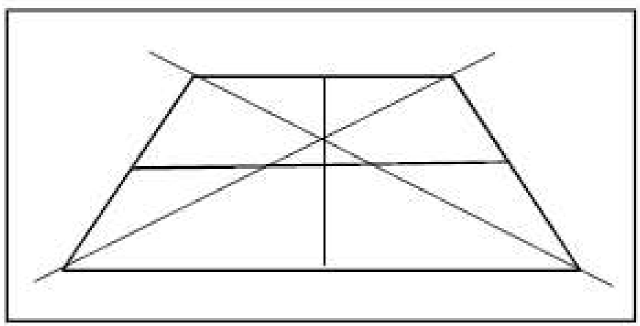
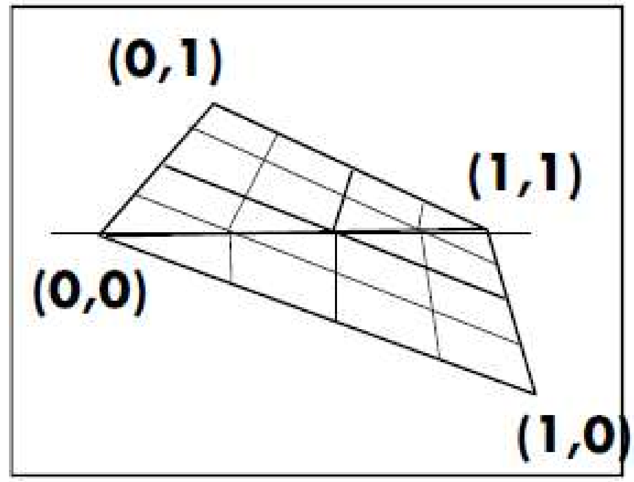
Linear Perspective



Correct Linear Perspective



Incorrect Perspective



Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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Texture Mapping Polygons

Forward transformation: linear projective map

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} s \\ t \\ r \end{bmatrix}$$

Backward transformation: linear projective map

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

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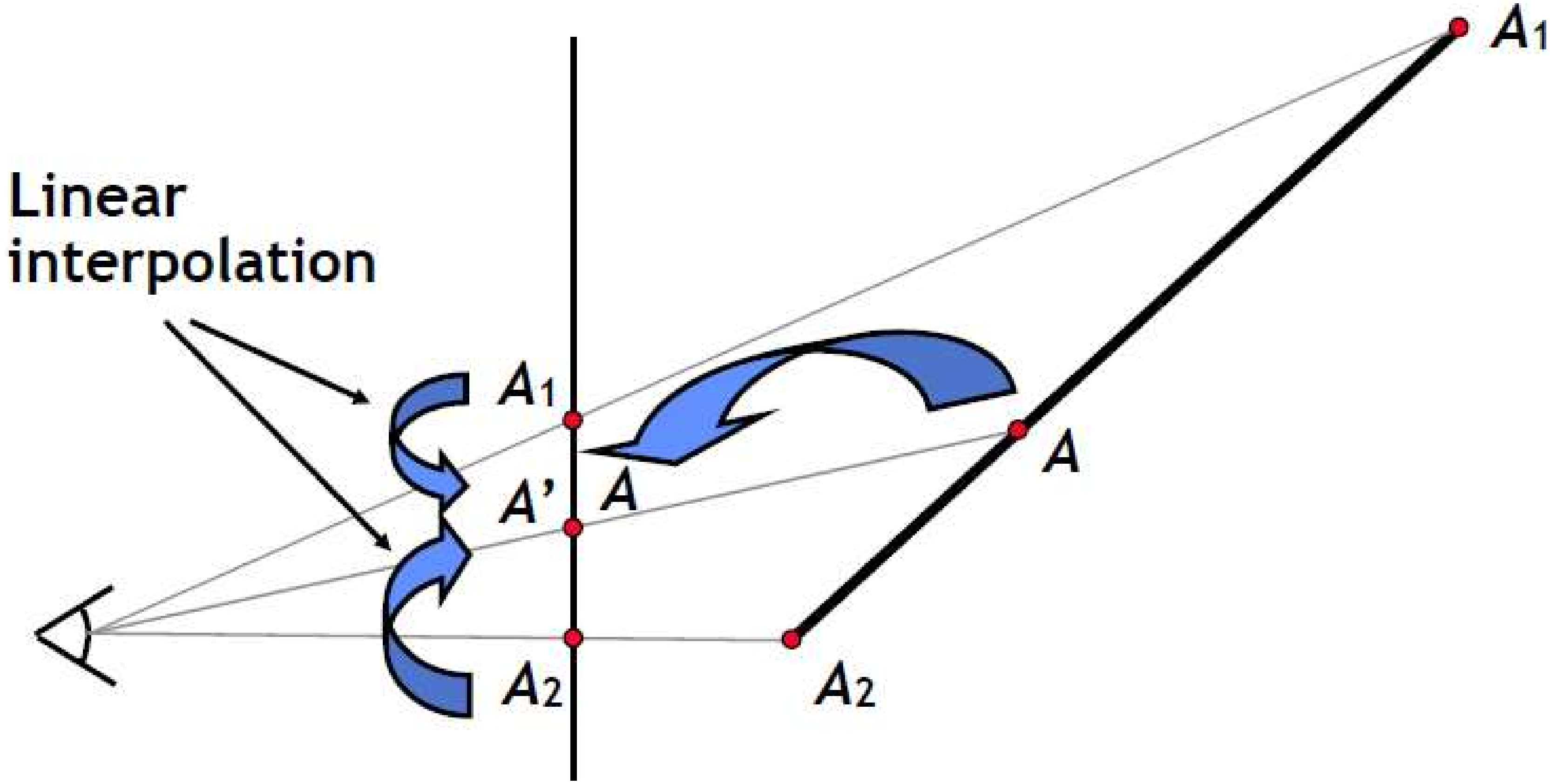
Incorrect attribute interpolation



$\boxed{A' \neq A\,!}$

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Linear interpolation

Compute intermediate attribute value

- Along a line: $A = aA_1 + bA_2, \quad a+b=1$
- On a plane: $A = aA_1 + bA_2 + cA_3, \quad a+b+c=1$

Only projected values interpolate linearly in screen space (straight lines project to straight lines)

- x and y are projected (divided by w)
- Attribute values are not naturally projected

Choice for attribute interpolation in screen space

- Interpolate unprojected values
  - Cheap and easy to do, but gives wrong values
  - Sometimes OK for color, but
  - Never acceptable for texture coordinates
- Do it right

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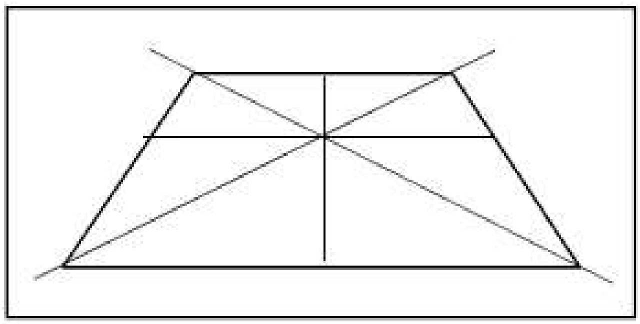
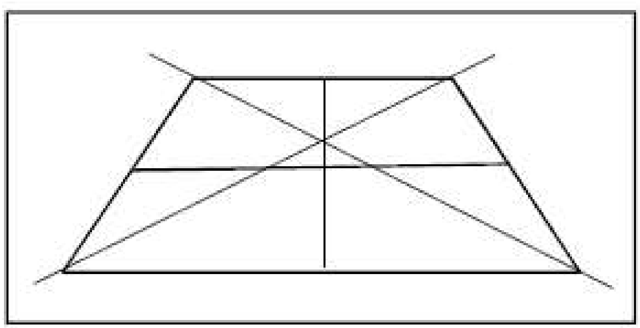
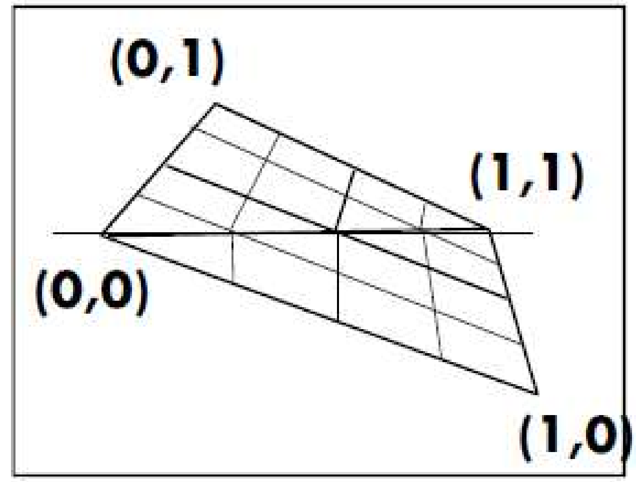
Linear Perspective



Correct Linear Perspective



Incorrect Perspective



Linear Interpolation, *Bad*

Perspective Interpolation, *Good*

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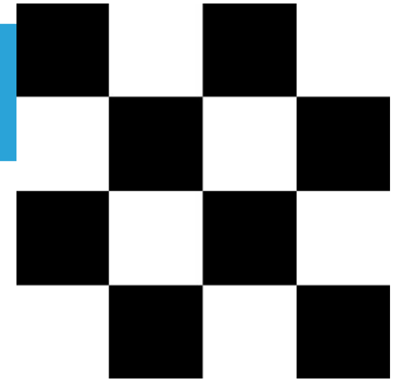
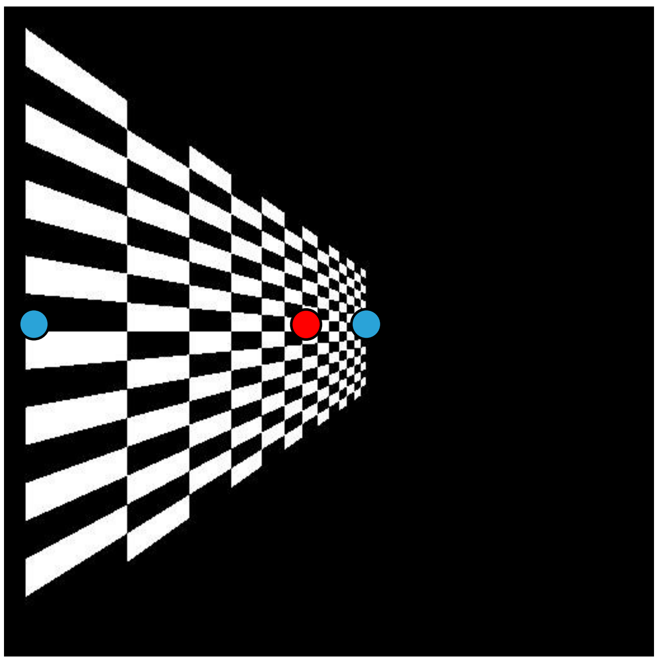
Perspective Texture Mapping

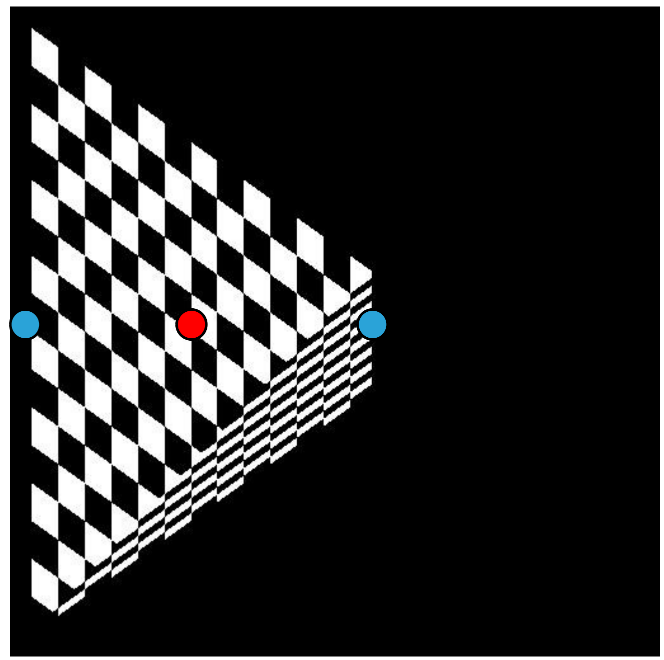


linear interpolation in object space

$$\frac{ax_1 + bx_2}{aw_1 + bw_2} \neq a\frac{x_1}{w_1} + b\frac{x_2}{w_2}$$

linear interpolation in screen space





$a = b = 0.5$

Early Perspective Texture Mapping in Games

Ultima Underworld (Looking Glass, 1992)

Early Perspective Texture Mapping in Games

DOOM (id Software, 1993)

Early Perspective Texture Mapping in Games

Quake (id Software, 1996)

Perspective-correct linear interpolation

Only projected values interpolate correctly, so project A

- Linearly interpolate A_1/w_1 and A_2/w_2

Also interpolate $1/w_1$ and $1/w_2$

- These also interpolate linearly in screen space

Divide interpolants at each sample point to recover A

- $(A/w) \,/\, (1/w) = A$
- Division is expensive (more than add or multiply), so
  - Recover w for the sample point (reciprocate), and
  - Multiply each projected attribute by w

Barycentric triangle parameterization:

$$A = \frac{aA_1/w_1 + bA_2/w_2 + cA_3/w_3}{a/w_1 + b/w_2 + c/w_3} \qquad a + b + c = 1$$

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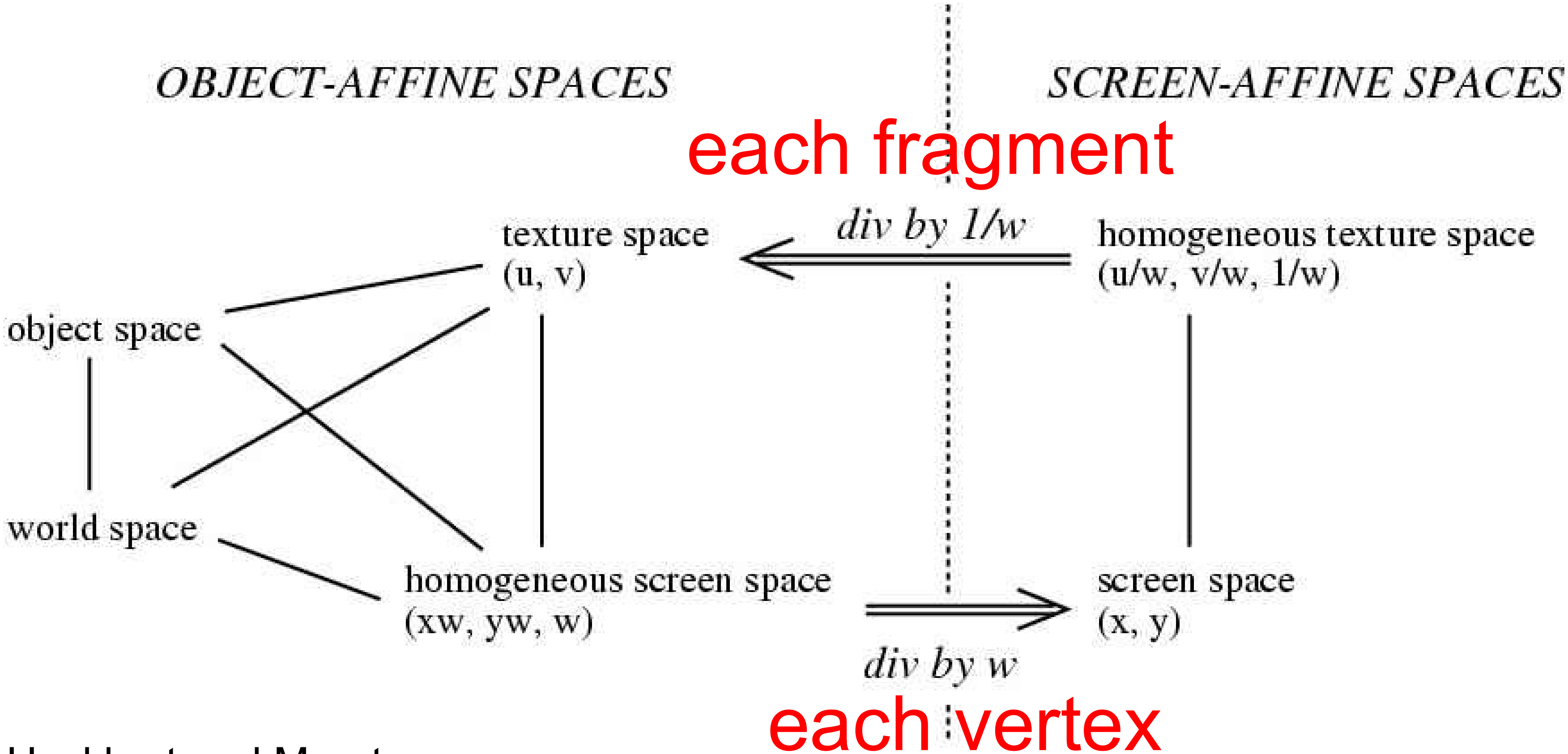
Perspective Texture Mapping

- Solution: interpolate (s/w, t/w, 1/w)
- (s/w) / (1/w) = s etc. at every fragment



Heckbert and Moreton

Perspective-Correct Interpolation Recipe

$$r_i(x, y) = \frac{r_i(x, y)/w(x, y)}{1/w(x, y)}$$

(1) Associate a record containing the n parameters of interest $(r_1, r_2, \cdots, r_n)$ with each vertex of the polygon.

(2) For each vertex, transform object space coordinates to homogeneous screen space using 4×4 object to screen matrix, yielding the values (xw, yw, zw, w).

(3) Clip the polygon against plane equations for each of the six sides of the viewing frustum, linearly interpolating all the parameters when new vertices are created.

(4) At each vertex, divide the homogeneous screen coordinates, the parameters r_i, and the number 1 by w to construct the variable list $(x, y, z, s_1, s_2, \cdots, s_{n+1})$, where $s_i = r_i/w$ for $i \leq n$, $s_{n+1} = 1/w$.

(5) Scan convert in screen space by linear interpolation of all parameters, at each pixel computing $r_i = s_i/s_{n+1}$ for each of the n parameters; use these values for shading.

Heckbert and Moreton

Thank you.