King Abdullah University of Science and Technology

KAUST

CS 247 – Scientific Visualization
Lecture 23: Vector Field / Flow Visualization, Pt.5

Markus Hadwiger, KAUST

Reading Assignment #14++

Read (required):

- Data Visualization book, Chapter 6.7
- J. van Wijk: *Image-Based Flow Visualization*,
  ACM SIGGRAPH 2002

  http://www.win.tue.nl/~vanwijk/ibfv/ibfv.pdf

Read (optional):

- T. Günther, A. Horvath, W. Bresky, J. Daniels, S. A. Buehler:
  Lagrangian Coherent Structures and Vortex Formation in High Spatiotemporal-Resolution Satellite Winds of an Atmospheric Karman Vortex Street, 2021

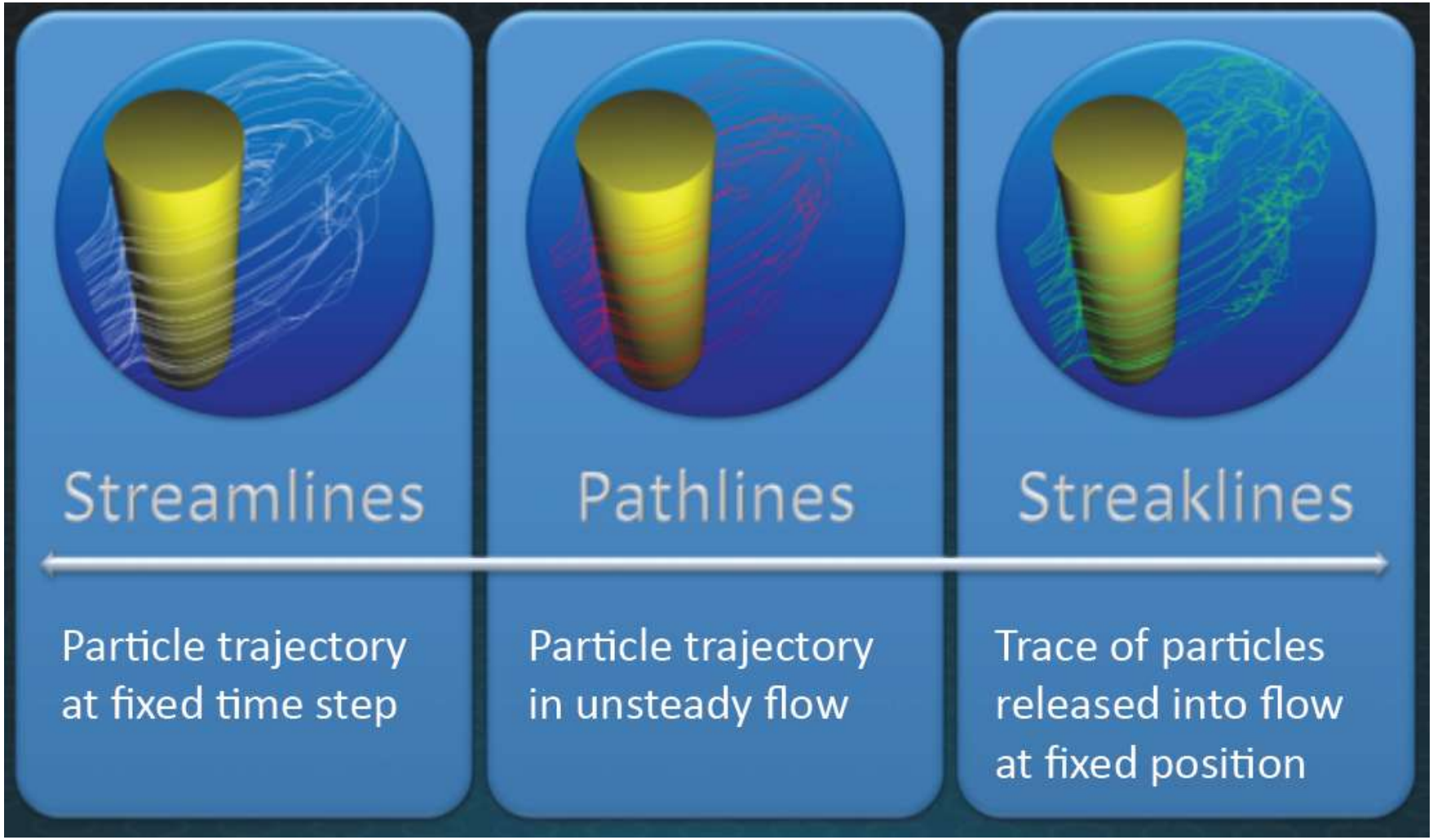
  https://www.essoar.org/doi/10.1002/essoar.10506682.2
- H. Bhatia, G. Norgard, V. Pascucci, P.-T. Bremer:
  The Helmholtz-Hodge Decomposition – A Survey, TVCG 19(8), 2013

  https://doi.org/10.1109/TVCG.2012.316
- Work through online tutorials of multi-variable partial derivatives, grad, div, curl, Laplacian:

  https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives
  https://www.youtube.com/watch?v=rB83DpBJQsE (3Blue1Brown)
- Matrix exponentials:

  https://www.youtube.com/watch?v=O85OWBJ2ayo (3Blue1Brown)

Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

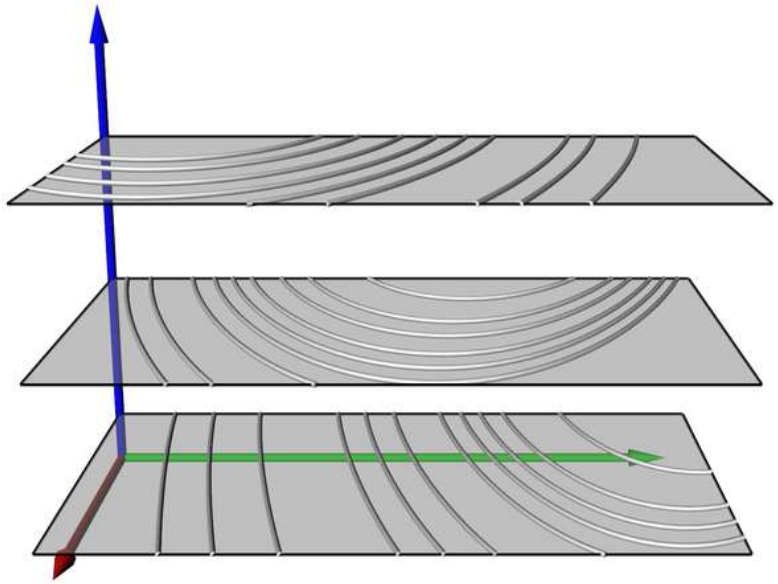
- Describes motion of a massless particle over time

Streakline

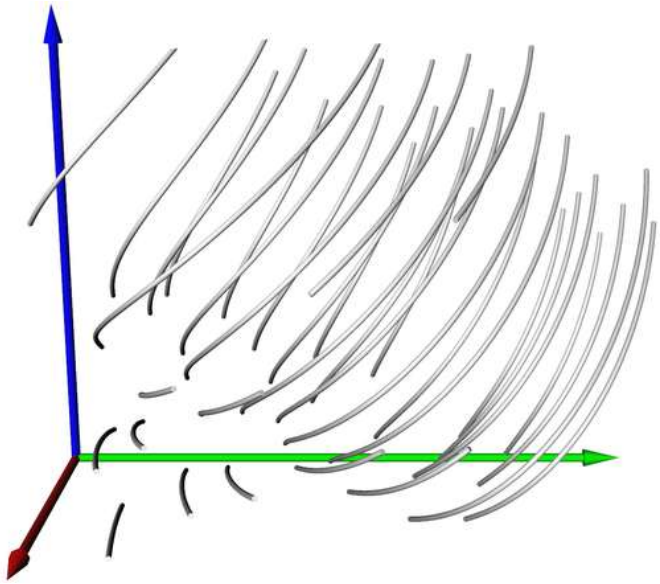
- Location of all particles released at a *fixed position* over time

Timeline

- Location of all particles released along a line at a *fixed time*

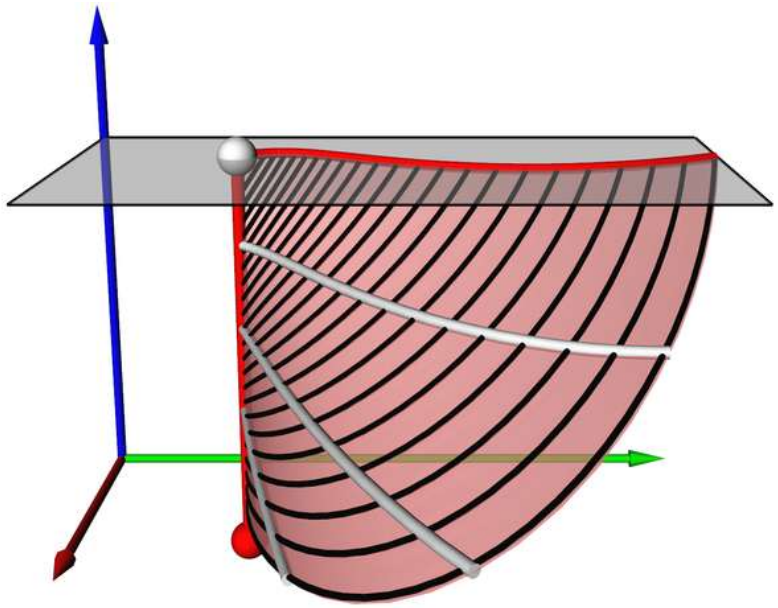


stream lines

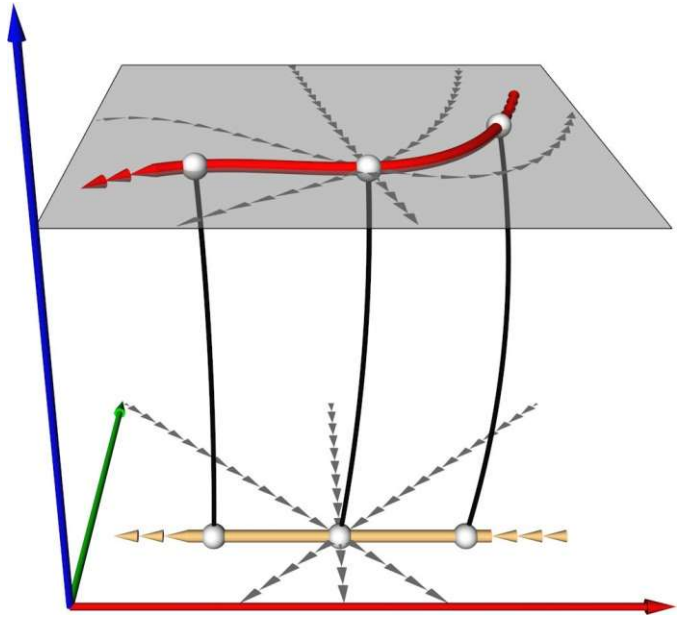


path lines

streak lines



time lines



Comparison of techniques:

(1) Pathlines:

- are physically meaningful
- allow comparison with experiment (observe marked particles)
- are well suited for dynamic visualization (of particles)

(2) Streamlines:

- are only geometrically, not physically meaningful
- are easiest to compute (no temporal interpolation, single IVP)
- are better suited for static visualization (prints)
- don't intersect (under reasonable assumptions)

(3) Streaklines:

- are physically meaningful
- allow comparison with experiment (dye injection)
- are well suited for static and dynamic visualization
- good choice for fast moving vortices
- can be approximated by set of disconnected particles

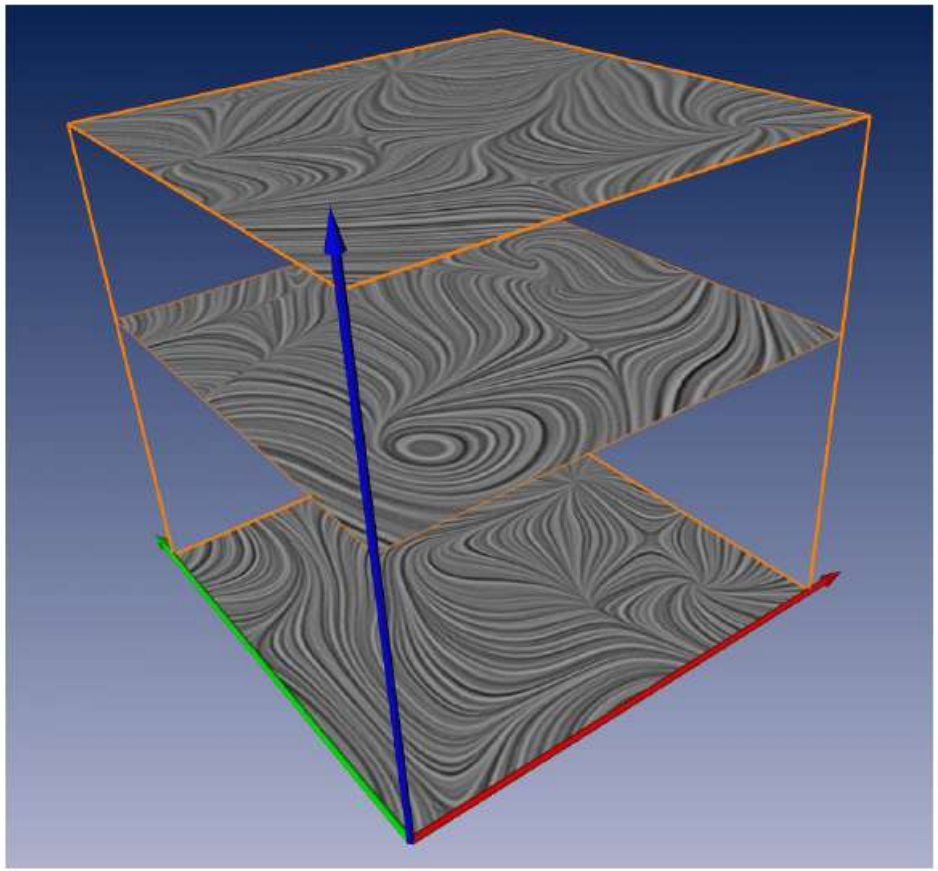
(4) Timelines:

- are physically meaningful
- are well suited for static and dynamic visualization
- can be approximated by set of disconnected particles

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

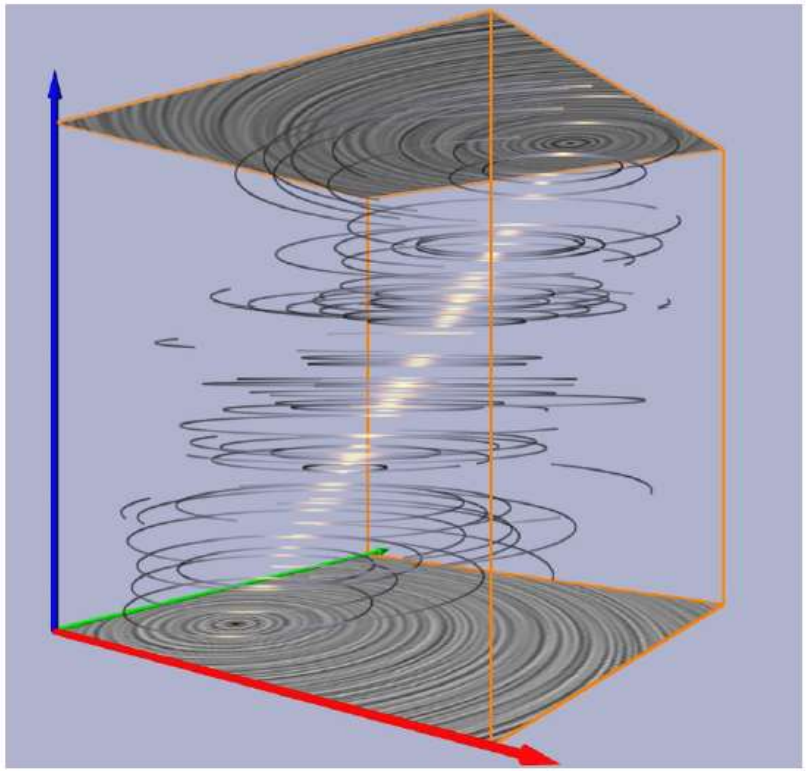


Tino Weinkauf

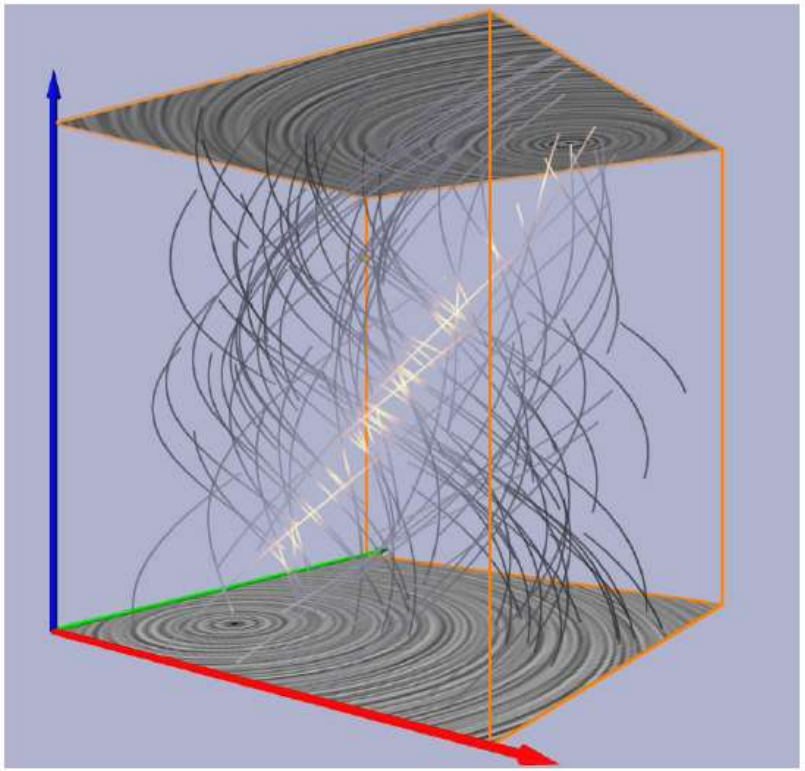
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Time

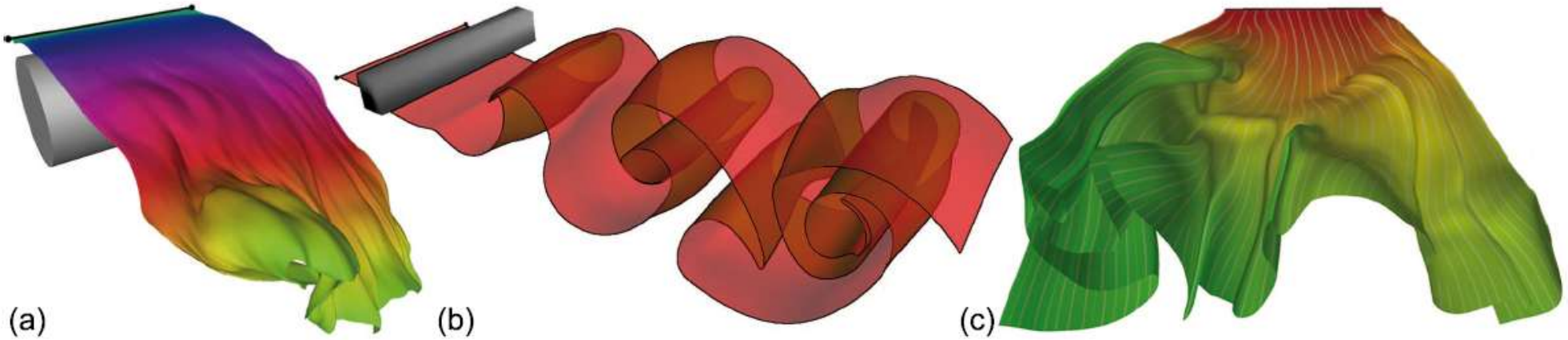
streak line

location of all particles set out at a fixed point at different times

Surfaces Instead of Lines

Seeding from a line instead of from a point

Example: streak surfaces



Volumes: seeding from a surface instead of a line

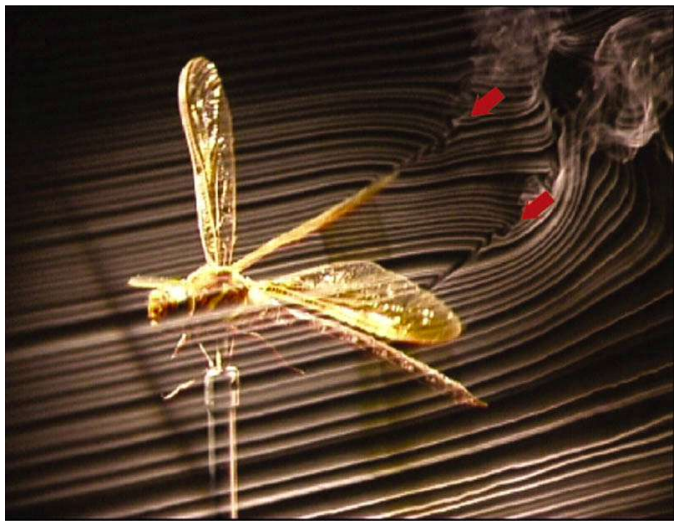
Real “Streak Surfaces”

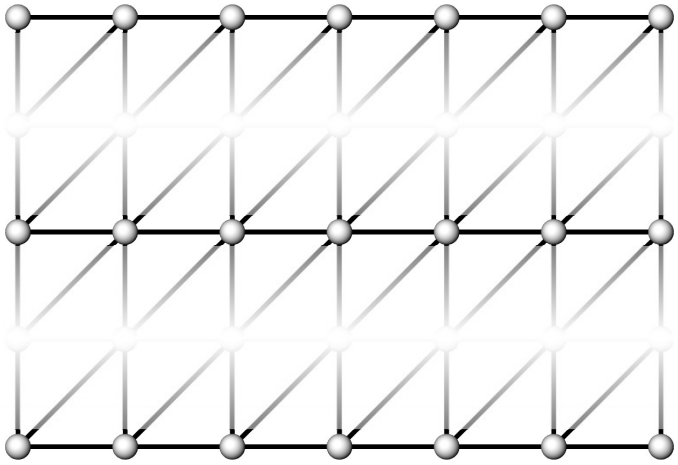
Artistic photographs of smoke

Time

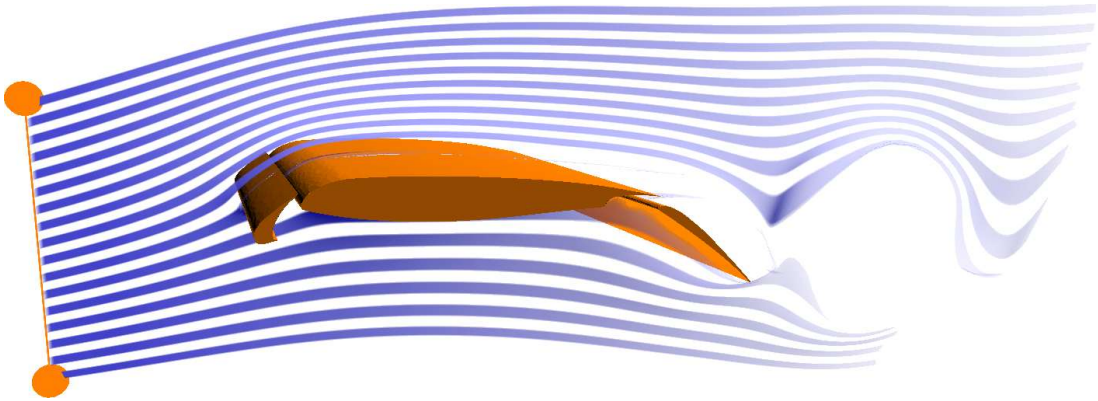
streak line

streak surface





fixed zero opacity rows



[Data courtesy of Günther (TU Berlin)]

break connectivity

Particle visualization

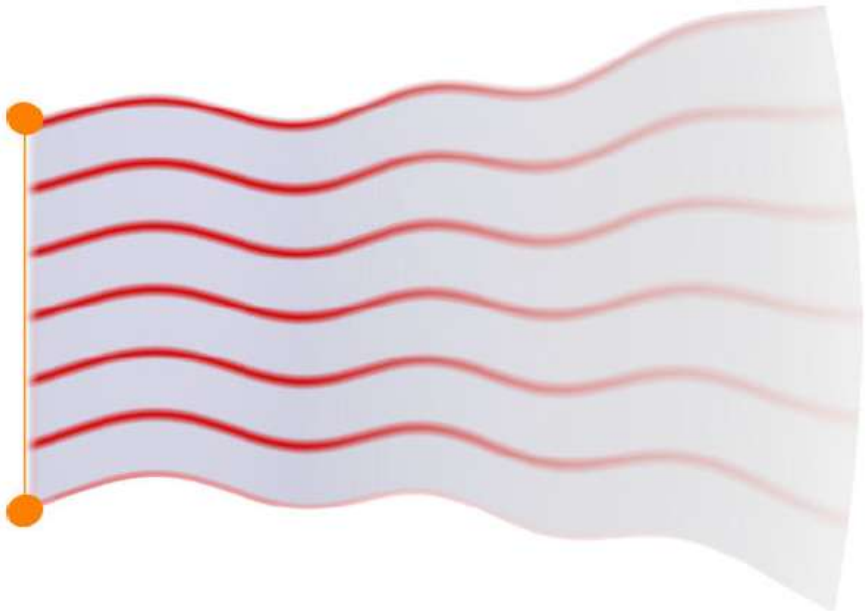
2D time-dependent flow around a cylinder

time line

location of all particles set out on a certain line at a fixed time

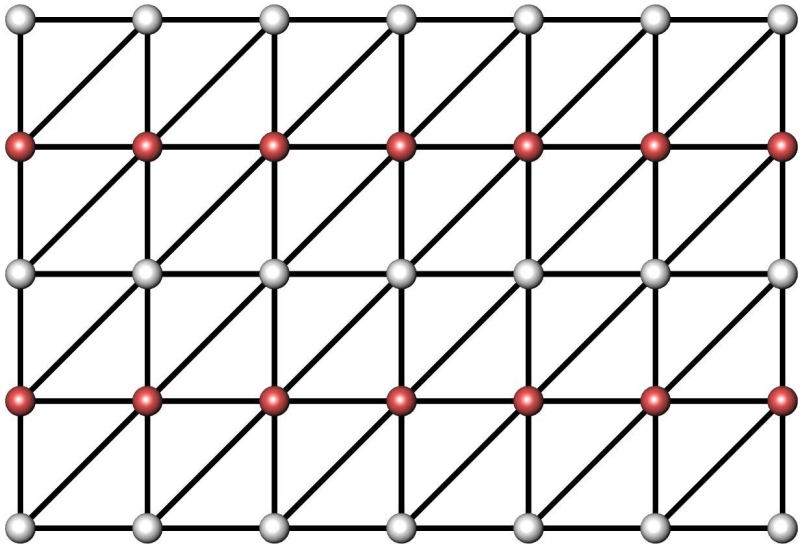
Streak Lines vs. Time Lines

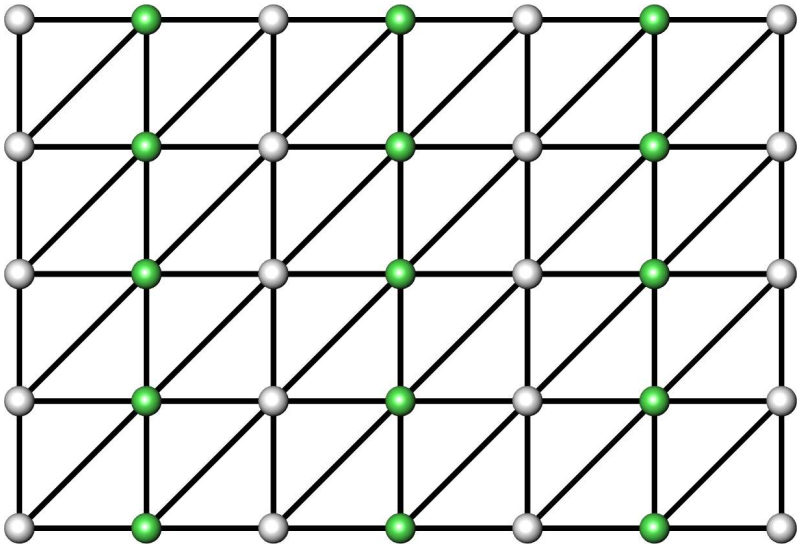
(on a streak surface)

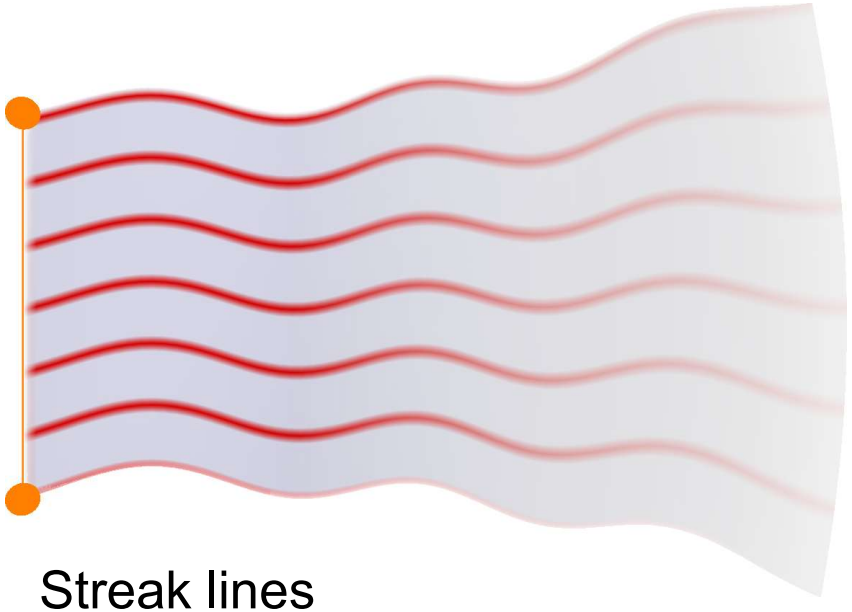


Streak Lines

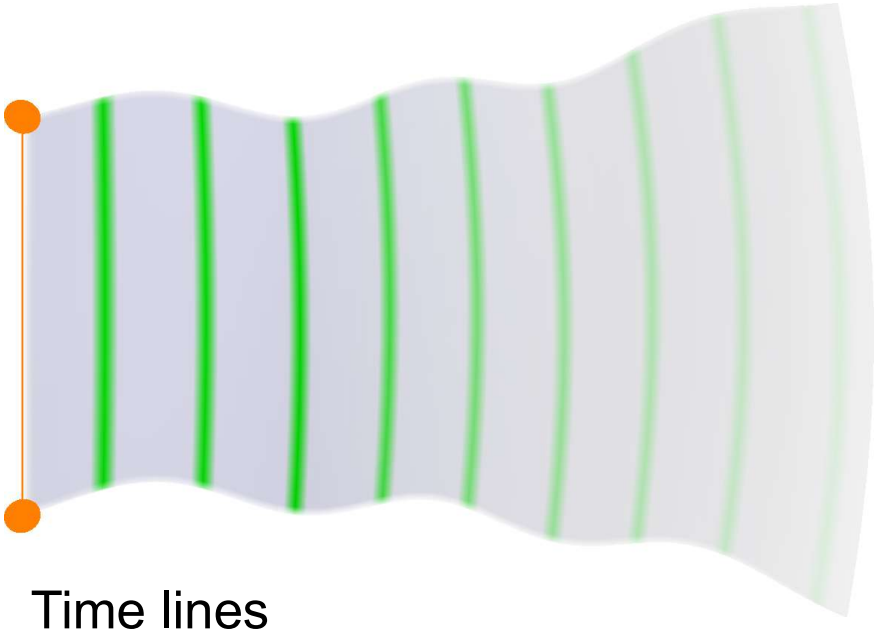
Time Lines







Streak lines



Time lines

The Flow / Flow Map of a Vector Field (1)

Flow of a *steady (time-independent)* vector field

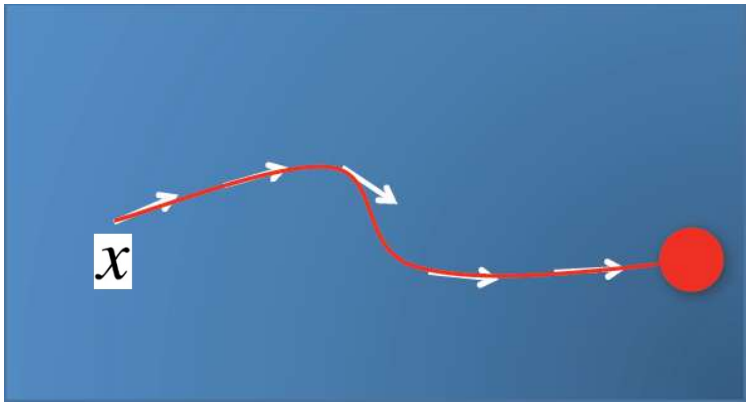
- Map source position *x* “forward” (*t>0*) or “backward” (*t<0*) by time *t*

$$\boxed{\phi(x,t)} \qquad \boxed{\phi_t(x)} \qquad \text{with} \qquad \phi_0(x) = x$$

$$\phi_s(\phi_t(x)) = \phi_{s+t}(x)$$

$$\phi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n, \qquad (x,t) \mapsto \phi(x,t).$$

$$\phi_t : \mathbb{R}^n \to \mathbb{R}^n, \qquad x \mapsto \phi_t(x).$$



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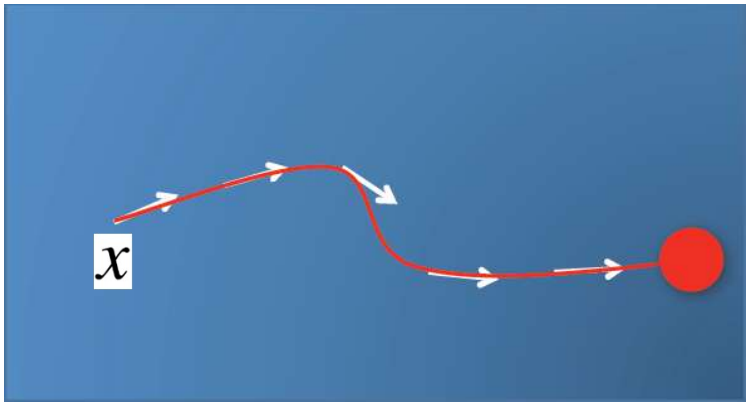

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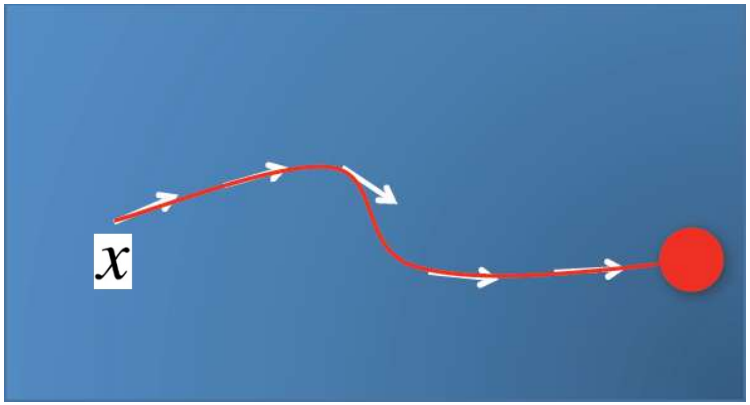
$$\phi(x,t) = x + \int_0^t \mathbf{v}\big(\phi(x,\tau)\big)\,\mathrm{d}\tau$$

(on a general manifold *M*, integration is performed in coordinate charts)



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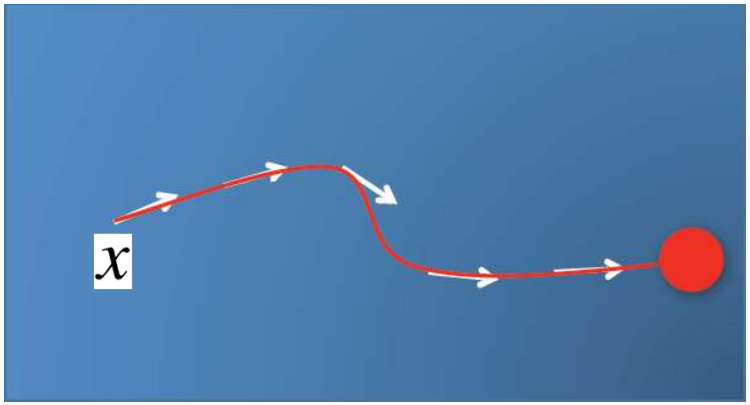
$$\phi : M \times \mathbb{R} \to M, \qquad (x,t) \mapsto \phi(x,t).$$

$$\phi_t : M \to M, \qquad x \mapsto \phi_t(x).$$

- Unsteady flow? Just fix arbitrary time T

$$\phi(x,t) = x + \int_0^t \mathbf{v}\big(\phi(x,\tau), \mathrm{T}\big)\,\mathrm{d}\tau$$

(on a general manifold *M*, integration is performed in coordinate charts)



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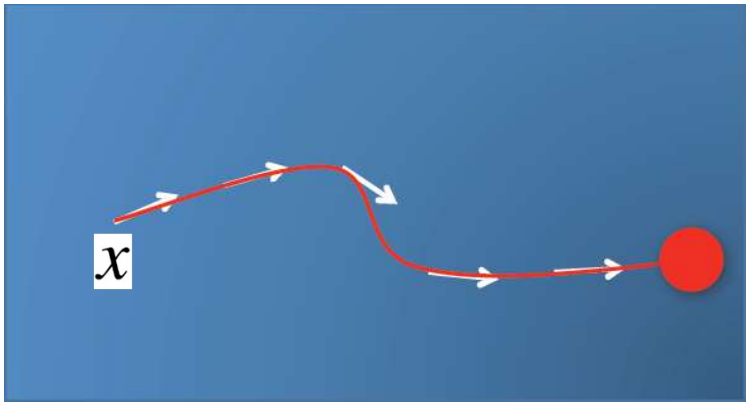
$$\phi : M \times \mathbb{R} \to M, \qquad (x,t) \mapsto \phi(x,t).$$

$$\phi_t : M \to M, \qquad x \mapsto \phi_t(x).$$

Can write explicitly as function of independent variable *t*, with *position x fixed*

$$t \mapsto \phi(x,t) \qquad t \mapsto \phi_t(x)$$

= stream line going through point *x*



The Flow / Flow Map of a Vector Field (2)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t* (t < s is allowed: map forward or backward in time)

$$\boxed{\psi_{t,s}(x)}$$

with

$$\psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\,\mathrm{d}\tau$$

$$\psi_{s,s}(x) = x$$

$$\psi_{t,r}\big(\psi_{r,s}(x)\big) = \psi_{t,s}(x)$$

The Flow / Flow Map of a Vector Field (3)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t* (t < s is allowed: map forward or backward in time)

$$\boxed{\psi_{t,s}(x)} \qquad \psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\,\mathrm{d}\tau$$

Can write explicitly as function of t, *with s and x fixed*

$t \mapsto \psi_{t,s}(x)$ → path line

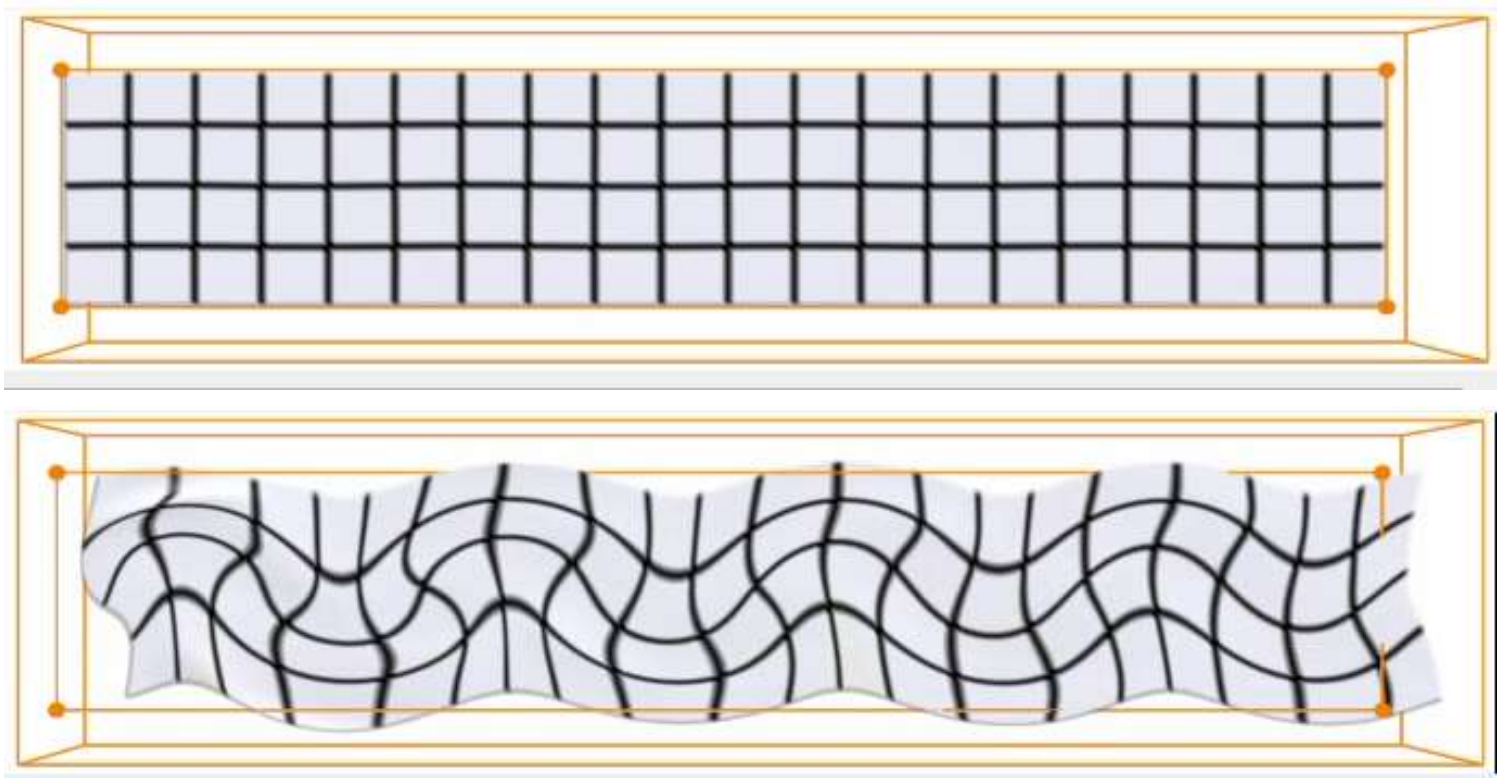
Can write explicitly as function of s, *with t and x fixed*

$s \mapsto \psi_{t,s}(x)$ → streak line

$\psi_{t,s}(x)$ is also often written as ***flow map*** $\phi_t^{\tau}(x)$ (with t:=s and either τ:=t or τ:=t-s)

The Flow / Flow Map of a Vector Field (4)

Can map a whole set of points (or the entire domain) through the flow map (this map is a *diffeomorphism*): $t \mapsto \psi_{t,s}(U)$



U

$= \psi_{s,s}(U)$

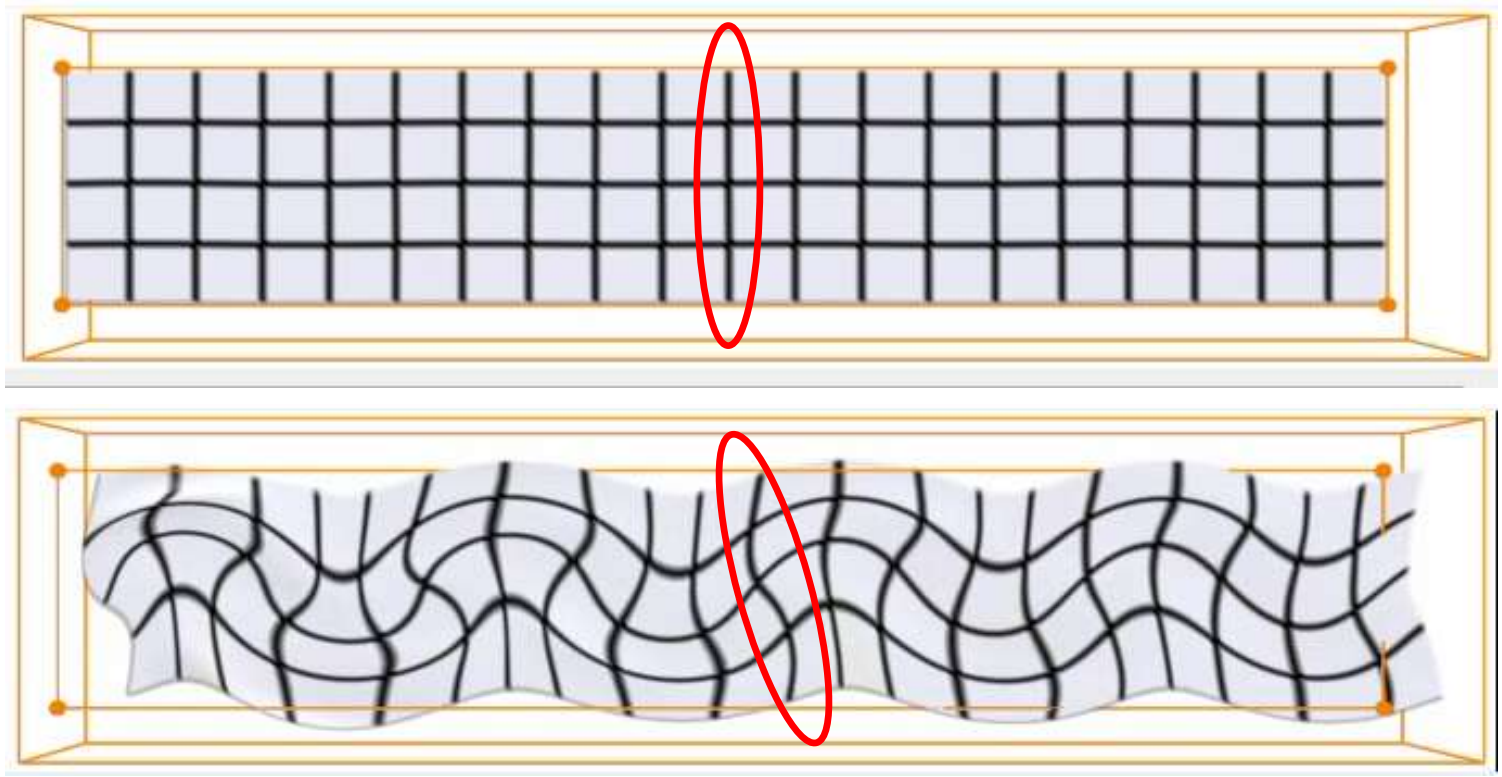
$\psi_{t,s}(U)$

(this is a *time surface!*)

The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (s) to another time (t)

$$t \mapsto \psi_{t,s}(c(\lambda))$$



$c(\lambda)$

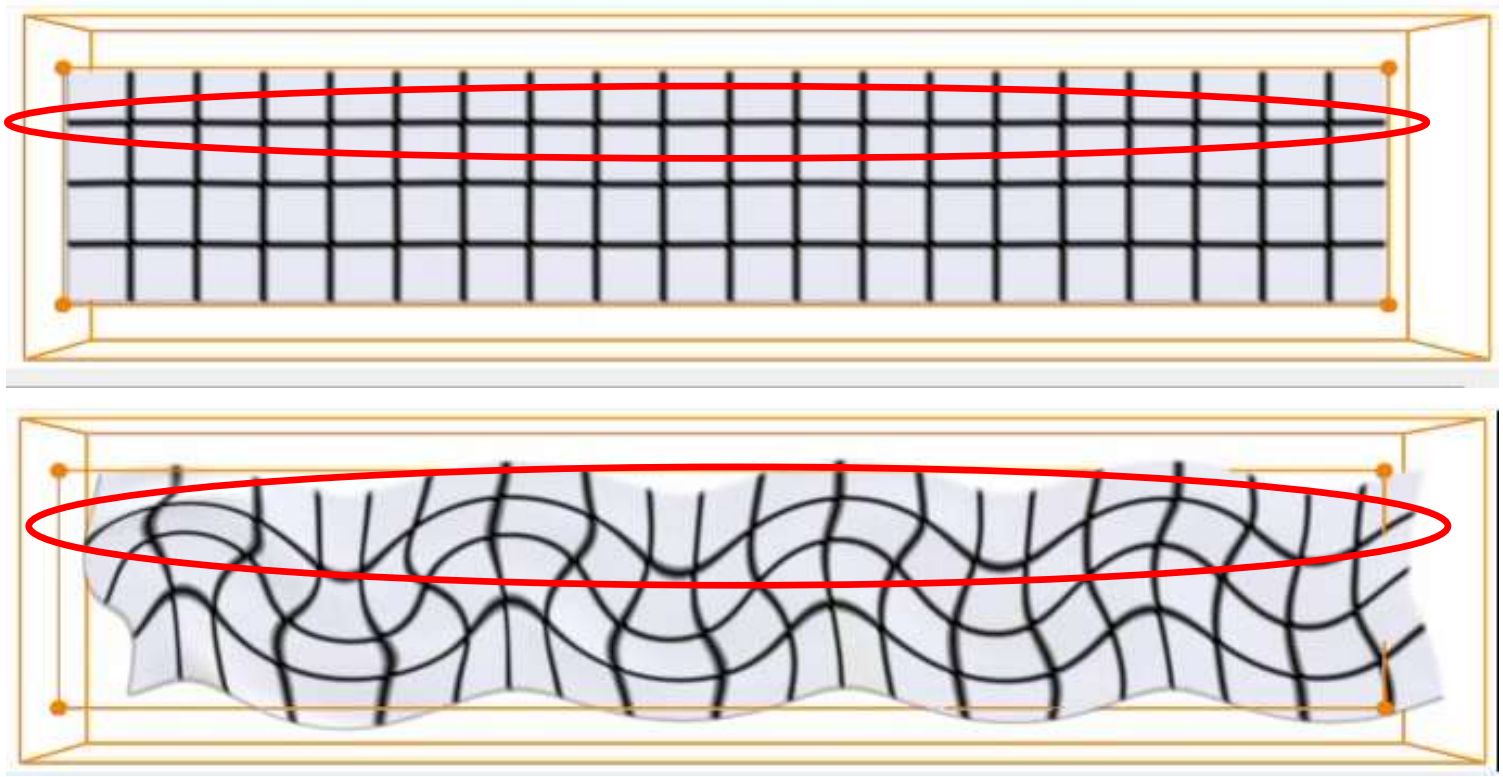
$= \psi_{s,s}(c(\lambda))$

$\psi_{t,s}(c(\lambda))$

The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (s) to another time (t)

$$t \mapsto \psi_{t,s}(c(\lambda))$$



$c(\lambda)$

$= \psi_{s,s}(c(\lambda))$

$\psi_{t,s}(c(\lambda))$

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama