King Abdullah University of Science and Technology

KAUST

CS 247 – Scientific Visualization
Lecture 22: Vector Field / Flow Visualization, Pt.4

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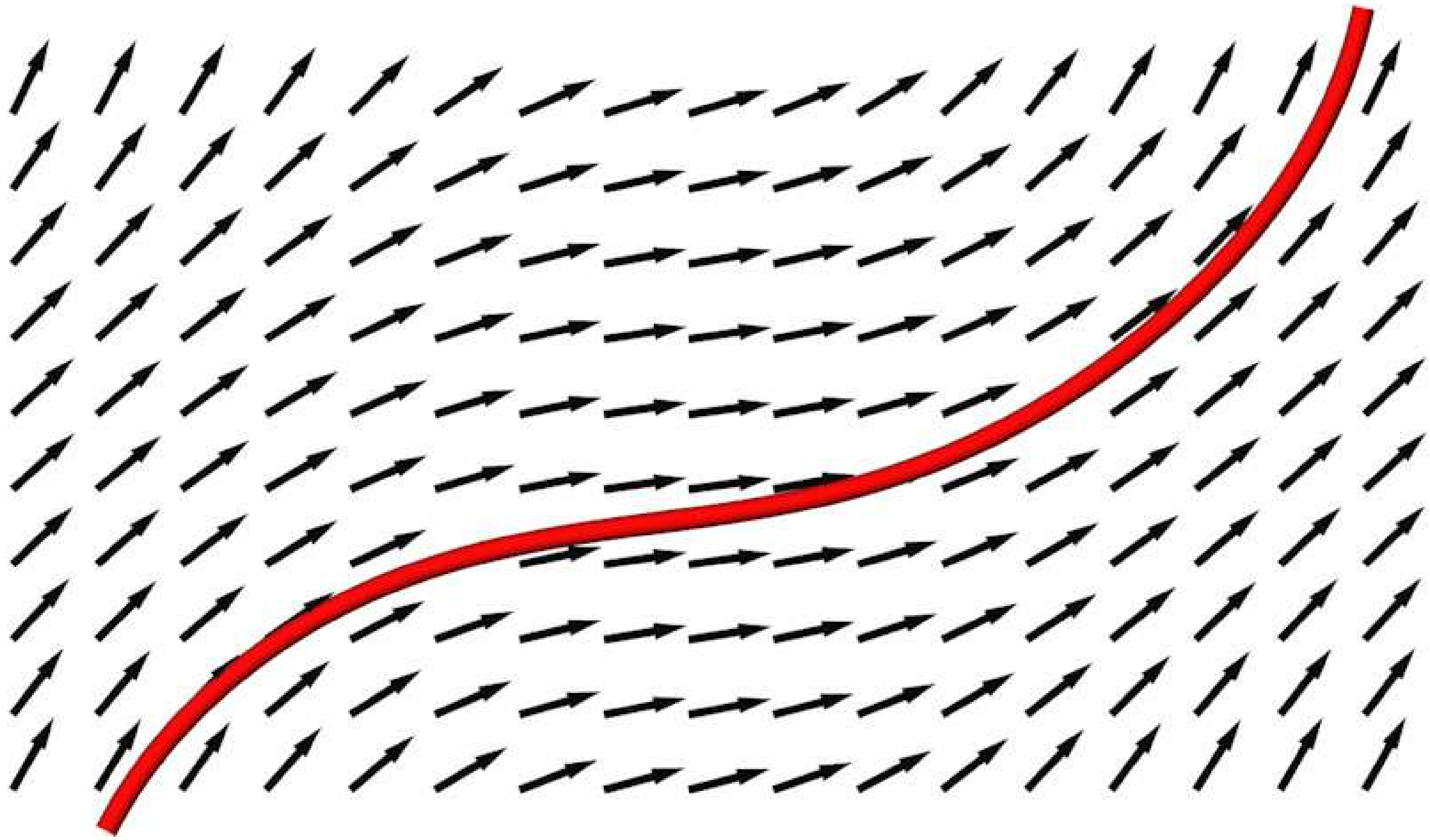
Reading Assignment #13 (until May 10)

Read (required):

- Data Visualization book
  - Chapter 6.1 (Divergence and Vorticity)
  - Chapter 6.6 (Texture-Based Vector Visualization)
- Diffeomorphisms / smooth deformations
  `https://en.wikipedia.org/wiki/Diffeomorphism`
- Learn how convolution (the convolution of two functions) works:
  `https://en.wikipedia.org/wiki/Convolution`
- B. Cabral, C. Leedom:
  Imaging Vector Fields Using Line Integral Convolution, SIGGRAPH 1993
  `http://dx.doi.org/10.1145/166117.166151`

Integral Curves / Stream Objects

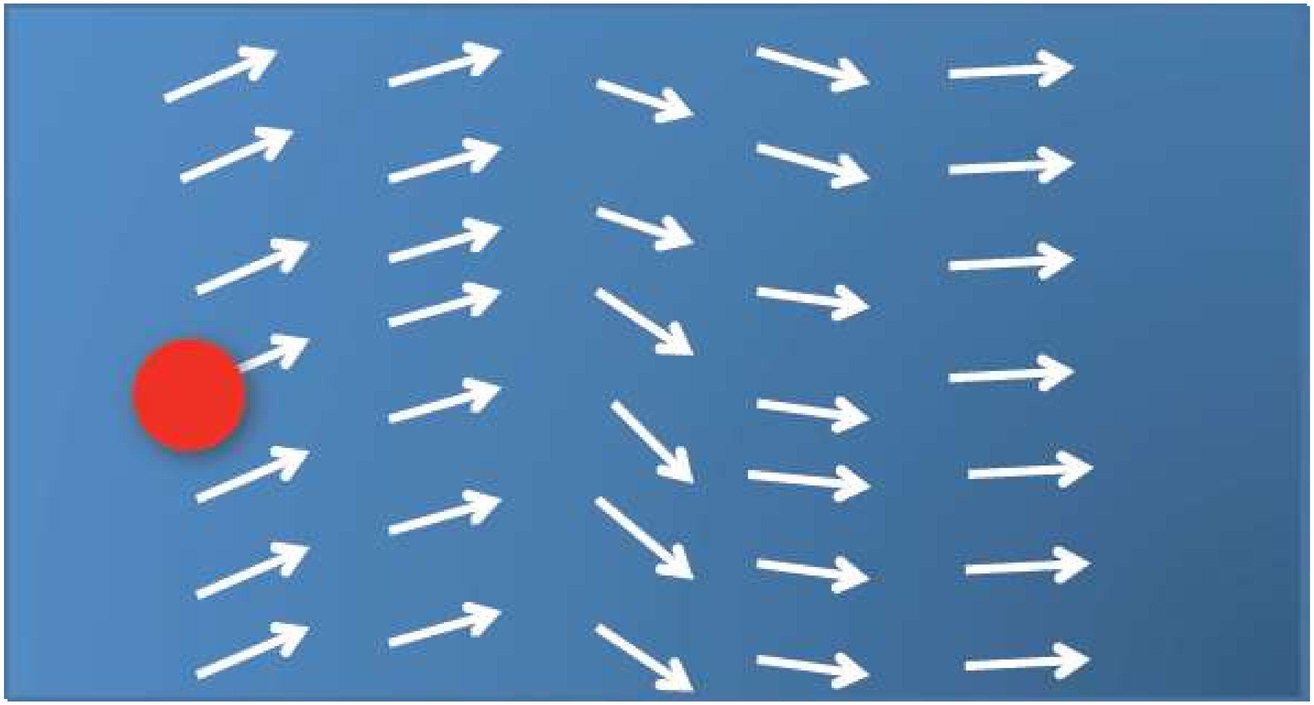
Integrating velocity over time yields spatial motion



Particle Trajectories

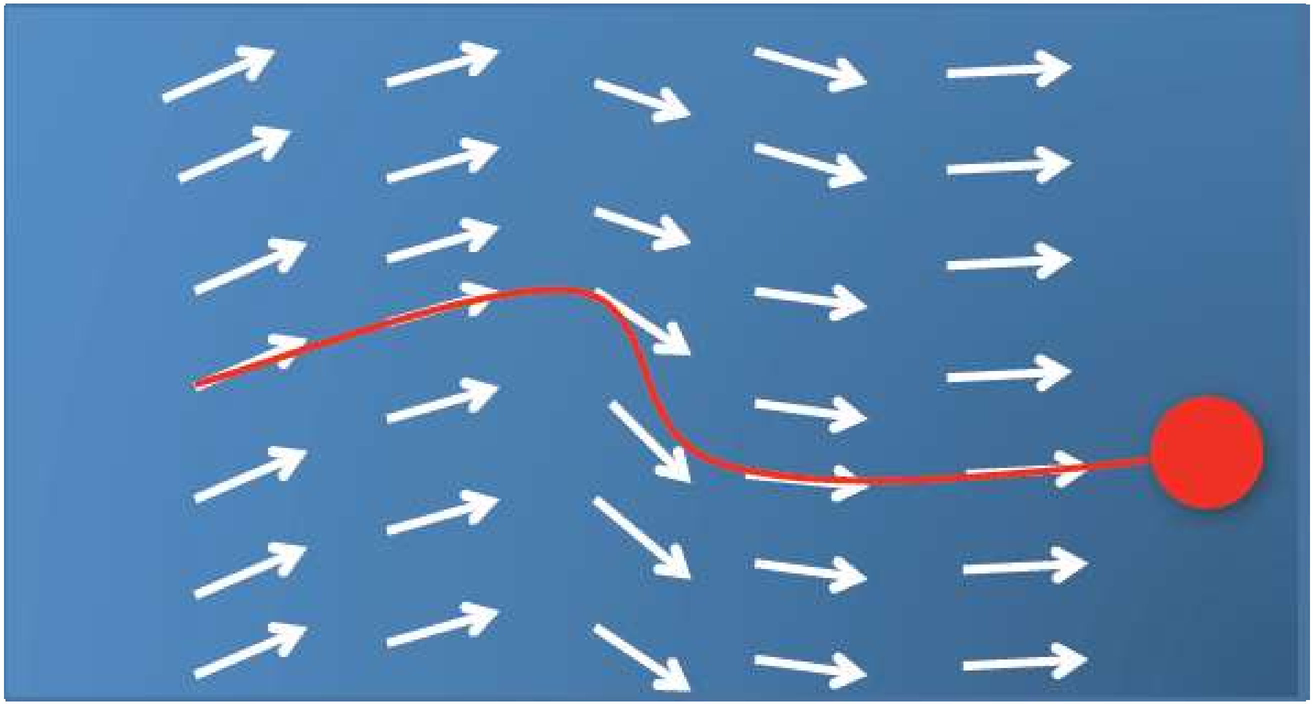
Courtesy Jens Krüger

Particle Trajectories



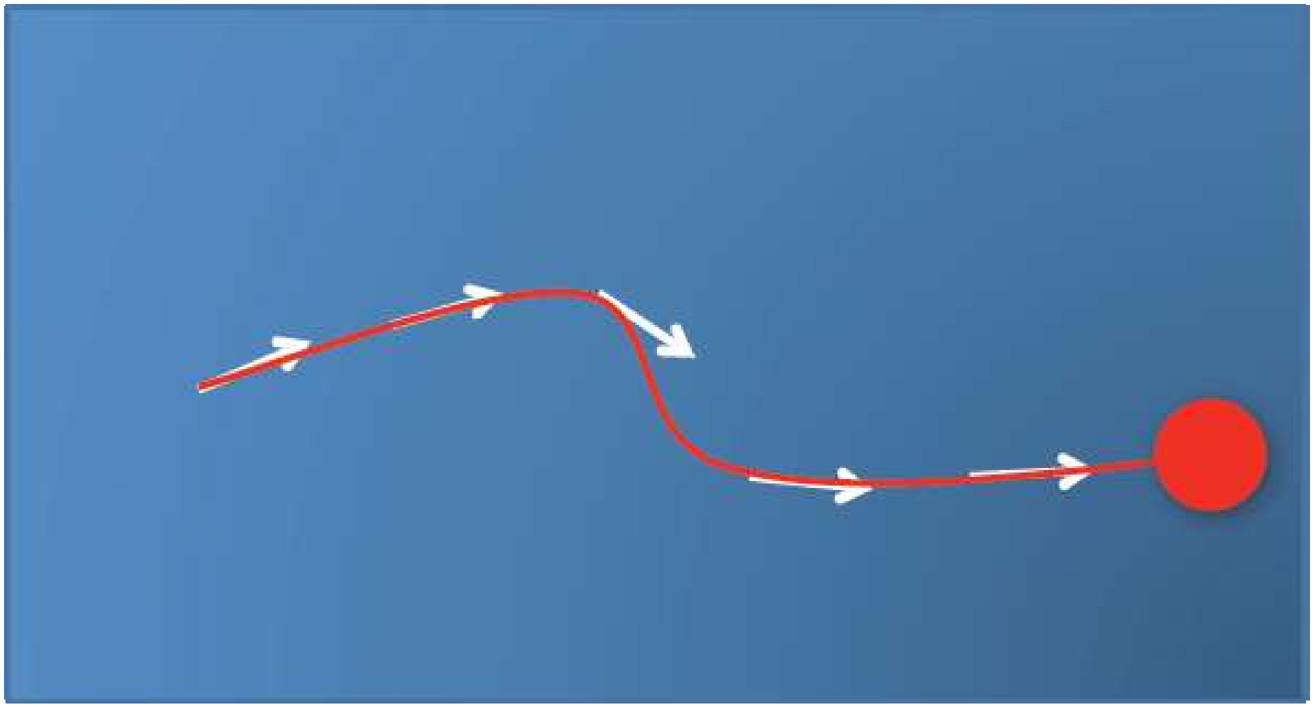
Courtesy Jens Krüger

Particle Trajectories



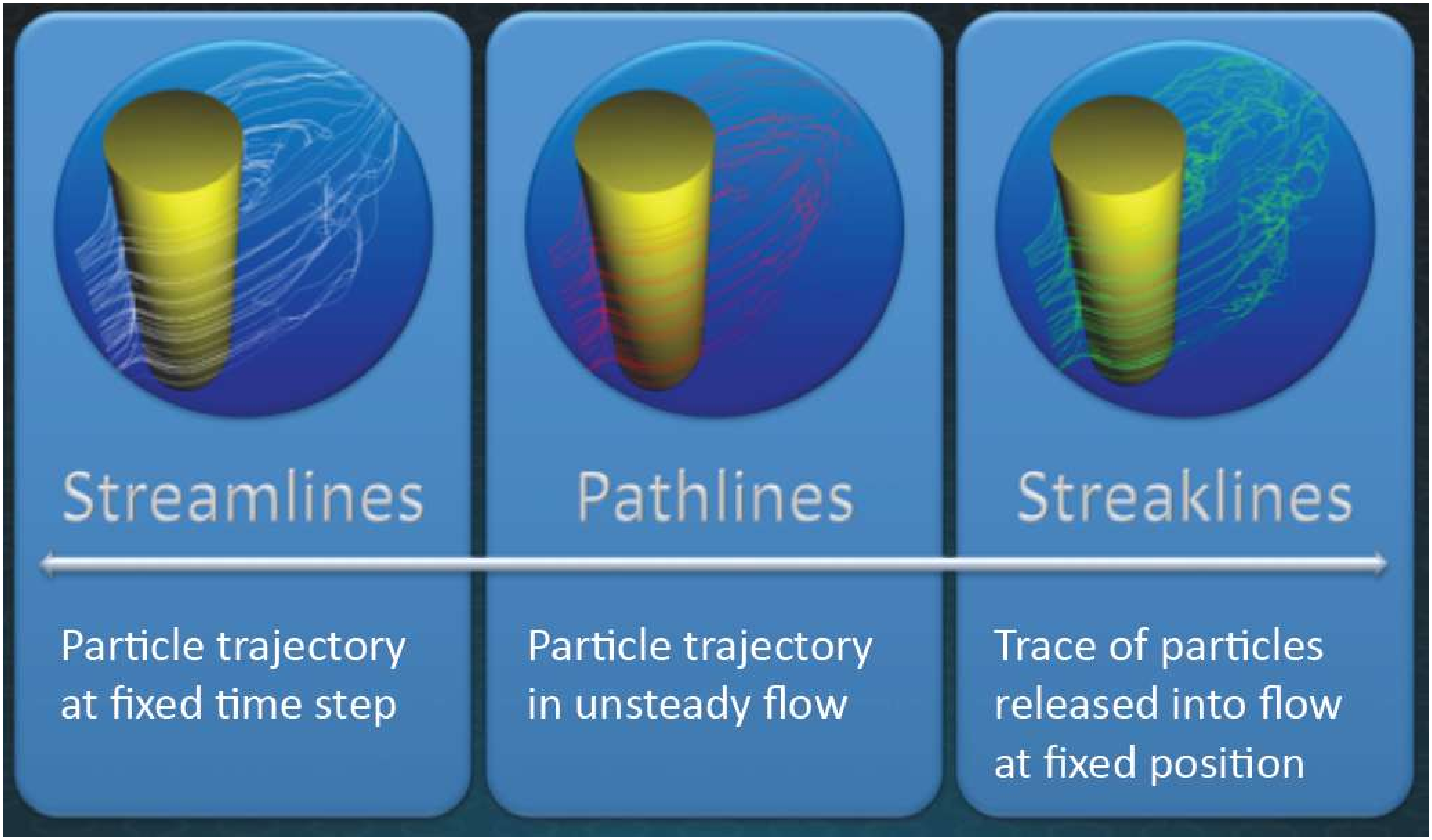
Courtesy Jens Krüger

Particle Trajectories



Courtesy Jens Krüger

Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

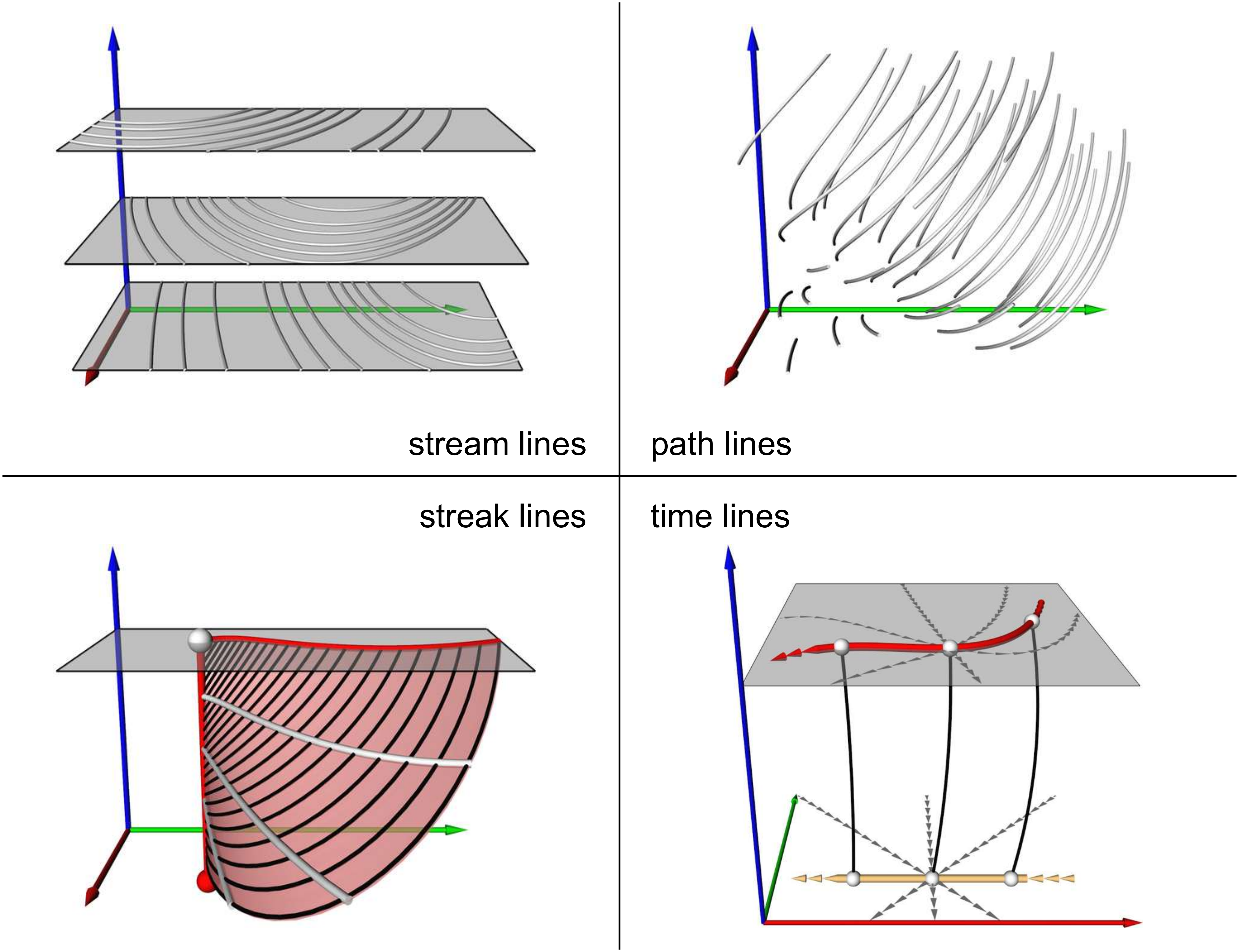
- Describes motion of a massless particle over time

Streakline

- Location of all particles released at a *fixed position* over time

Timeline

- Location of all particles released along a line at a *fixed time*


stream lines
path lines
streak lines
time lines

Comparison of techniques:

(1) Pathlines:

- are physically meaningful
- allow comparison with experiment (observe marked particles)
- are well suited for dynamic visualization (of particles)

(2) Streamlines:

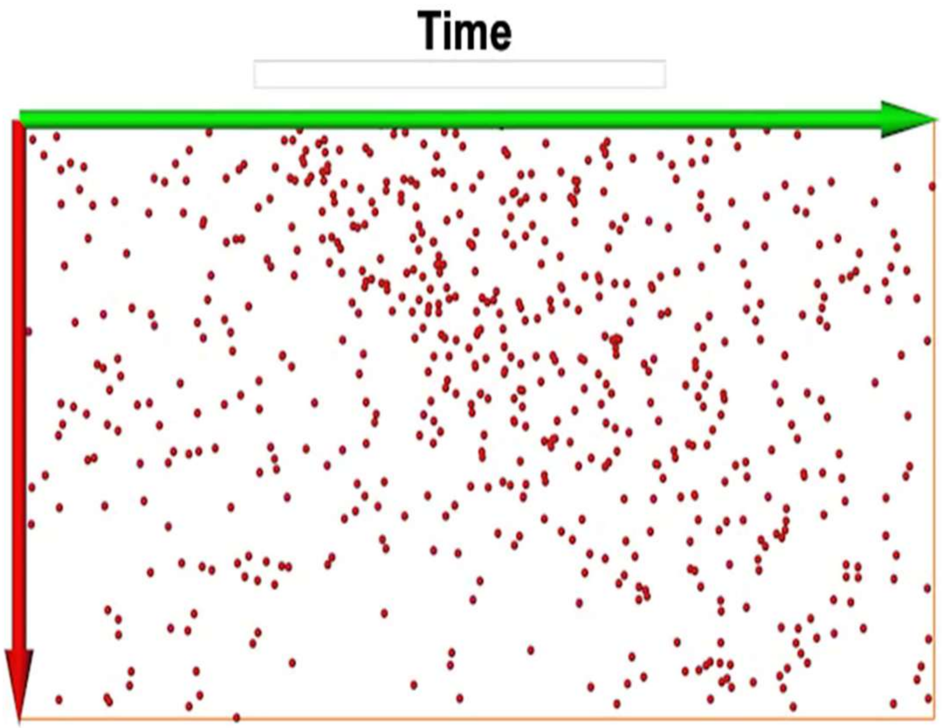
- are only geometrically, not physically meaningful
- are easiest to compute (no temporal interpolation, single IVP)
- are better suited for static visualization (prints)
- don't intersect (under reasonable assumptions)

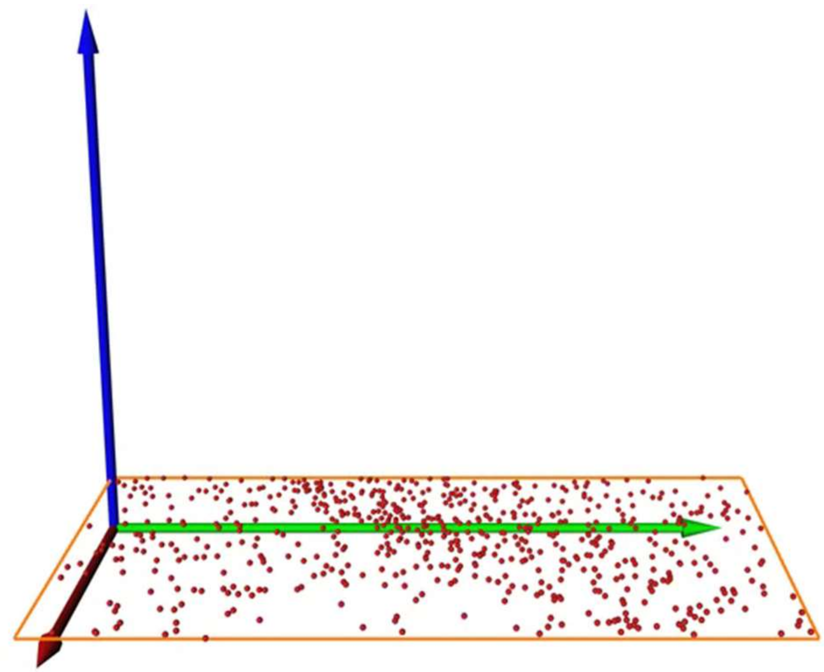
(3) Streaklines:

- are physically meaningful
- allow comparison with experiment (dye injection)
- are well suited for static and dynamic visualization
- good choice for fast moving vortices
- can be approximated by set of disconnected particles

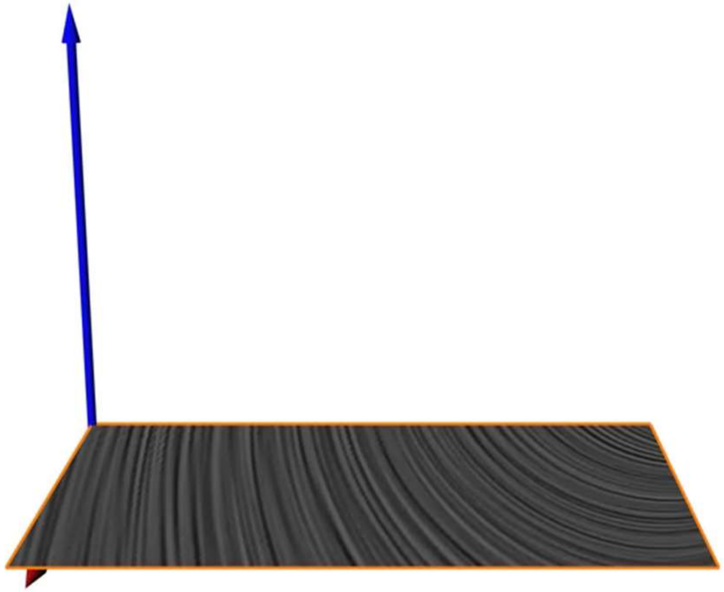
(4) Timelines:

- are physically meaningful
- are well suited for static and dynamic visualization
- can be approximated by set of disconnected particles

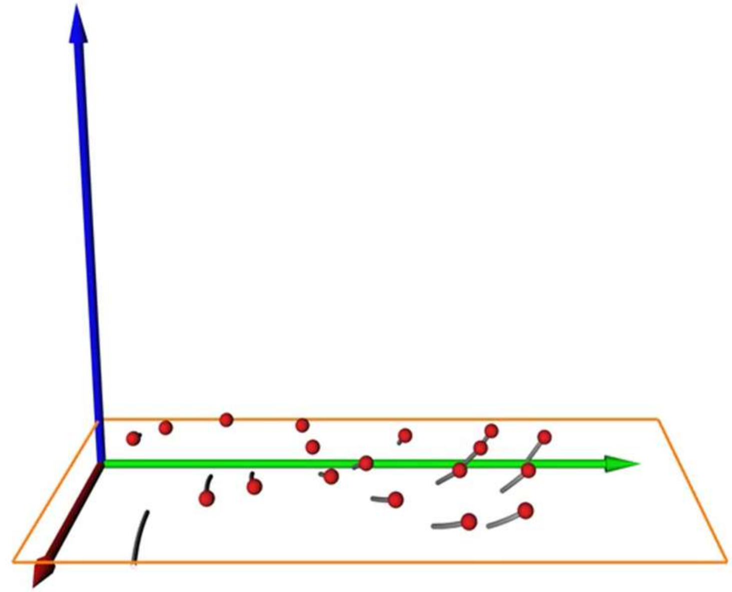




2D time-dependent vector field
particle visualization



stream lines



path lines

curve parallel to the vector field in each point for a **fixed time**

describes motion of a massless particle in an **steady** flow field

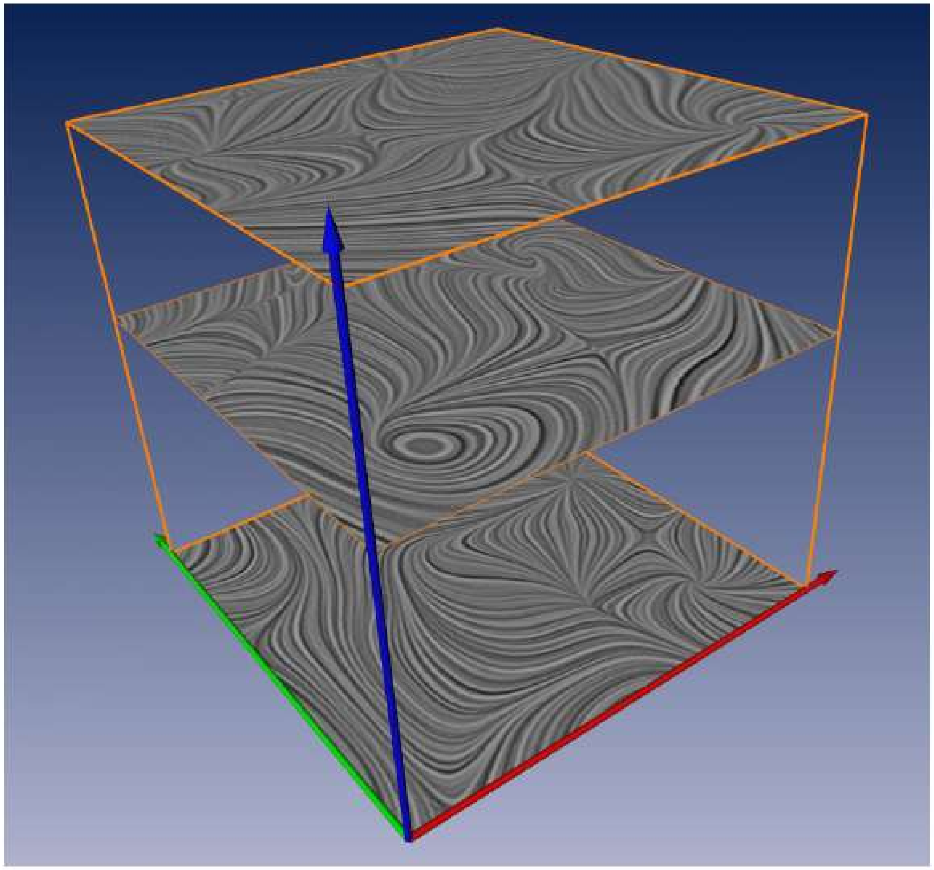
curve parallel to the vector field in each point **over time**

describes motion of a massless particle in an **unsteady** flow field

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

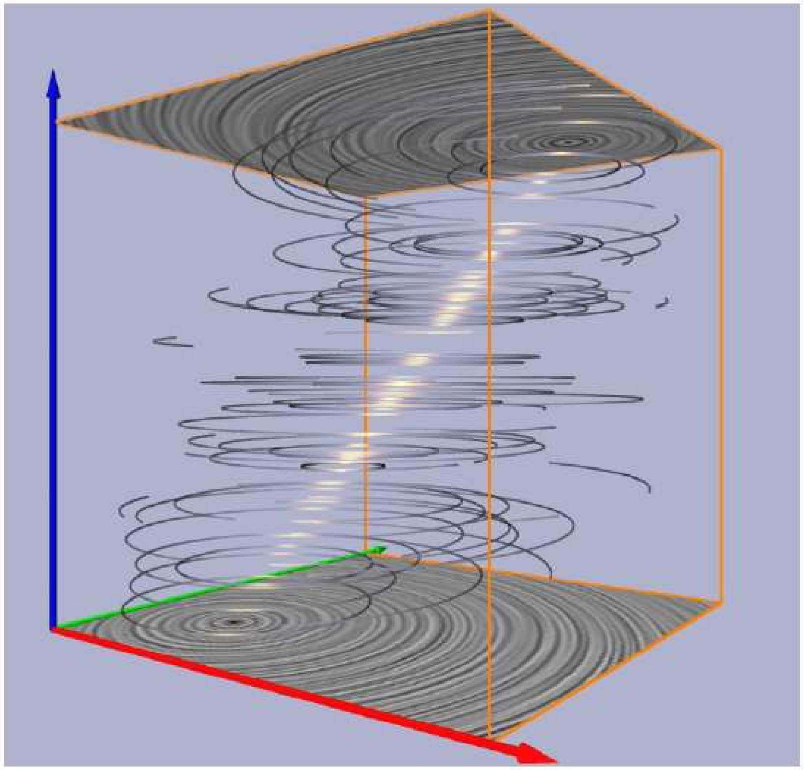


Tino Weinkauf

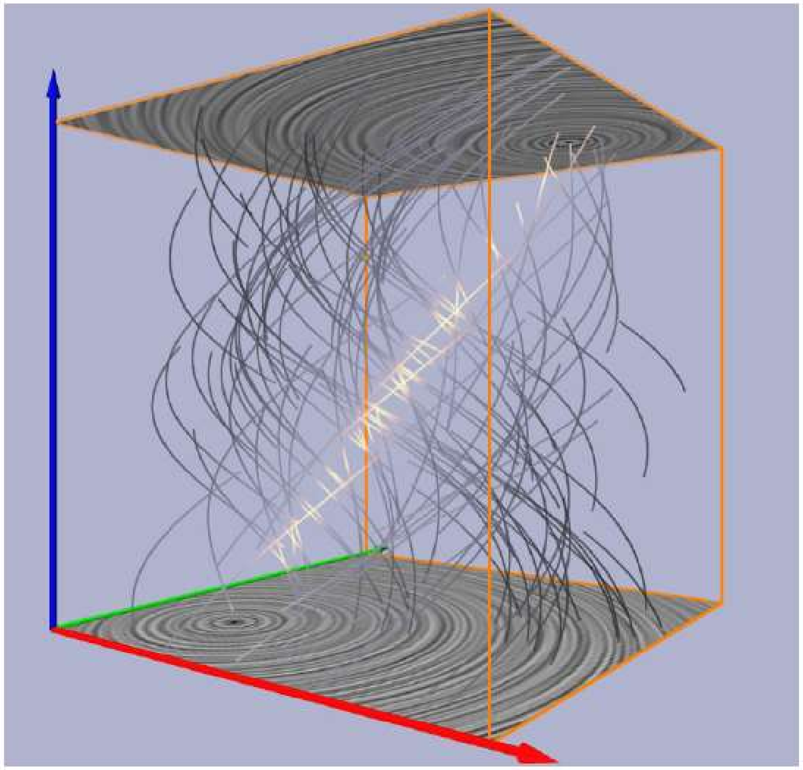
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Vector fields as ODEs

For simplicity, the vector field is now interpreted as a velocity field.

Then the field $\mathbf{v}(\mathbf{x}, t)$ describes the connection between location and velocity of a (massless) particle.

It can equivalently be expressed as an ordinary differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t), t)$$

This ODE, together with an initial condition

$$\mathbf{x}(t_0) = \mathbf{x}_0 ,$$

is a so-called initial value problem (IVP).

Its solution is the integral curve (or trajectory)

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau), \tau)\, d\tau$$

The integral curve is a pathline, describing the path of a massless particle which was released at time t_0 at position x_0.

Remark: $t < t_0$ is allowed.

For static fields, the ODE is autonomous:

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t))$$

and its integral curves

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau))\, d\tau$$

are called field lines, or (in the case of velocity fields) streamlines.

In static vector fields, pathlines and streamlines are identical.

In time-dependent vector fields, instantaneous streamlines can be computed from a "snapshot" at a fixed time T (which is a static vector field)

$$\mathbf{v}_T(\mathbf{x}) = \mathbf{v}(\mathbf{x}, T)$$

In practice, time-dependent fields are often given as a dataset per time step. Each dataset is then a snapshot.

Streamline integration

Outline of algorithm for numerical streamline integration (with obvious extension to pathlines):

Inputs:

- static vector field $\mathbf{v}(\mathbf{x})$
- seed points with time of release $(\mathbf{x}_0, t_0)$
- control parameters:
  - step size (temporal, spatial, or in local coordinates)
  - step count limit, time limit, etc.
  - order of integration scheme

Output:

- streamlines as "polylines", with possible attributes (interpolated field values, time, speed, arc length, etc.)

Preprocessing:

- set up search structure for point location
- for each seed point:
  - global point location: Given a point **x**,
    find the cell containing **x** and the local coordinates (ξ, η, ζ)
    or ir the grid is structured:
    find the computational space coordinates $(i + \xi,\ j + \eta,\ k + \zeta)$
  - If **x** is not found in a cell, remove seed point

Integration loop, for each seed point **x**:

- interpolate ***v*** trilinearly to local coordinates (ξ, η, ζ)
- do an integration step, producing a new point **x'**
- incremental point location: For position **x'** find cell and local coordinates (ξ', η', ζ') making use of information (coordinates, local coordinates, cell) of old point **x**

Termination criteria:

- grid boundary reached
- step count limit reached
- optional: velocity close to zero
- optional: time limit reached
- optional: arc length limit reached

Integration step: widely used integration methods:

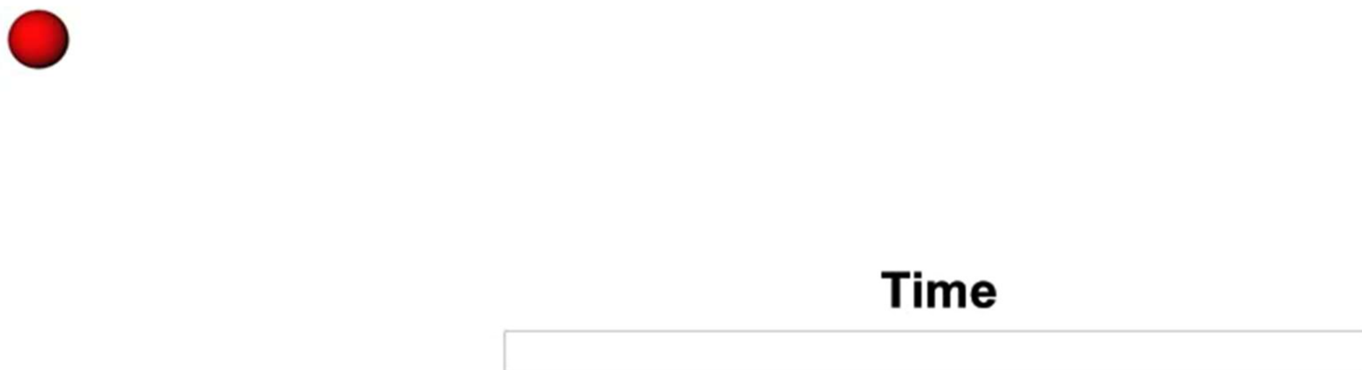
- Euler (used only in special speed-optimized techniques, e.g. GPU-based texture advection)

$$\mathbf{x}_{new} = \mathbf{x} + \mathbf{v}(\mathbf{x}, t) \cdot \Delta t$$

- Runge-Kutta, 2nd or 4th order

Higher order than 4th?

- often too slow for visualization
- study (Yeung/Pope 1987) shows that, when using standard trilinear interpolation, interpolation errors dominate integration errors.



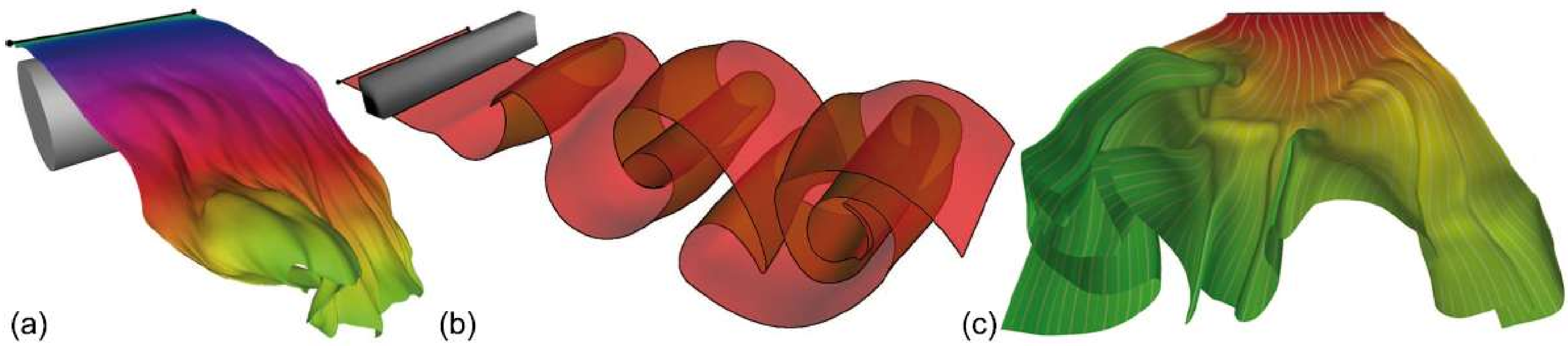
streak line

location of all particles set out at a fixed point at different times

Surfaces Instead of Lines

Seeding from a line instead of from a point

Example: streak surfaces



Volumes: seeding from a surface instead of a line

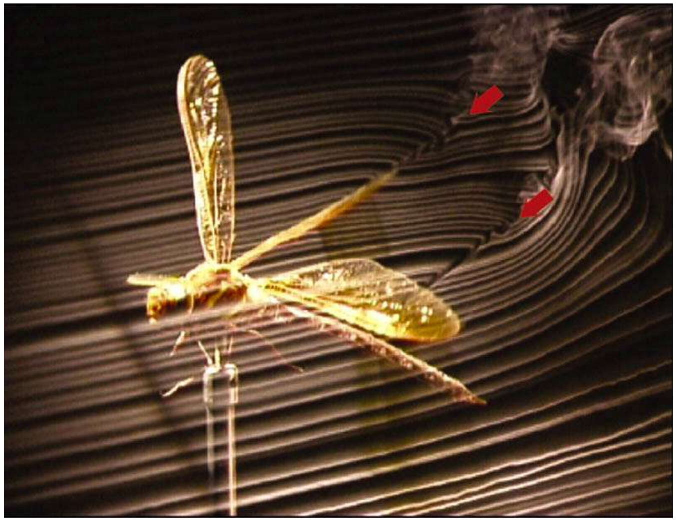
Real "Streak Surfaces"

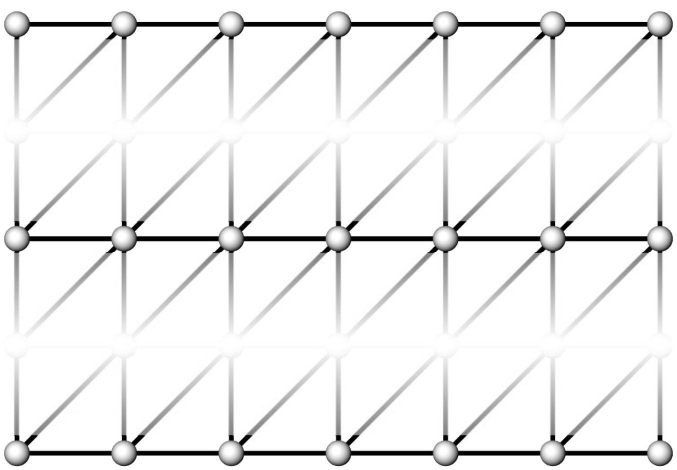
Artistic photographs of smoke

Time

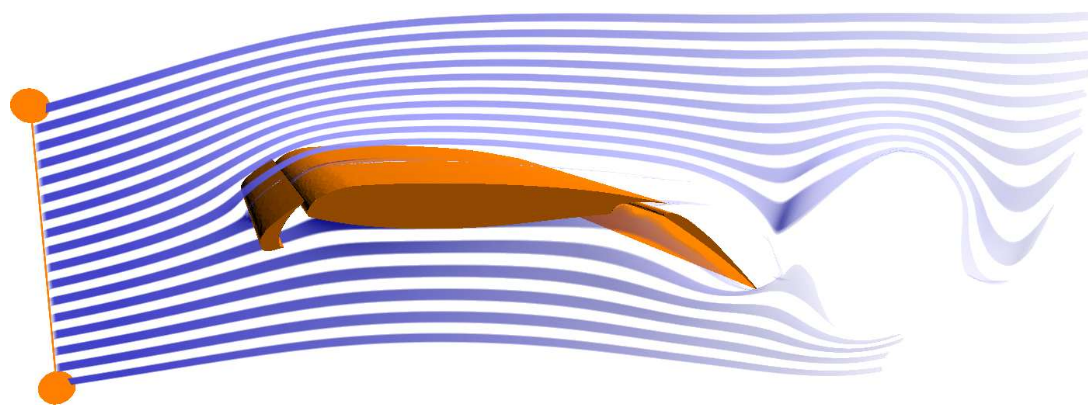
streak line

streak surface



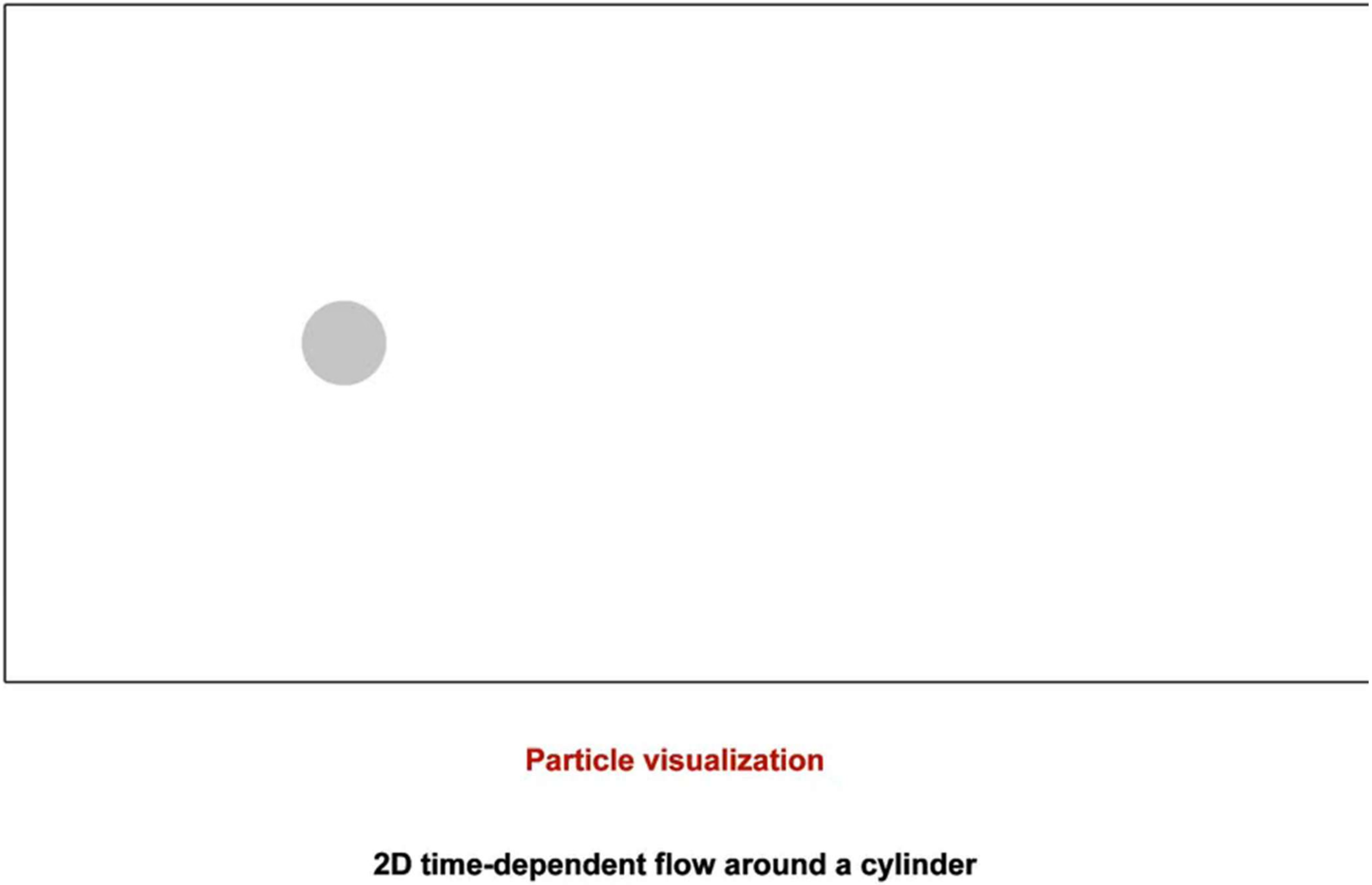


fixed zero opacity rows



[Data courtesy of Günther (TU Berlin)]

break connectivity

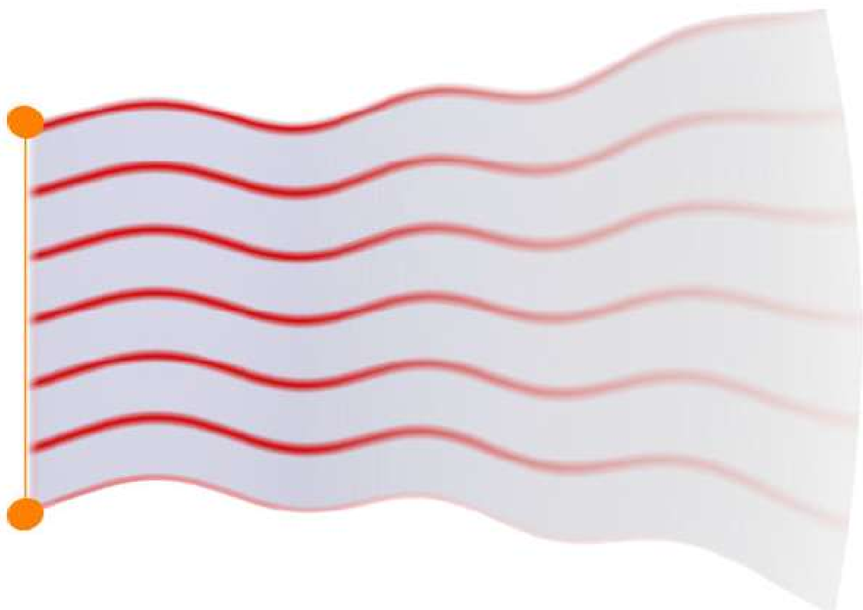


time line

location of all particles set out on a certain line at a fixed time

Streak Lines vs. Time Lines

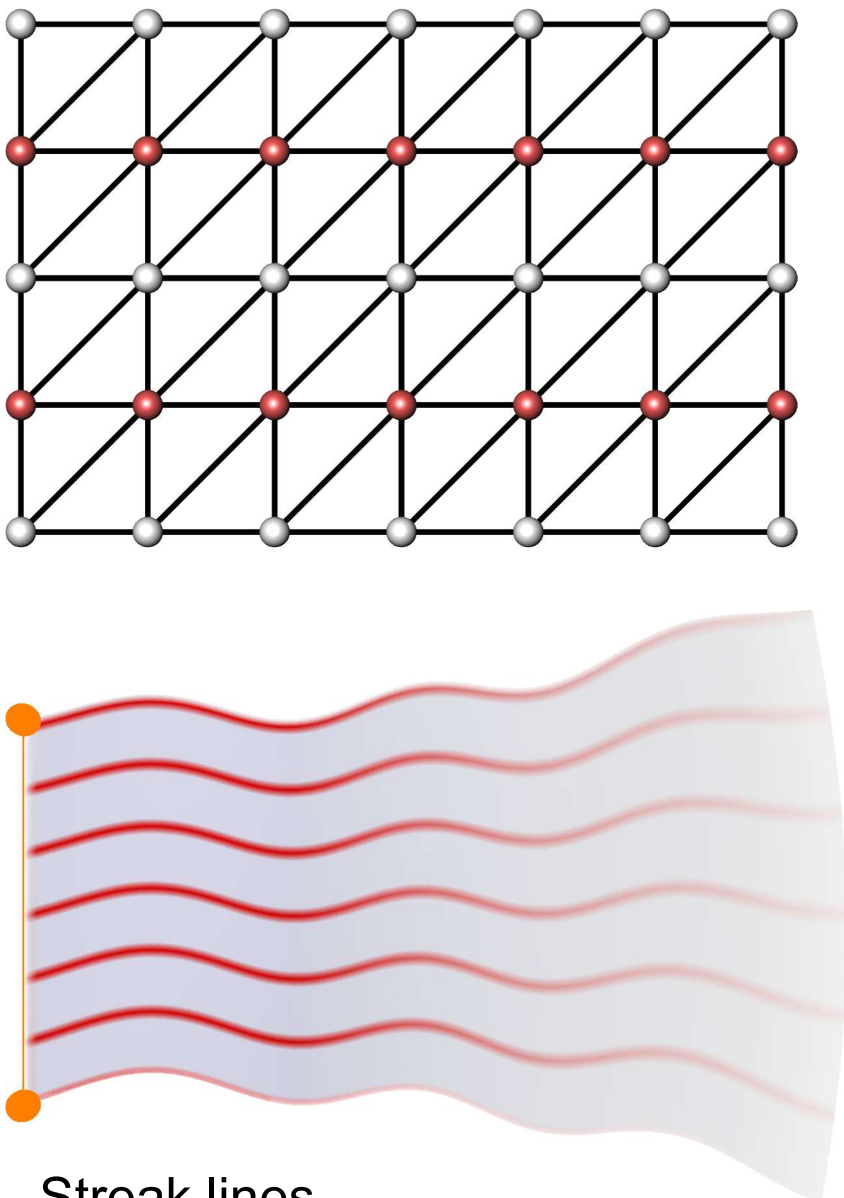
(on a streak surface)



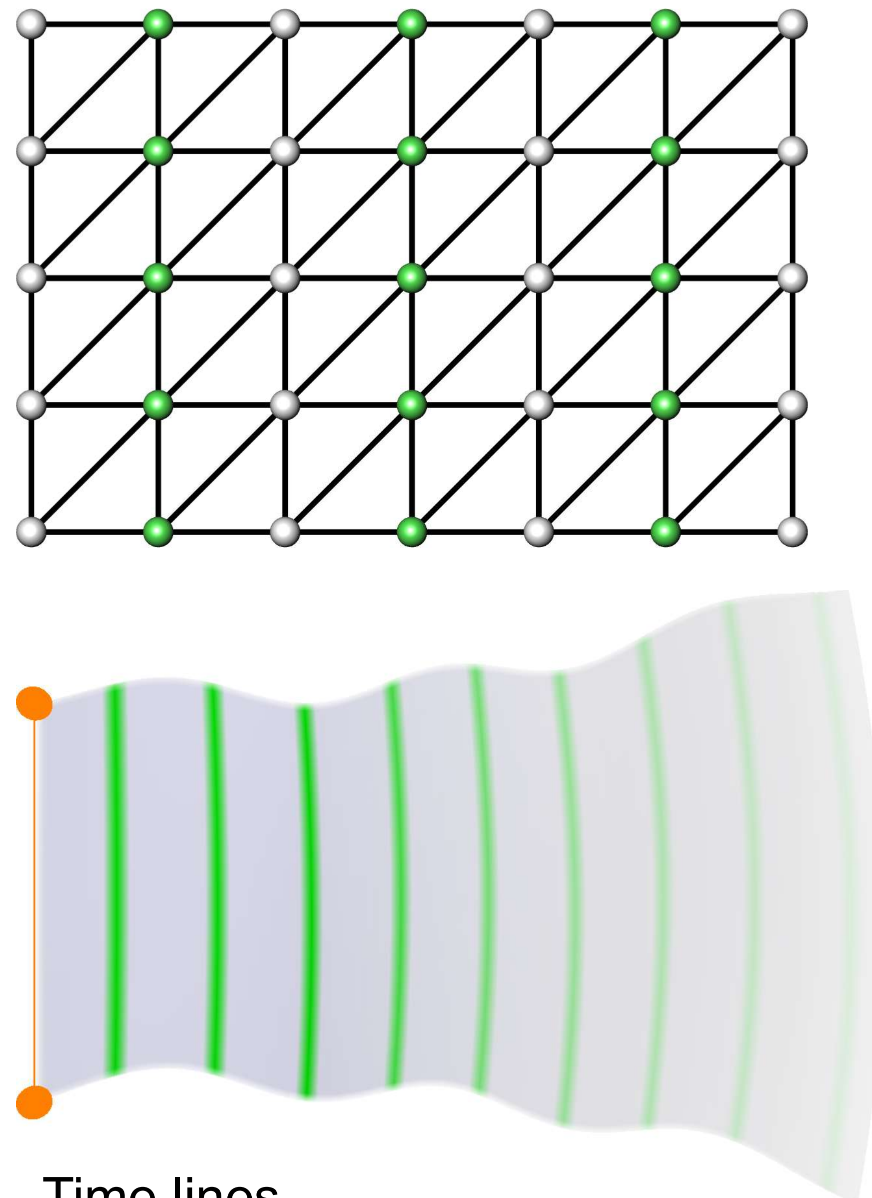
Streak Lines

Time Lines

Streak and Time Lines



Streak lines



Time lines

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama