King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 20: Vector Field / Flow Visualization, Pt.2

Markus Hadwiger, KAUST

Reading Assignment #11 (until Apr 26)

Read (required):

- Data Visualization book
  - Chapter 6 (Vector Visualization)
    - Beginning (before 6.1)
    - Chapters 6.2, 6.3, 6.5
- More general vector field basics (the book is not very precise on the basics)

  `https://en.wikipedia.org/wiki/Vector_field`

Read (optional):

- Paper:
  Bruno Jobard and Wilfrid Lefer
  Creating Evenly-Spaced Streamlines of Arbitrary Density,

  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.9498`

Next CS 247 Lectures

Lecture #21: Sunday, April 26

no lecture on April 29!

Lecture #22: Sunday, May 3

Lecture #23: Wednesday, May 6

Lecture #24: Sunday, May 10

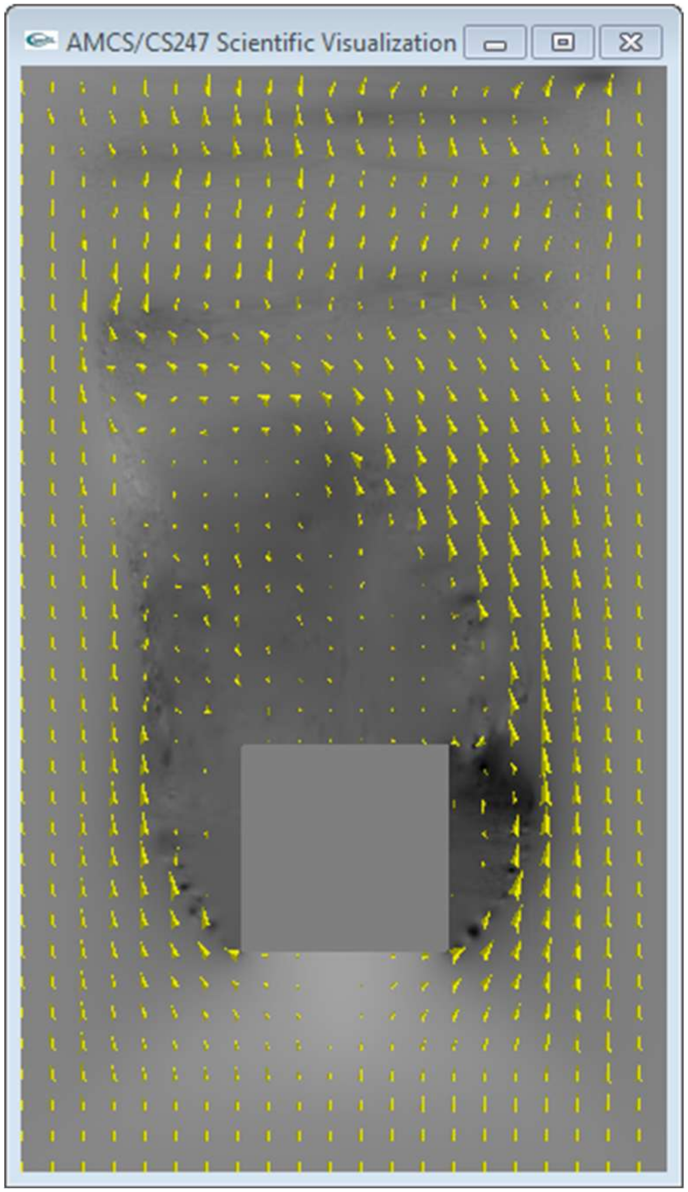
Lecture #25: Wednesday, May 13 (last lecture)

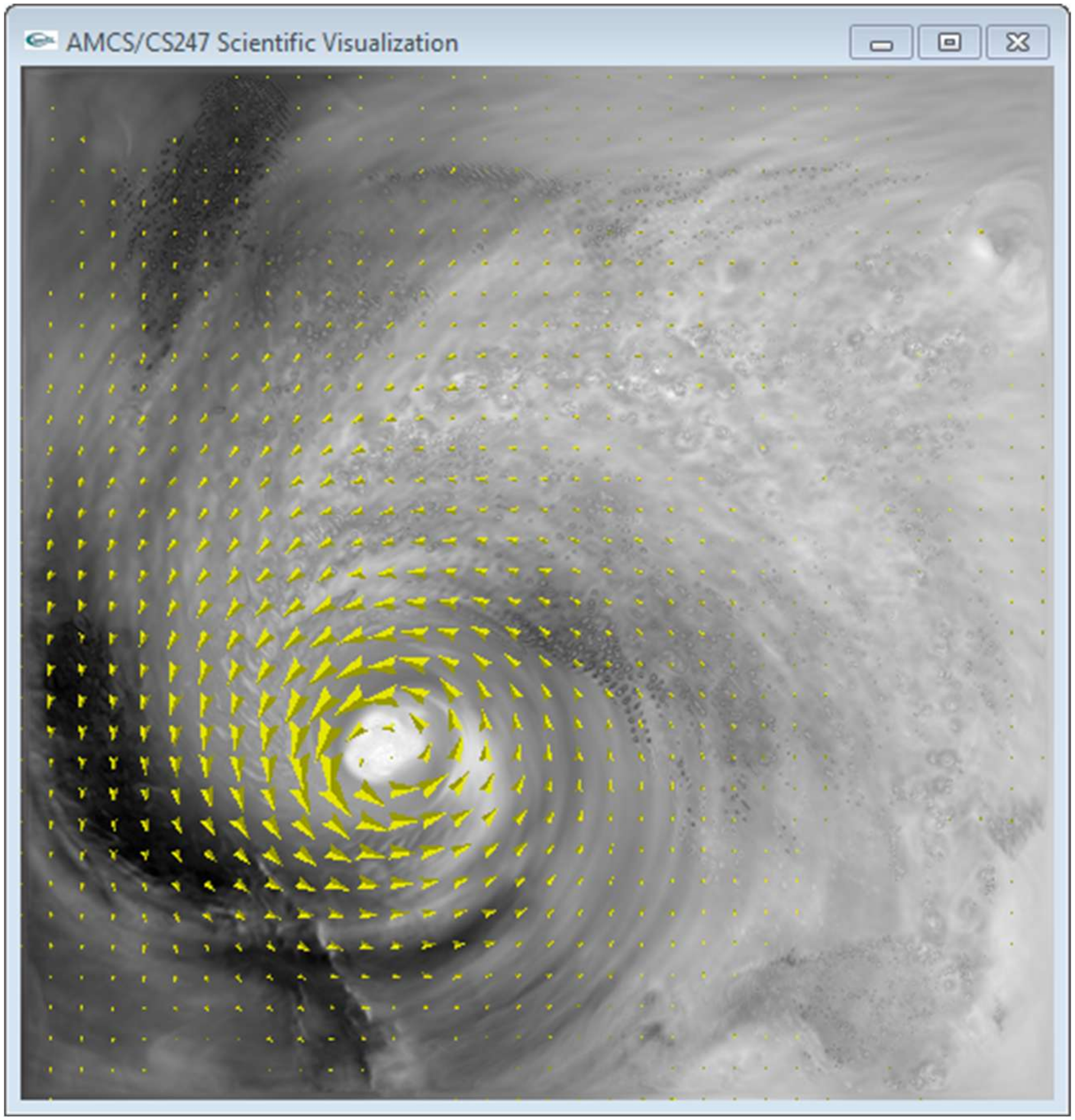
No Quiz #3 !

Programming Assignments Schedule (tentative)

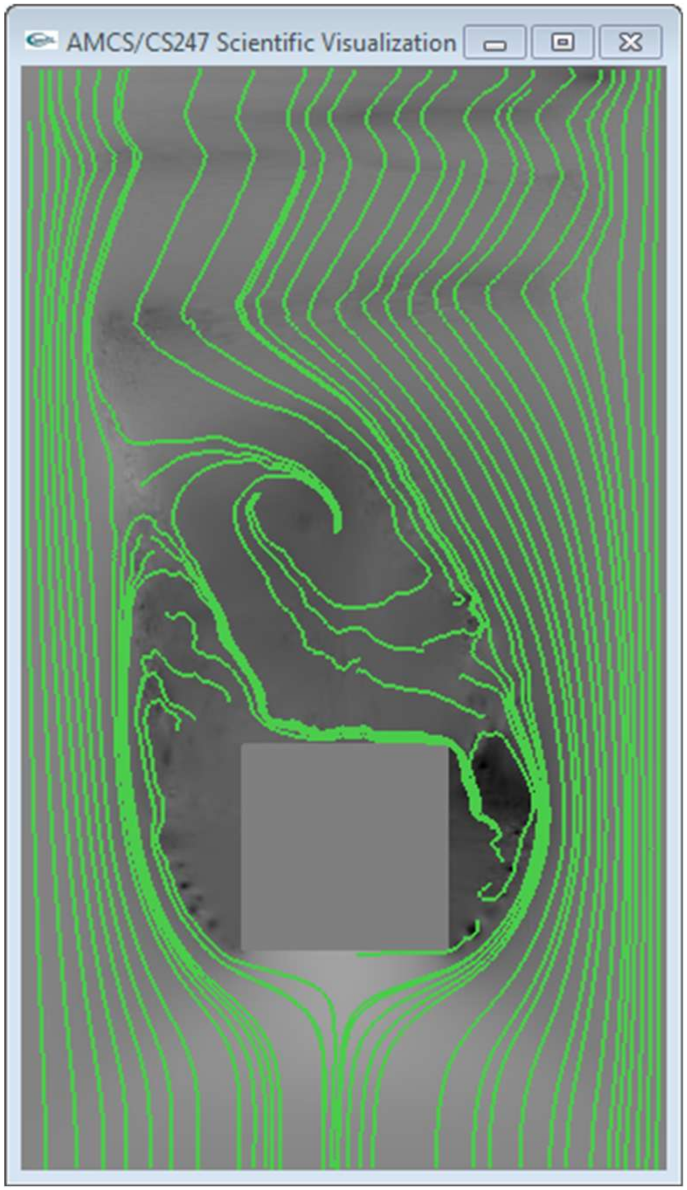
Assignment 0:	Lab sign-up: join discord, setup github account + get repo	until	**Feb 1**
	Basic OpenGL example [we will offer a tutorial!]		
Assignment 1:	Volume slice viewer	until	**Feb 15**
Assignment 2:	Iso-contours (marching squares)	until	**Mar 1**
Assignment 3:	Iso-surface rendering (marching cubes)	until	**Mar 15**
Assignment 4:	Volume ray-casting, part 1	until	**Apr 12**
	Volume ray-casting, part 2	until	**Apr 19**
Assignment 5:	Flow vis, part 1+2 (hedgehog, streamlines, pathlines, colors)	until	**May 12**
Assignment 6:	Flow vis, part 2 (LIC with color coding)	until	**May 13**

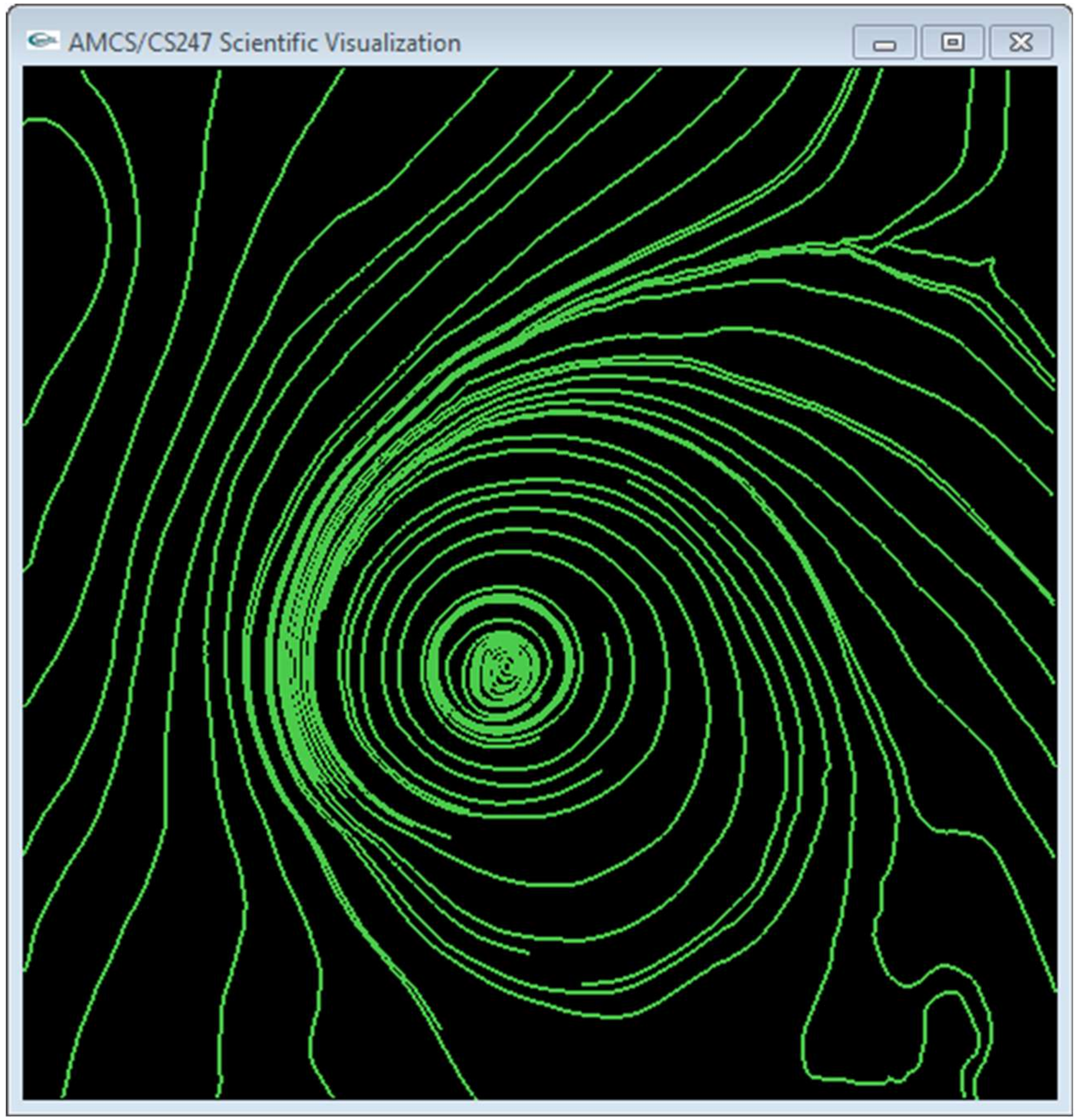
Programming Assignment #5: Flow Vis 1+2



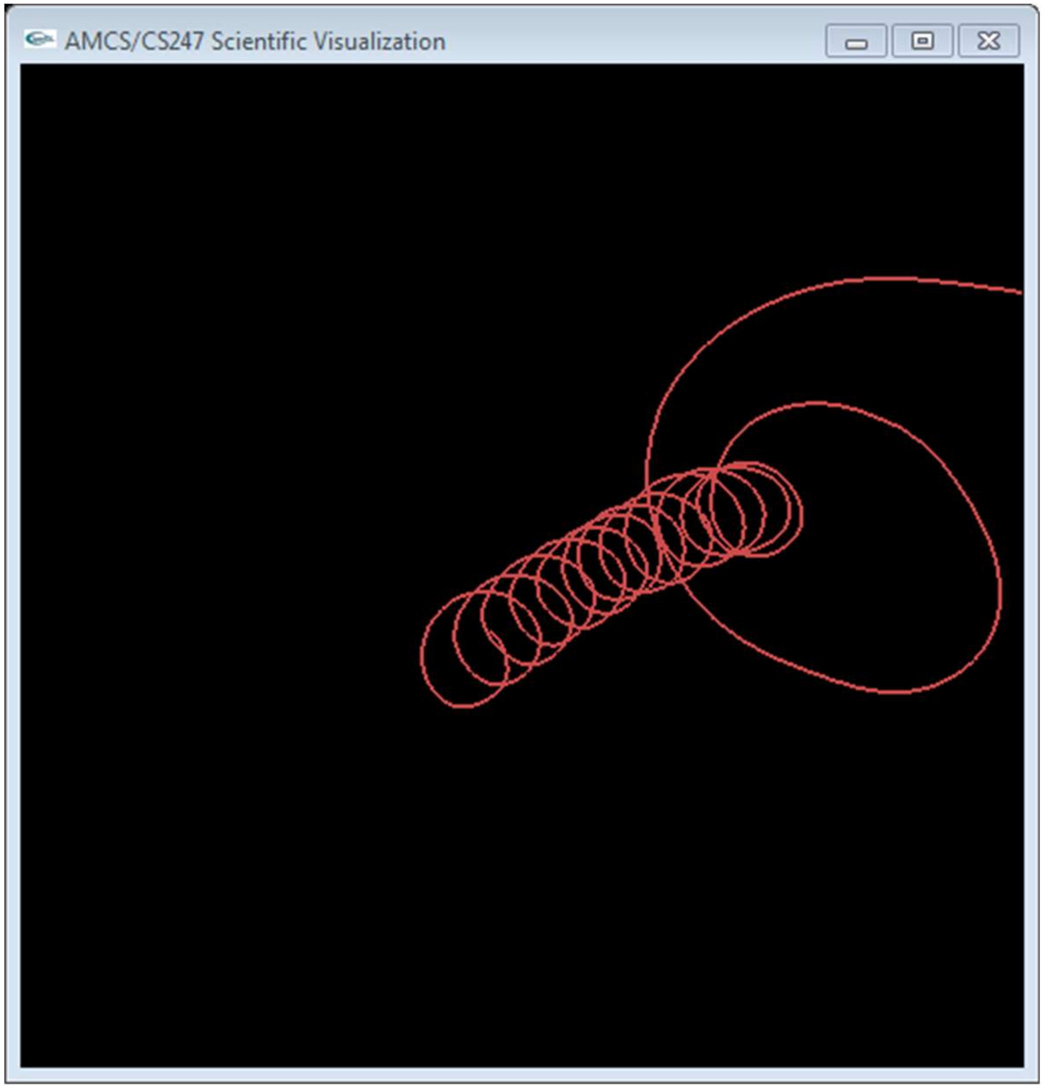


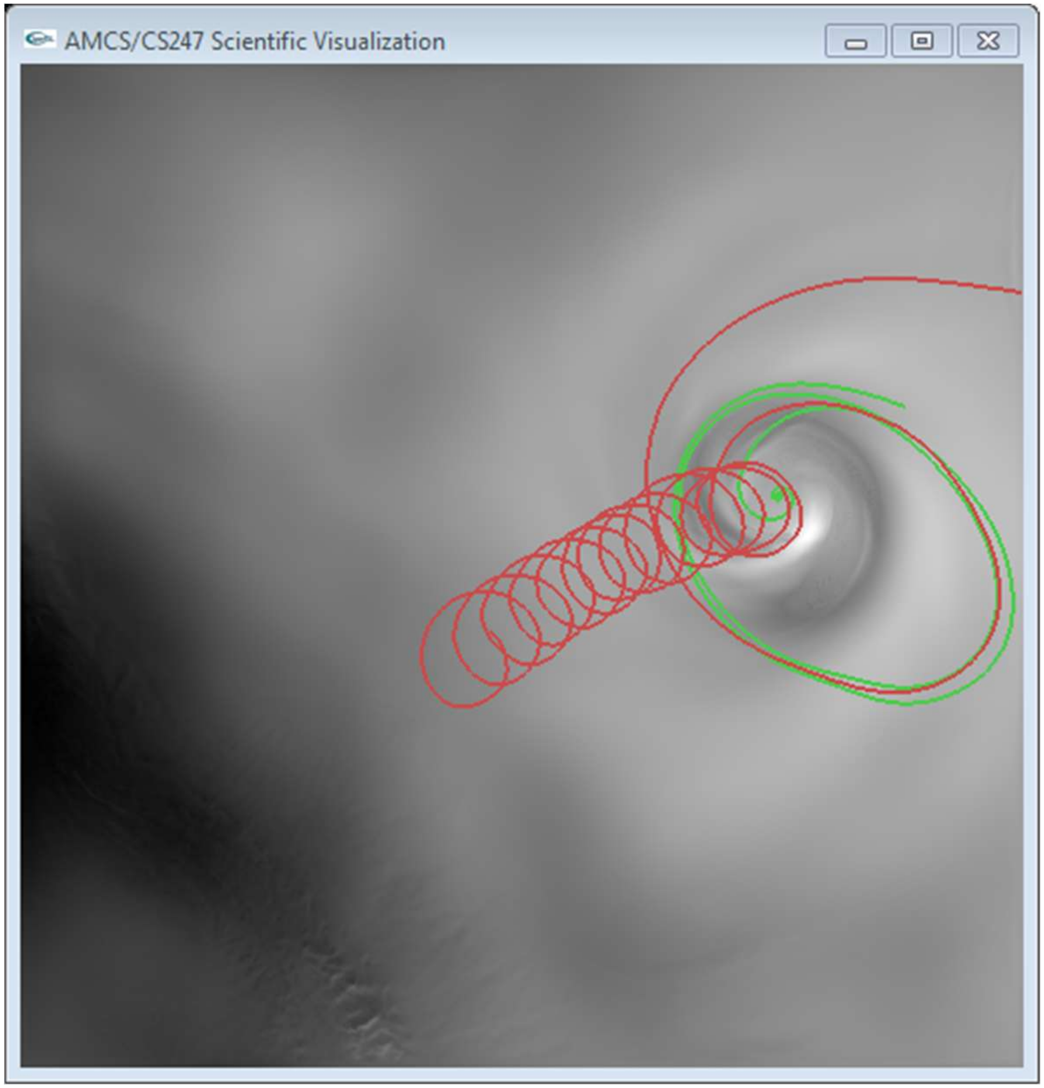
Programming Assignment #5: Flow Vis 1+2





Programming Assignment #5: Flow Vis 1+2





Vector fields

A static vector field $\mathbf{v}(\mathbf{x})$ is a vector-valued function of space.

A time-dependent vector field $\mathbf{v}(\mathbf{x},t)$ depends also on time.

In the case of velocity fields, the terms steady and unsteady flow are used.

The dimensions of **x** and **v** are equal, often 2 or 3, and we denote components by *x,y,z* and *u,v,w*:

$$\mathbf{x} = (x,y,z), \ \mathbf{v} = (u,v,w)$$

Sometimes a vector field is defined on a surface $\mathbf{x}(i,j)$. The vector field is then a function of parameters and time:

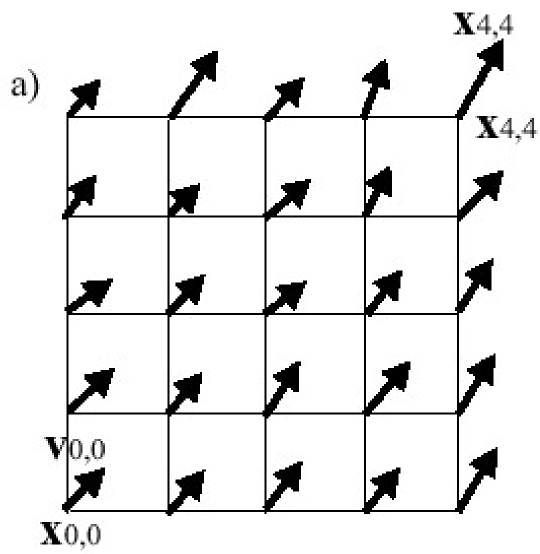
$$\mathbf{v}(i,j,t)$$

Vector Fields

Each vector is usually thought of as a velocity vector

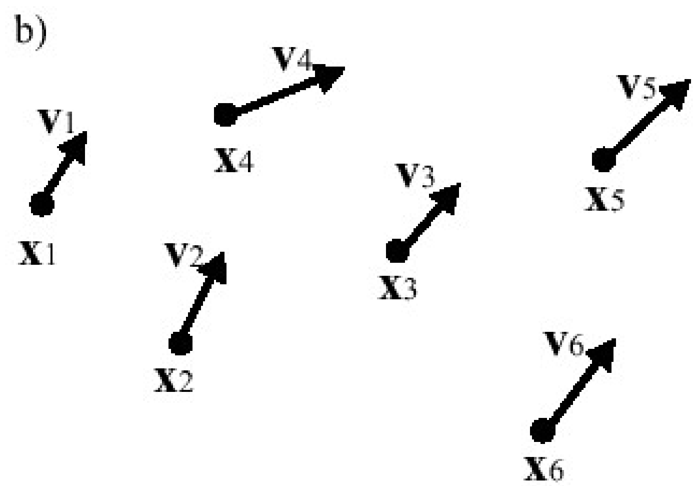
- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

Eulerian specification:



vectors given at grid points
(grid points do not move)

Lagrangian specification:



vectors given at particle positions
(particle positions do move)

Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
  - Example: laminar flows

$$\mathbf{v}: \mathbb{R}^n \to \mathbb{R}^n, \quad x \mapsto \mathbf{v}(x).$$

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x}, t)$
  - Example: turbulent flows

$$\mathbf{v}: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n, \quad (x,t) \mapsto \mathbf{v}(x,t).$$

(here just for Euclidean domain; analogous on general manifolds)

Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
  - Example: laminar flows

$$\mathbf{v}: M \to \mathbb{R}^n, \quad x \mapsto \mathbf{v}(x).$$

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x}, t)$
  - Example: turbulent flows

$$\mathbf{v}: M \times \mathbb{R} \to \mathbb{R}^n, \quad (x,t) \mapsto \mathbf{v}(x,t).$$

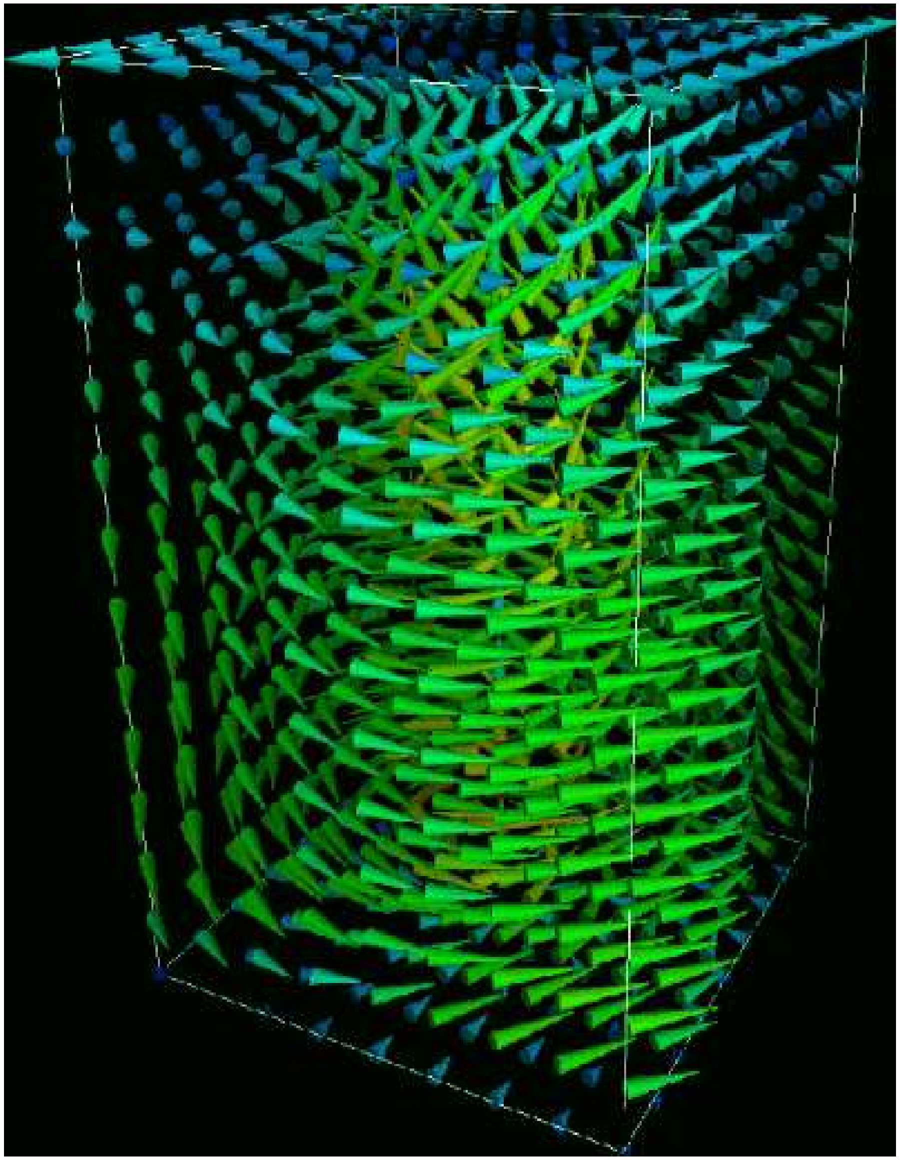
(here now for general manifolds)

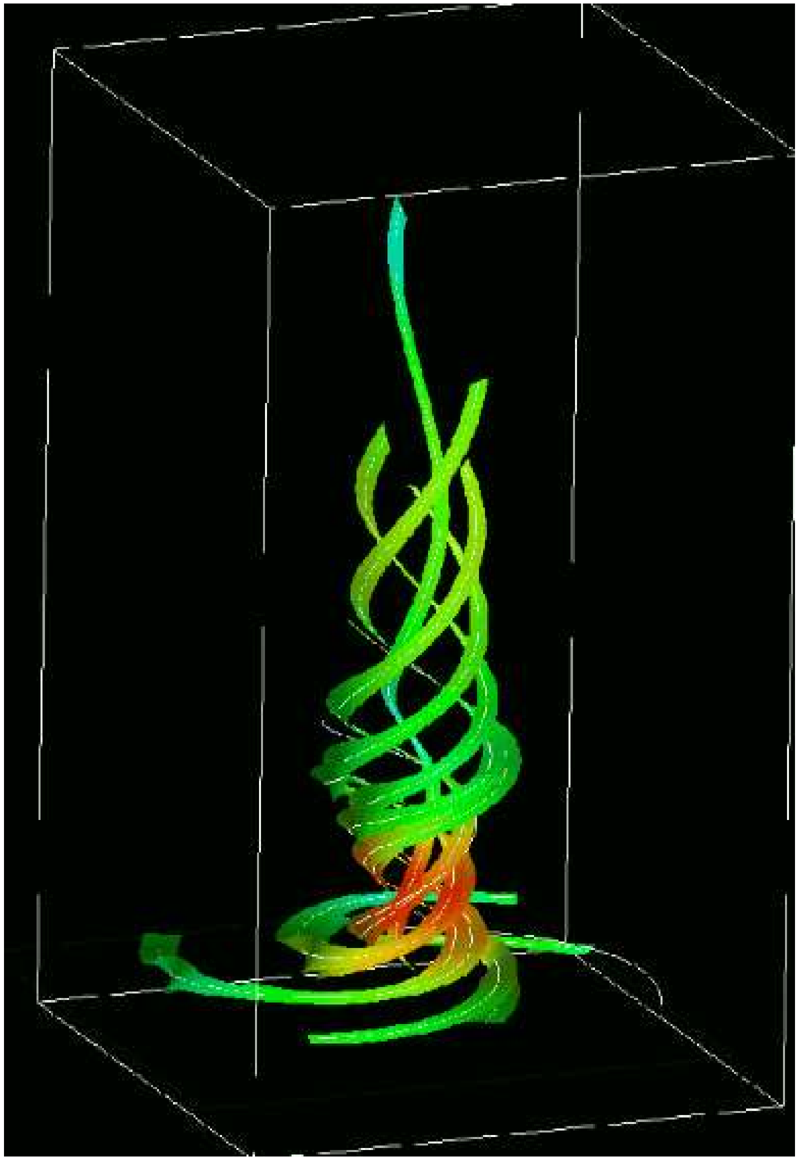
Direct vs. Indirect Flow Visualization

- Direct flow visualization
  - Overview of current flow state
  - Visualization of vectors: arrow plots (“hedgehog” plots)

- Indirect flow visualization
  - Use intermediate representation: vector field integration over time
  - Visualization of temporal evolution
  - Integral curves: streamlines, pathlines, streaklines, timelines
  - Integral surfaces: streamsurfaces, pathsurfaces, streaksurfaces

Direct vs. Indirect Flow Visualization

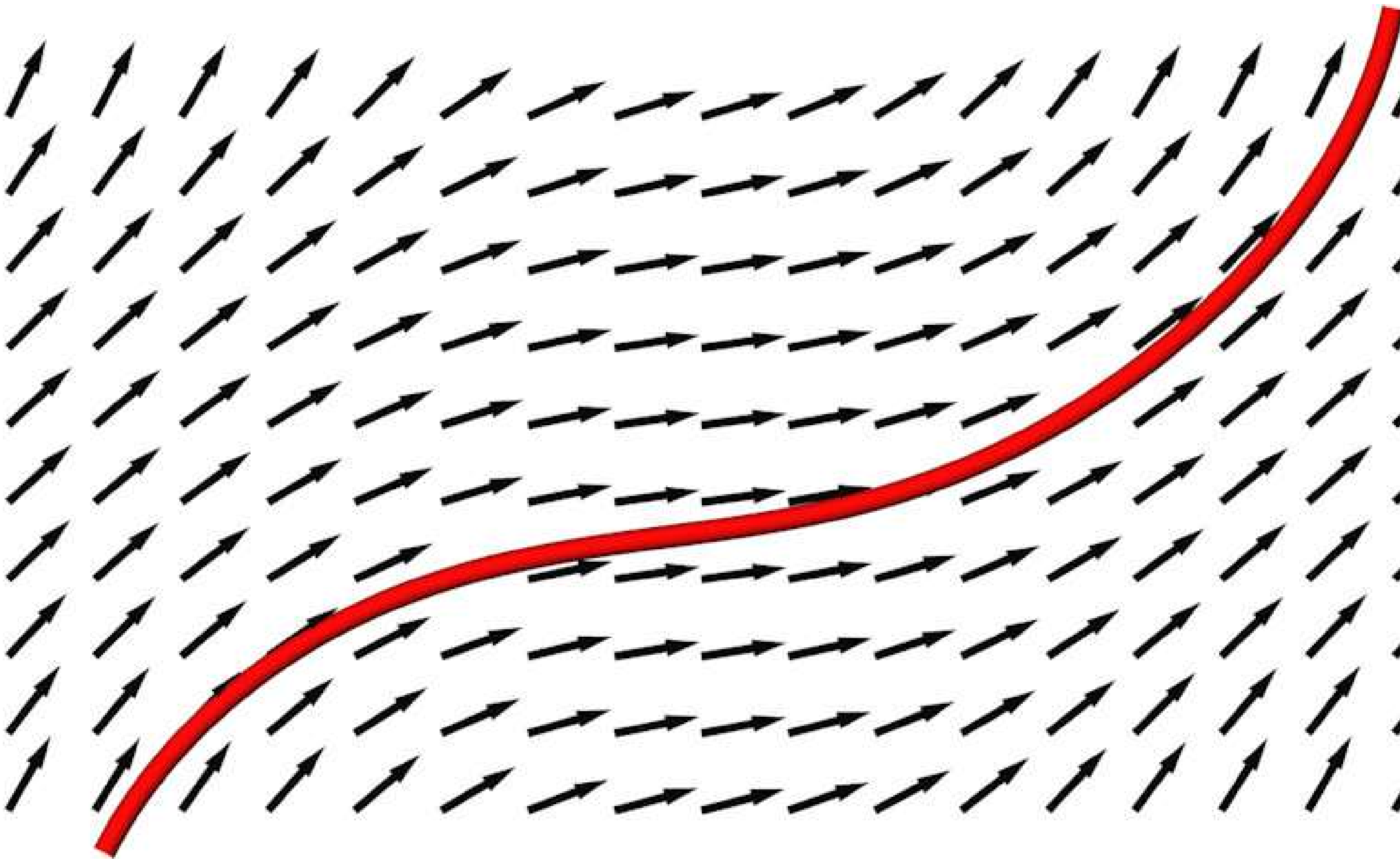




Integral Curves: Intro

Integral Curves / Stream Objects

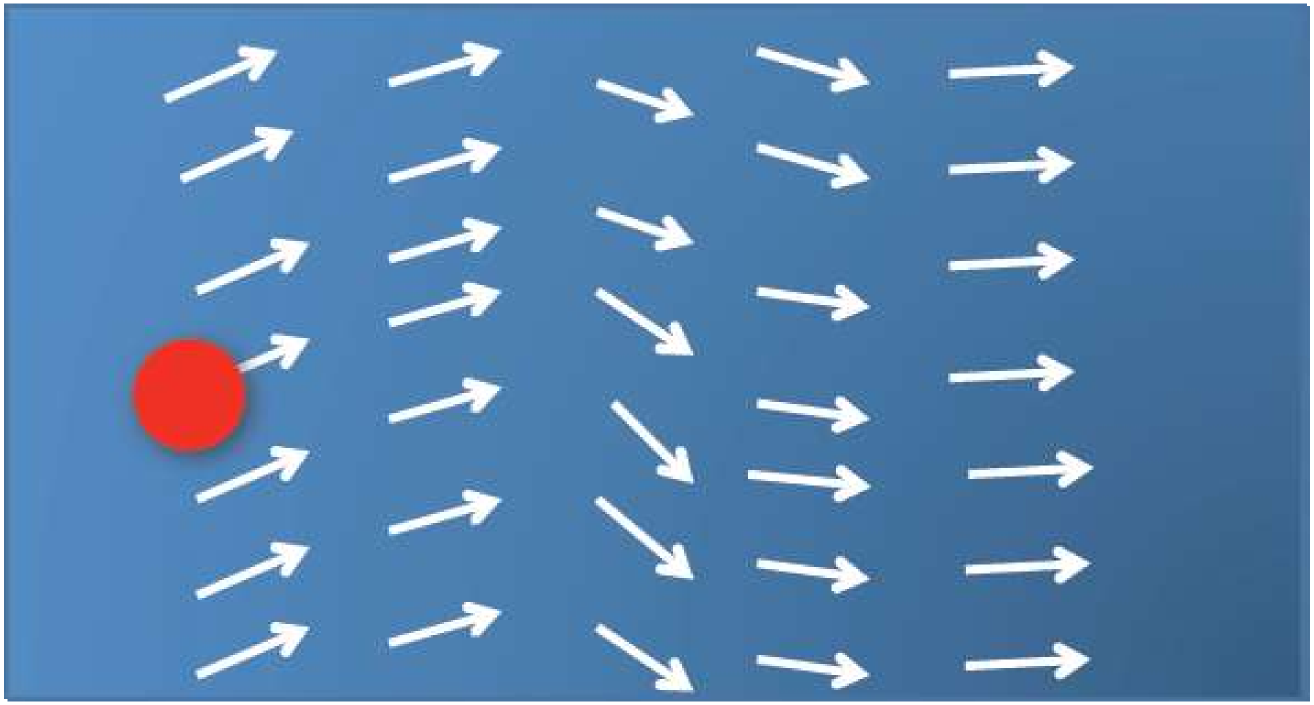
Integrating velocity over time yields spatial motion



Particle Trajectories

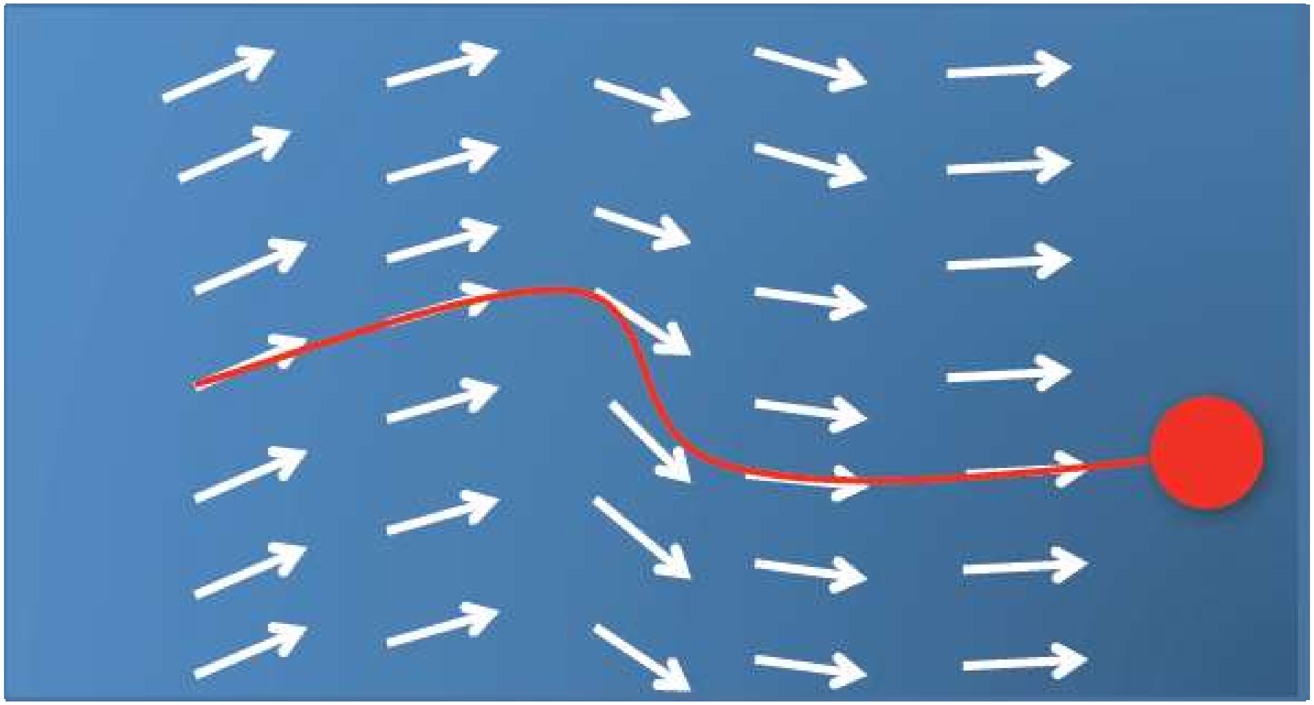
Courtesy Jens Krüger

Particle Trajectories



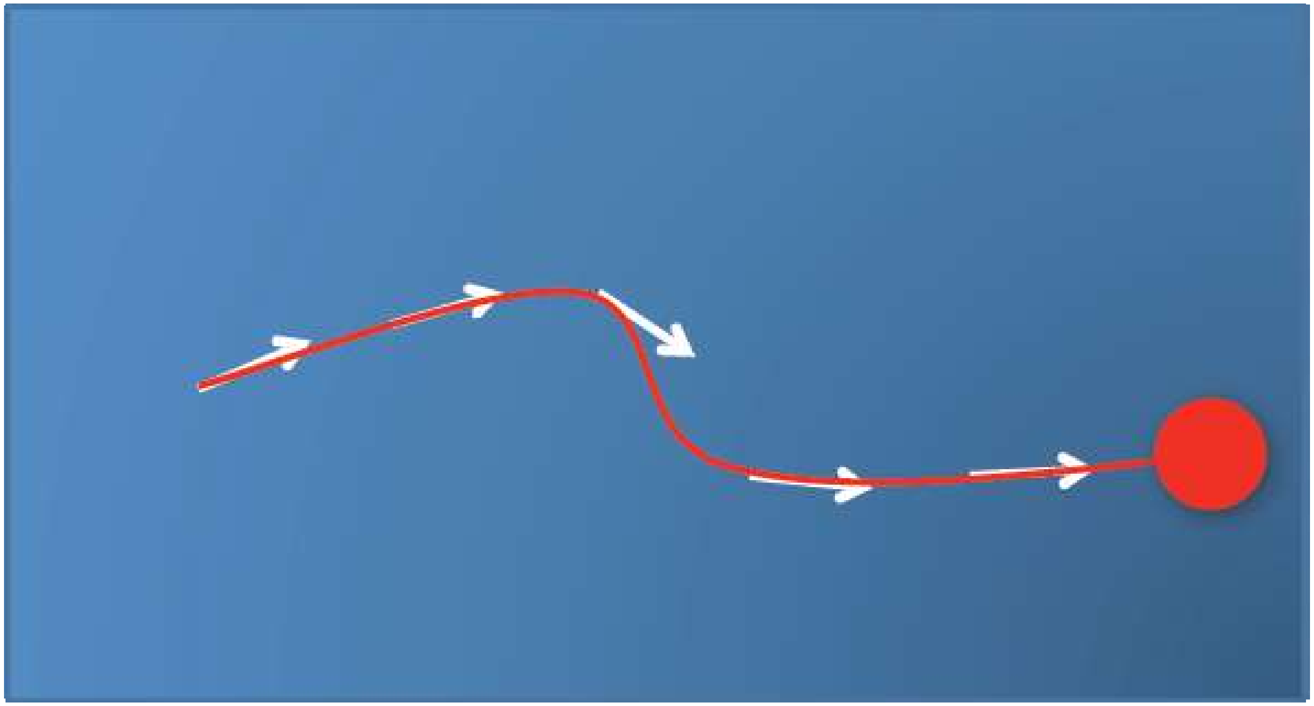
Courtesy Jens Krüger

Particle Trajectories



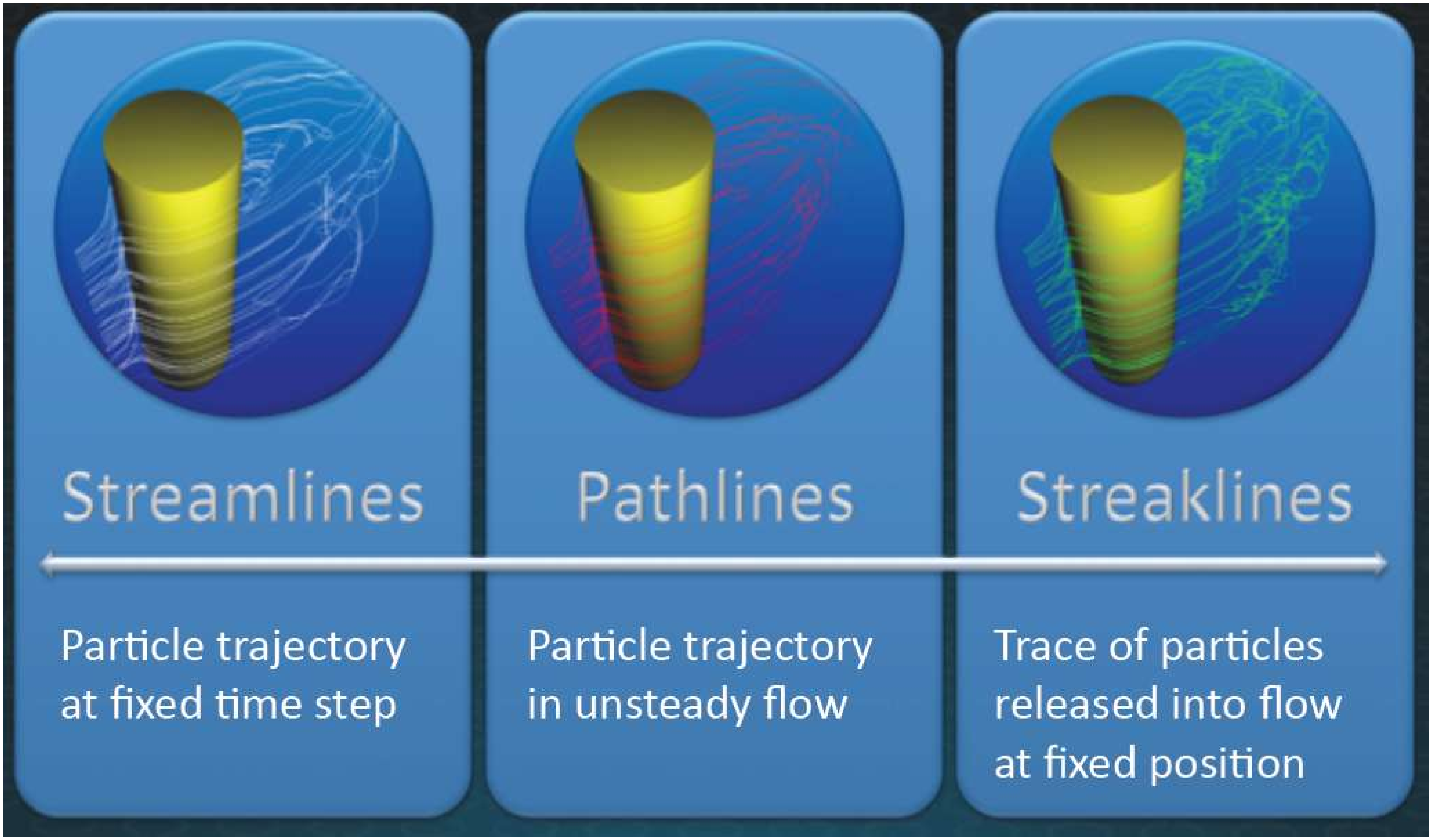
Courtesy Jens Krüger

Particle Trajectories



Courtesy Jens Krüger

Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

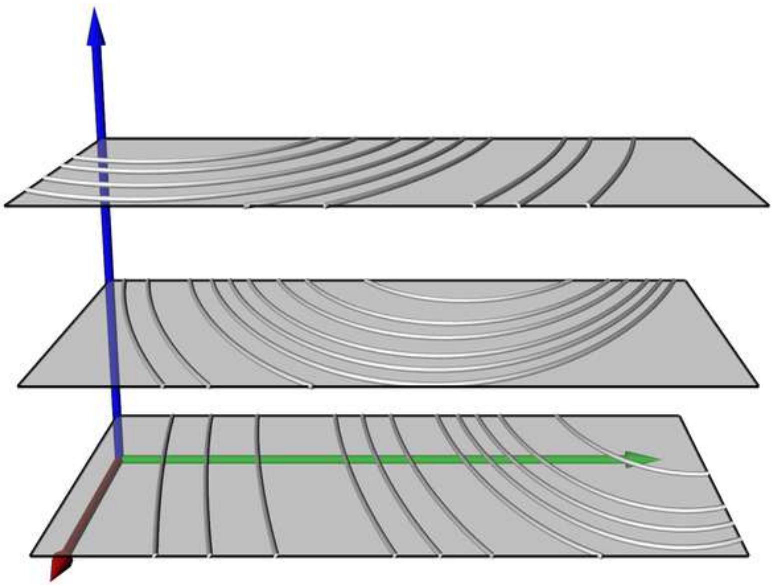
- Describes motion of a massless particle over time

Streakline

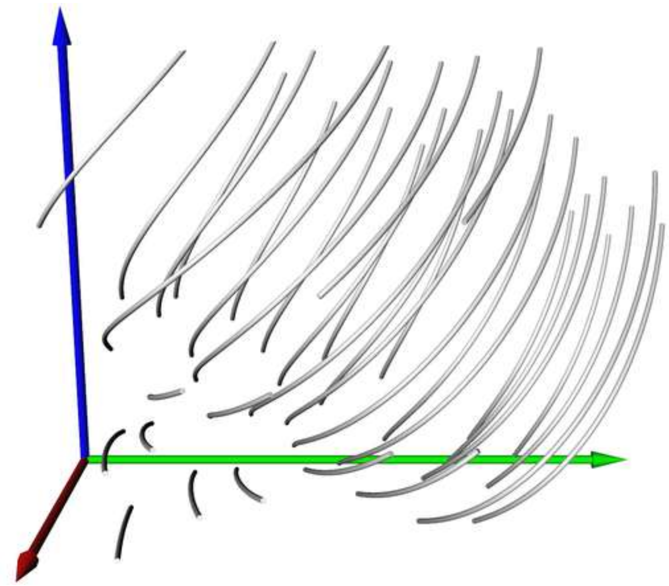
- Location of all particles released at a *fixed position* over time

Timeline

- Location of all particles released along a line at a *fixed time*

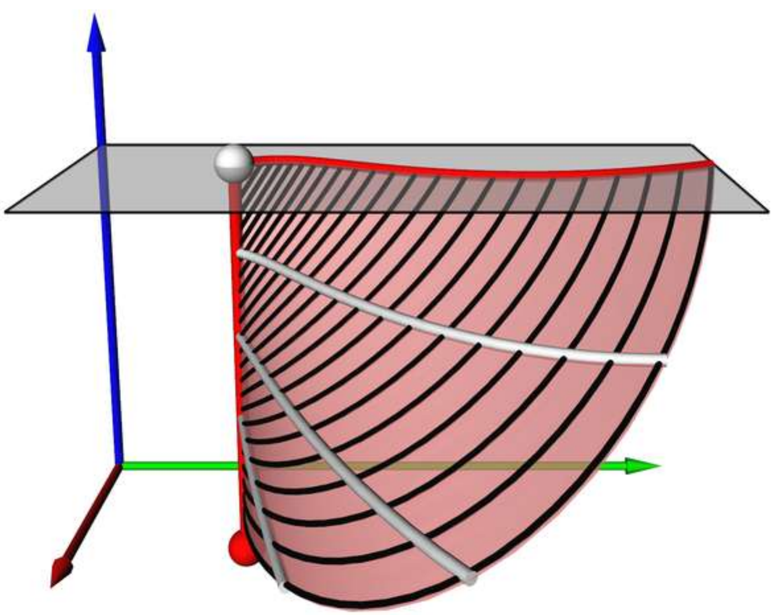


stream lines

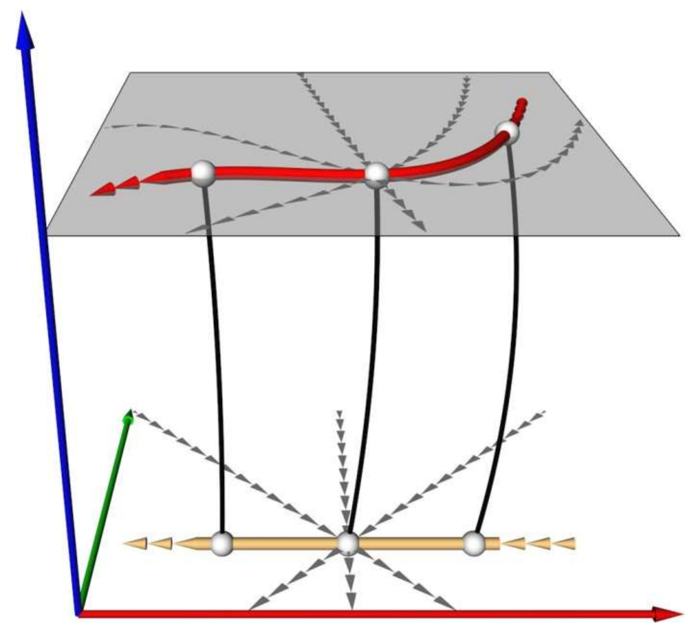


path lines

streak lines



time lines



Comparison of techniques:

(1) Pathlines:

- are physically meaningful
- allow comparison with experiment (observe marked particles)
- are well suited for dynamic visualization (of particles)

(2) Streamlines:

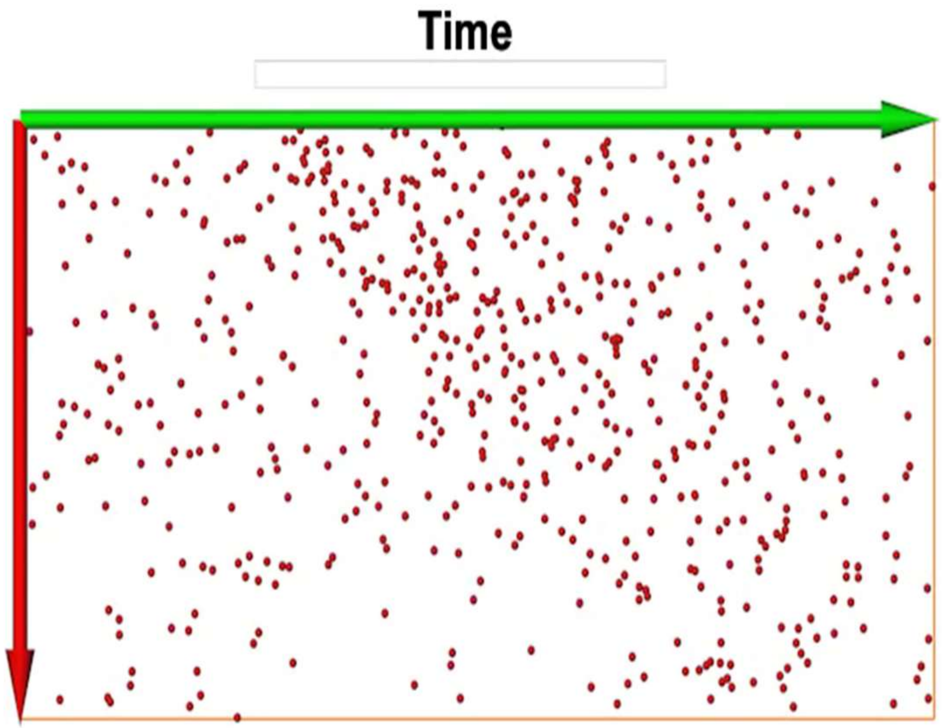
- are only geometrically, not physically meaningful
- are easiest to compute (no temporal interpolation, single IVP)
- are better suited for static visualization (prints)
- don't intersect (under reasonable assumptions)

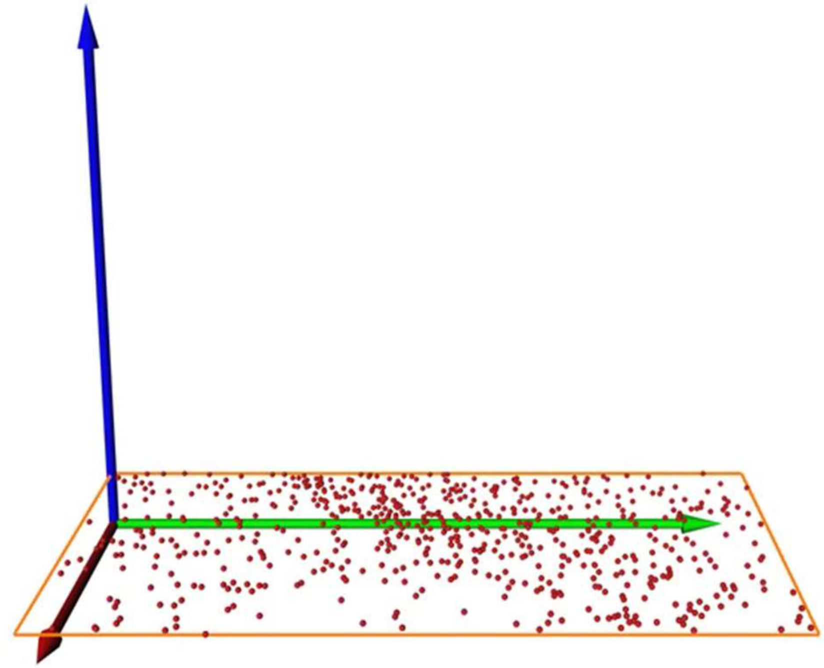
(3) Streaklines:

- are physically meaningful
- allow comparison with experiment (dye injection)
- are well suited for static and dynamic visualization
- good choice for fast moving vortices
- can be approximated by set of disconnected particles

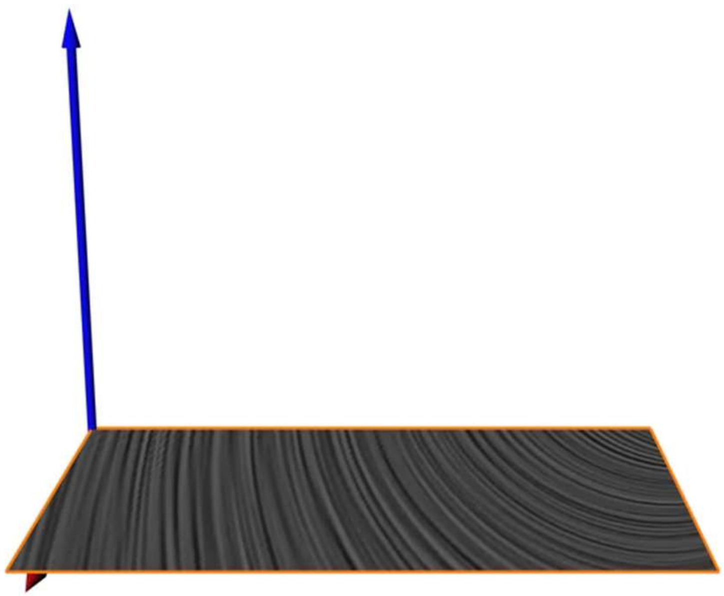
(4) Timelines:

- are physically meaningful
- are well suited for static and dynamic visualization
- can be approximated by set of disconnected particles

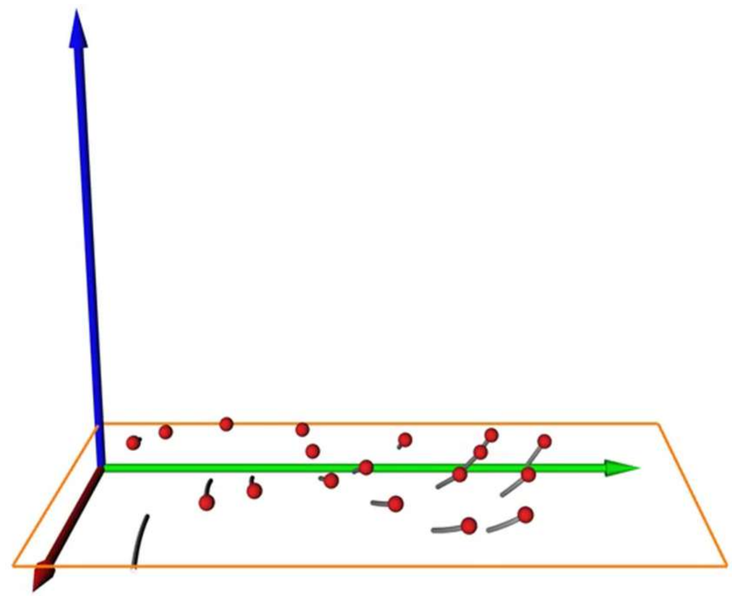




2D time-dependent vector field
particle visualization



stream lines



path lines

curve parallel to the vector field in each point for a **fixed time**

describes motion of a massless particle in an **steady** flow field

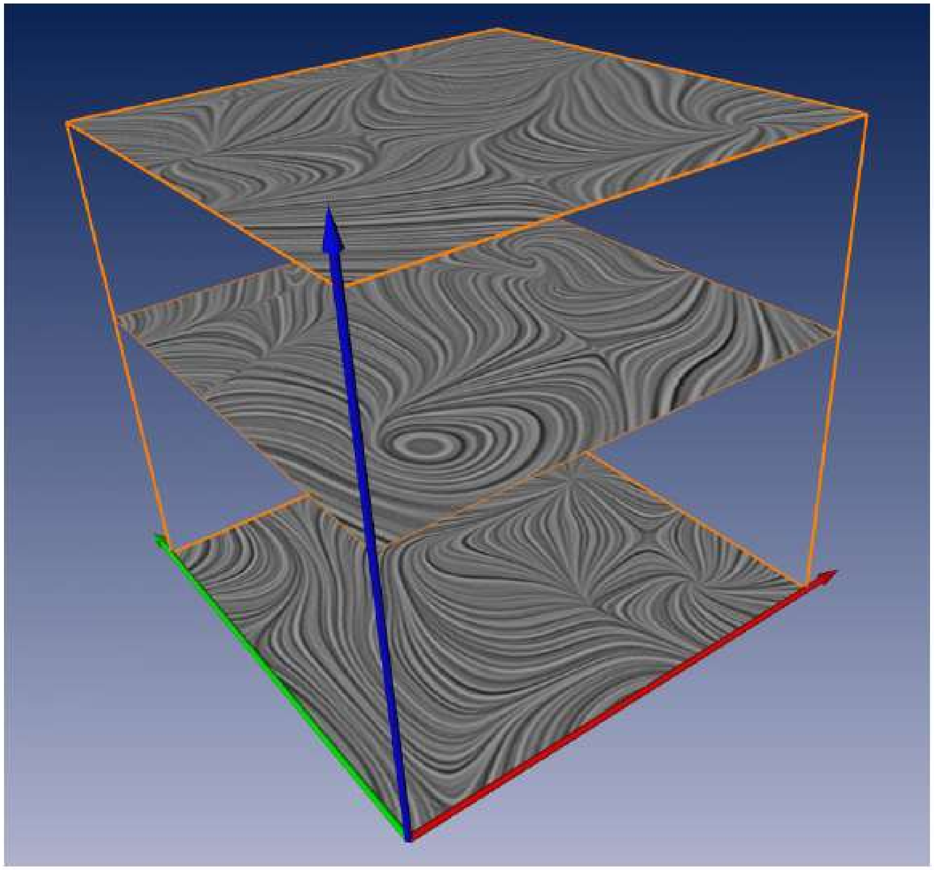
curve parallel to the vector field in each point **over time**

describes motion of a massless particle in an **unsteady** flow field

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

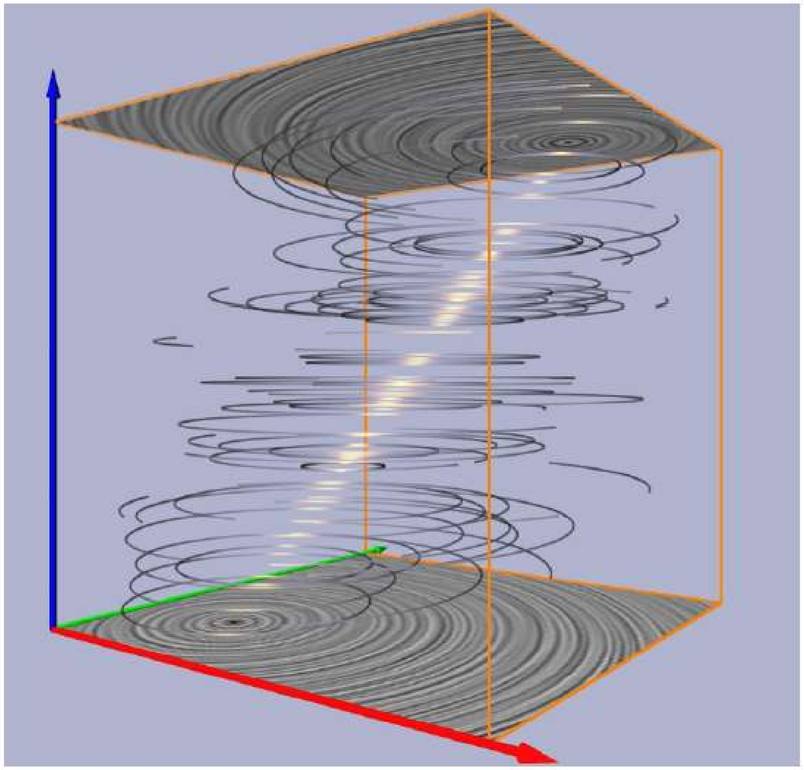


Tino Weinkauf

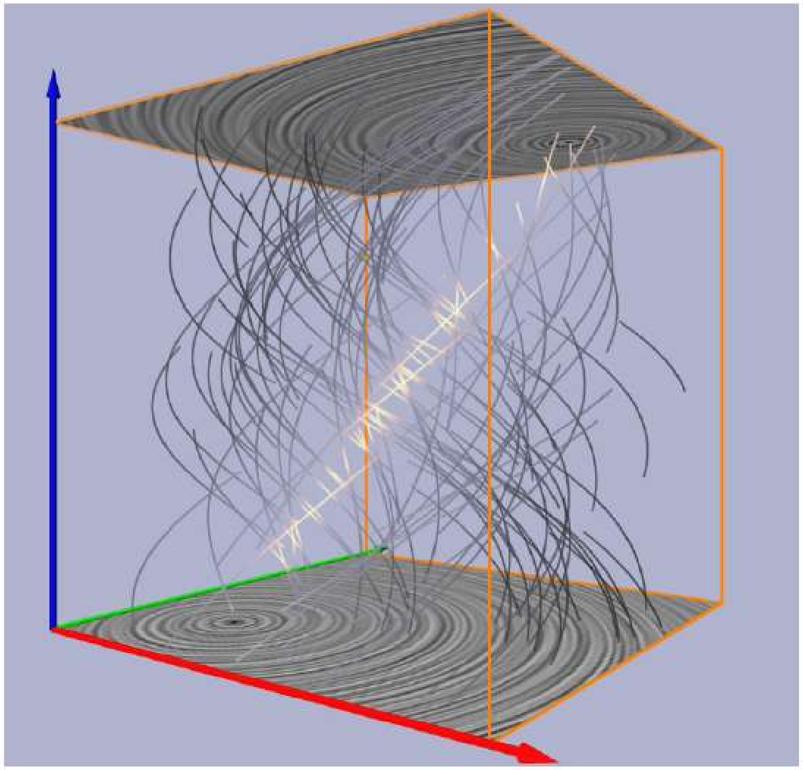
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Vector fields as ODEs

For simplicity, the vector field is now interpreted as a velocity field.

Then the field $\mathbf{v}(\mathbf{x}, t)$ describes the connection between location and velocity of a (massless) particle.

It can equivalently be expressed as an ordinary differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t), t)$$

This ODE, together with an initial condition

$$\mathbf{x}(t_0) = \mathbf{x}_0 \quad ,$$

is a so-called initial value problem (IVP).

Its solution is the integral curve (or trajectory)

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau), \tau)\, d\tau$$

The integral curve is a pathline, describing the path of a massless particle which was released at time t_0 at position x_0.

Remark: $t < t_0$ is allowed.

For static fields, the ODE is autonomous:

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}(t))$$

and its integral curves

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{v}(\mathbf{x}(\tau))\, d\tau$$

are called field lines, or (in the case of velocity fields) streamlines.

In static vector fields, pathlines and streamlines are identical.

In time-dependent vector fields, instantaneous streamlines can be computed from a "snapshot" at a fixed time T (which is a static vector field)

$$\mathbf{v}_T(\mathbf{x}) = \mathbf{v}(\mathbf{x}, T)$$

In practice, time-dependent fields are often given as a dataset per time step. Each dataset is then a snapshot.

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama