King Abdullah University of Science and Technology

KAUST

CS 247 – Scientific Visualization
Lecture 15: Volume Rendering, Pt. 2

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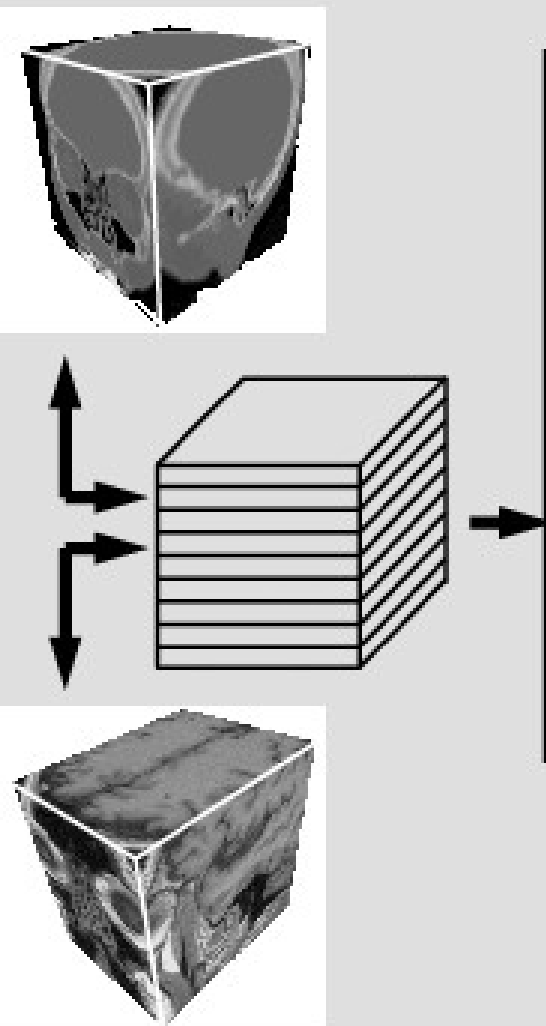
Reading Assignment #8 (until Apr 5)

Read (required):

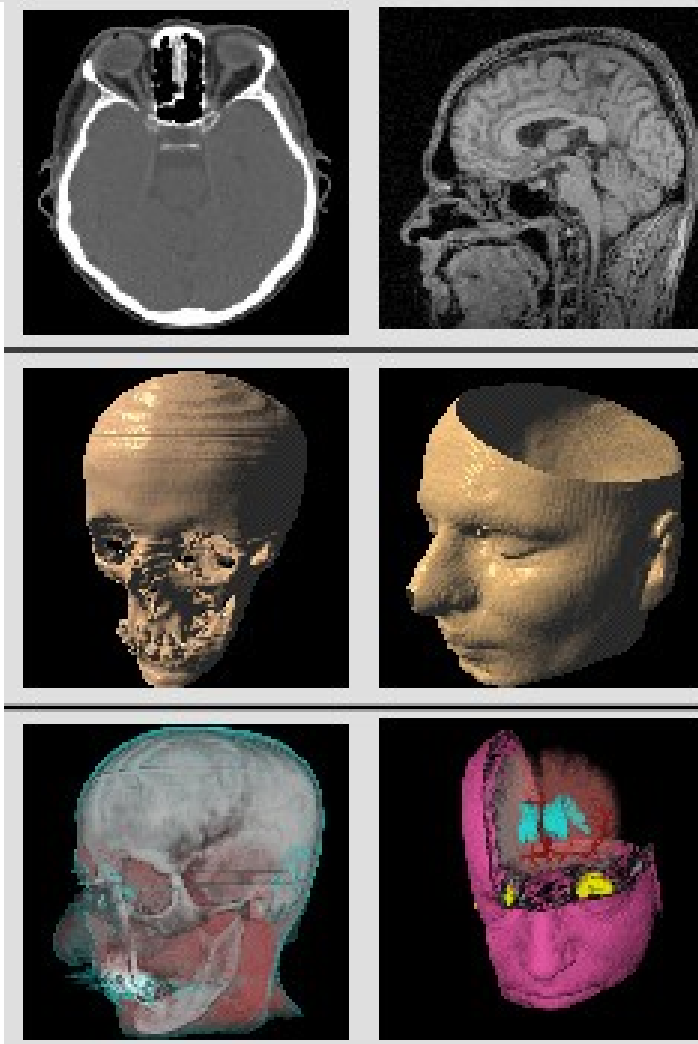
- Real-Time Volume Graphics, Chapter 7 (GPU-Based Ray Casting)
- Real-Time Volume Graphics, Chapter 4.5 – 4.8

Theory

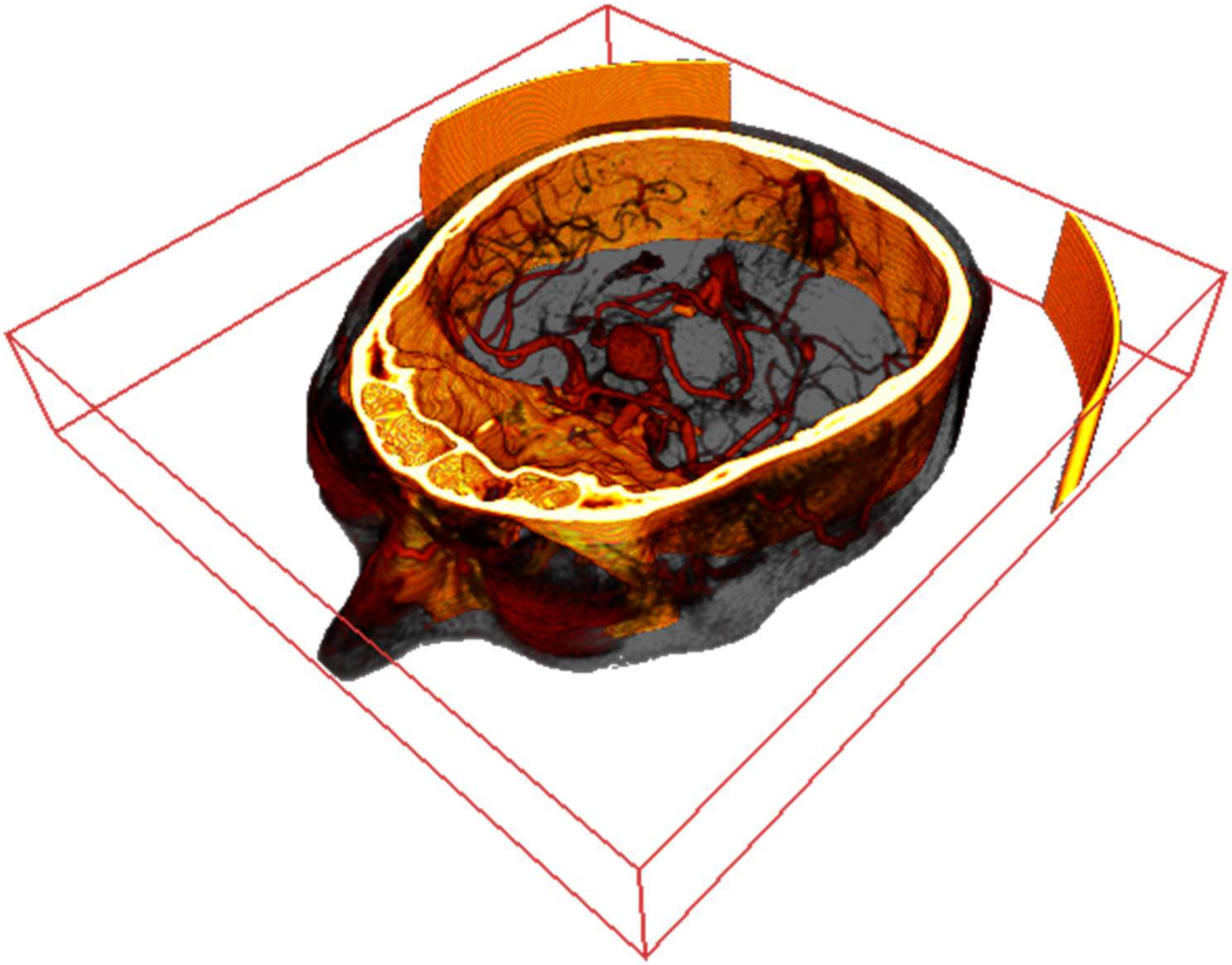
Volume Visualization



- 2D visualization
  slice images
  (or multi-planar
  reformatting MPR)
- *Indirect*
  3D visualization
  isosurfaces
  (or surface-shaded
  display: SSD)
- *Direct*
  3D visualization
  (direct volume
  rendering: DVR)

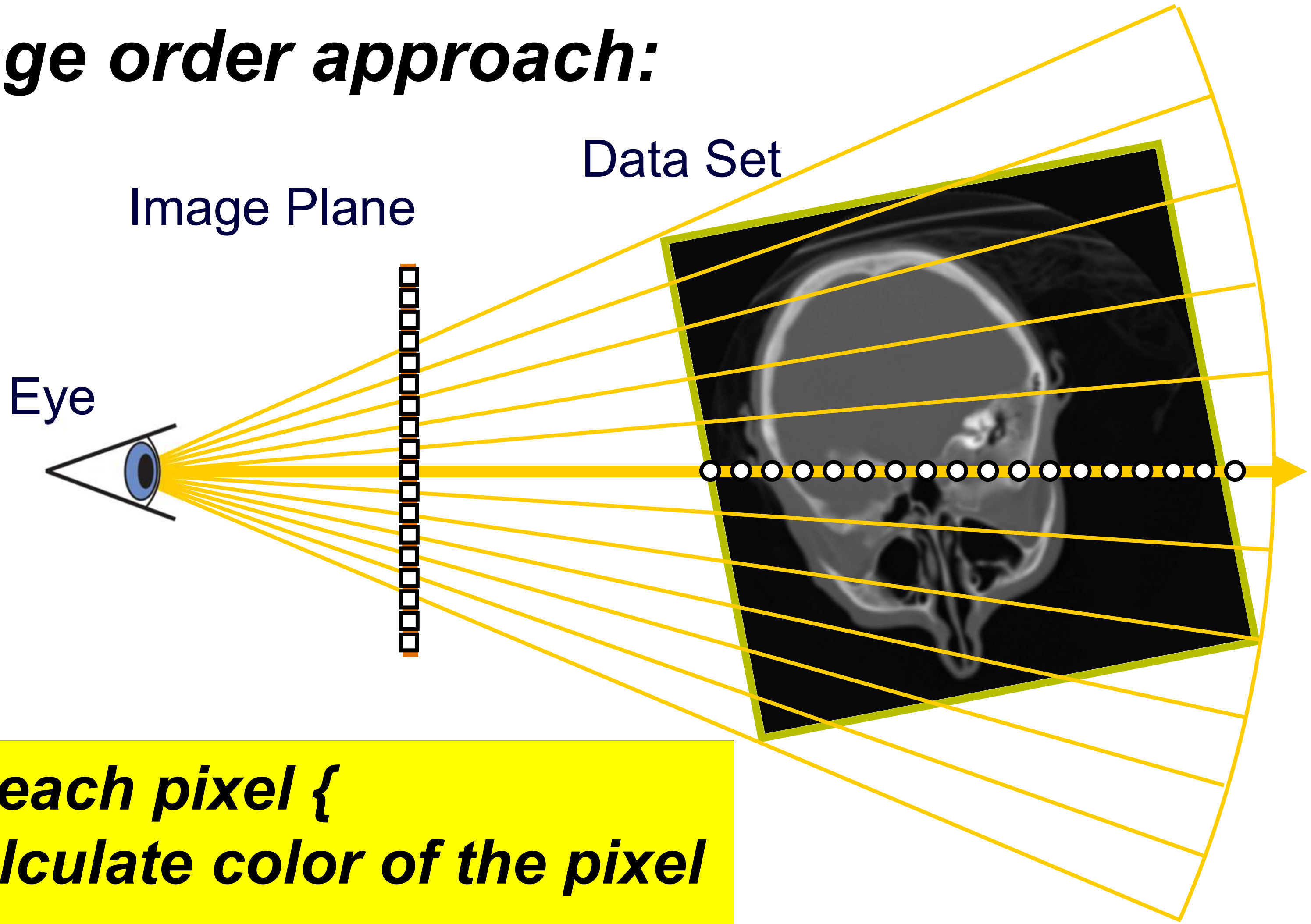


Direct Volume Rendering



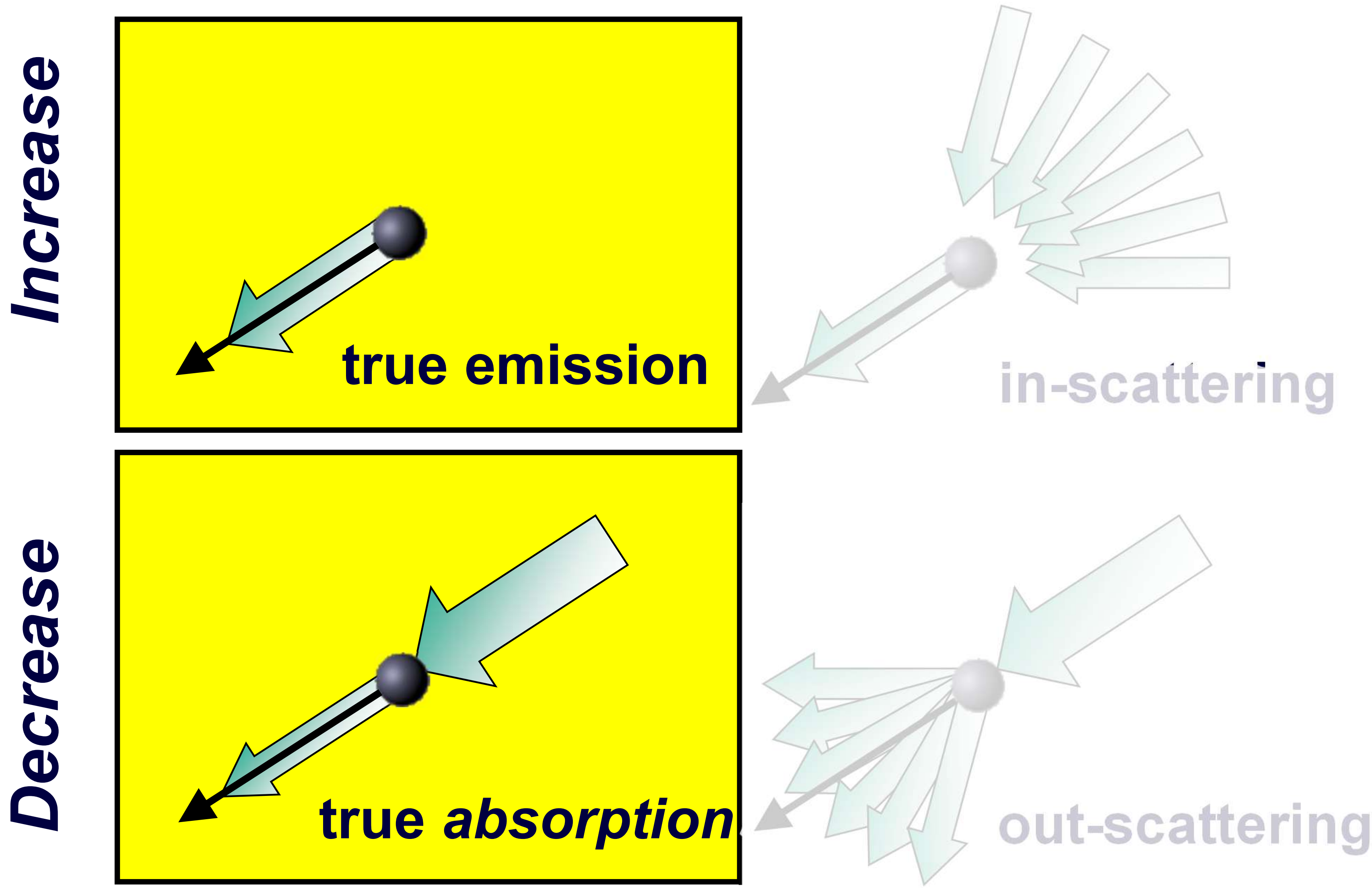
Direct Volume Rendering

Image order approach:



For each pixel {
calculate color of the pixel
}

Physical Model of Radiative Transfer



Optical Models: Physical Model gives ODE

Optical Models for Direct Volume Rendering, Nelson Max

Emission-Absorption optical model

$$\frac{dI}{ds}(s) = q(s) - \kappa(s) I(s)$$

Right-hand side: *Rates of change* (derivatives) of light intensity along ray

Absorption rate is proportional to light intensity: Solution is exponential

Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model

true emission **true *absorption***

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) \;=\; \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

Back-to-front compositing

$$C'_i \;=\; C_i + (1 - A_i)C'_{i-1}$$

here, all colors are *associated* colors!

Front-to-back compositing

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1})A_i$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

Initial intensity at s_0

$$I(s) = I(s_0)$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

Initial intensity at s_0

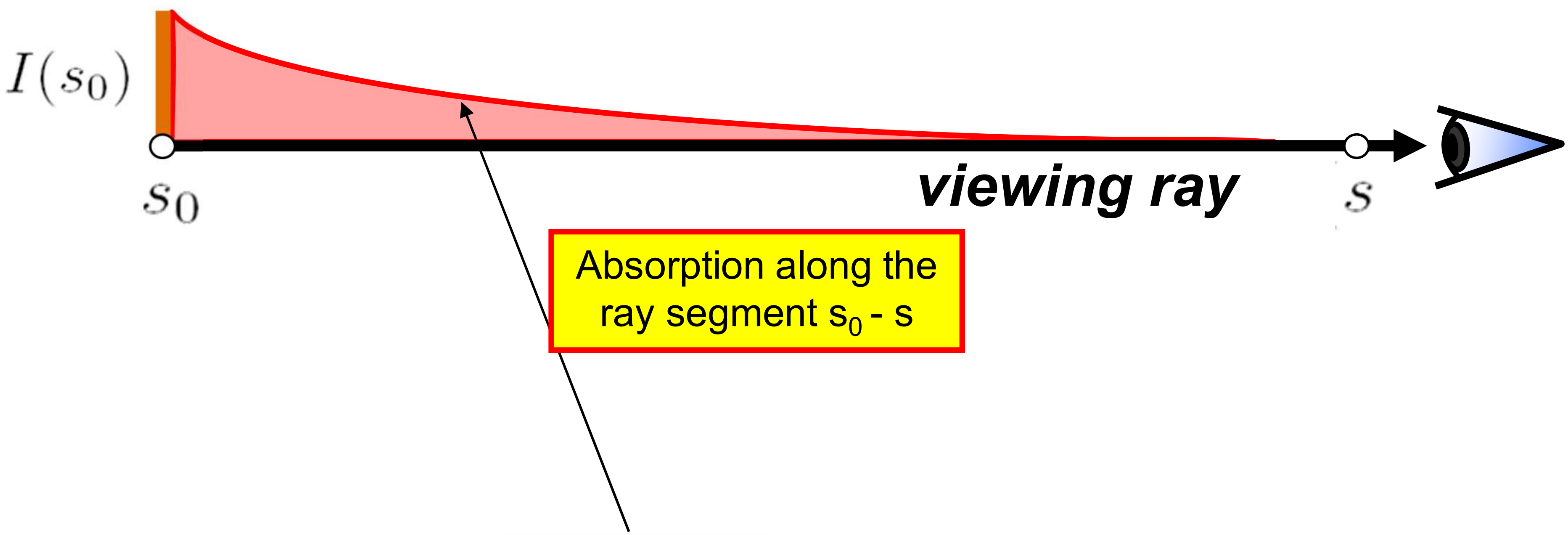
$$I(s) = \boxed{I(s_0)}$$

Without absorption all the initial radiant energy would reach the point ***s.***

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

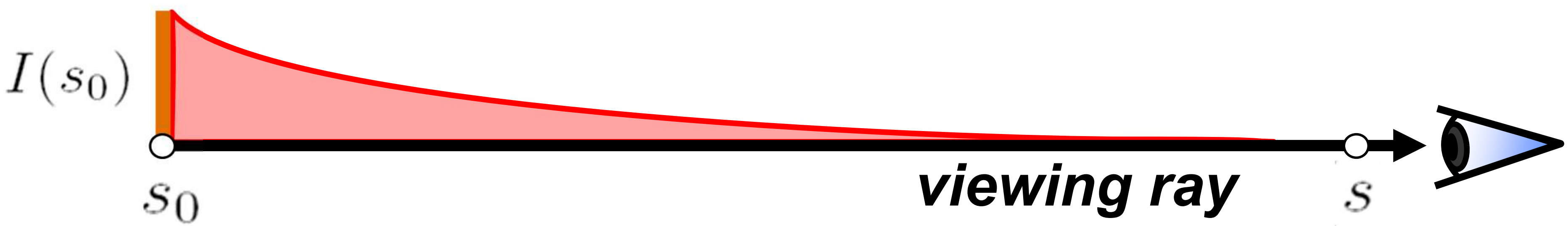


$$I(s) = I(s_0)\boxed{e^{-\tau(s_0,s)}}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

Optical depth τ

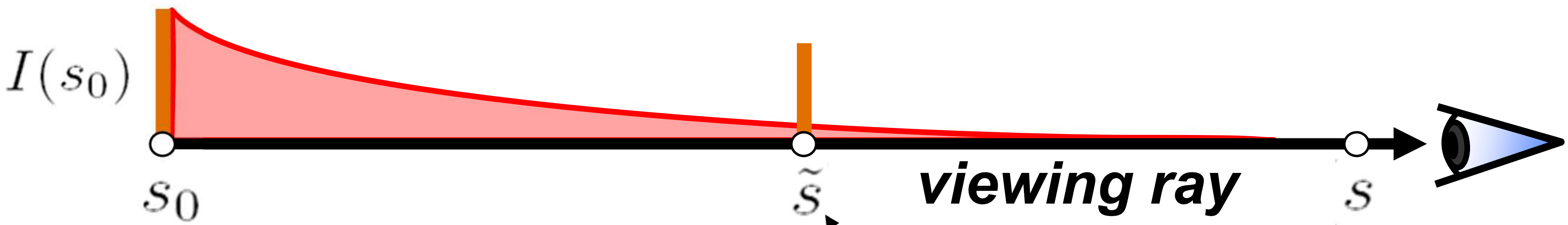
Absorption κ

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

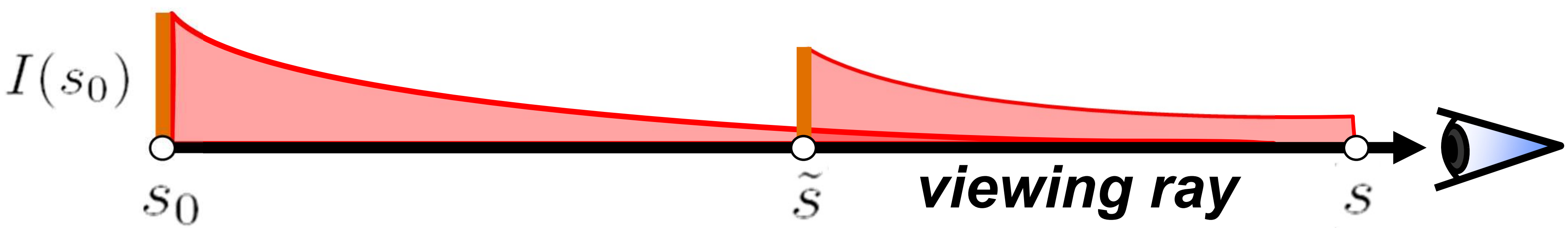
Active emission at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)} + q(\tilde{s})$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

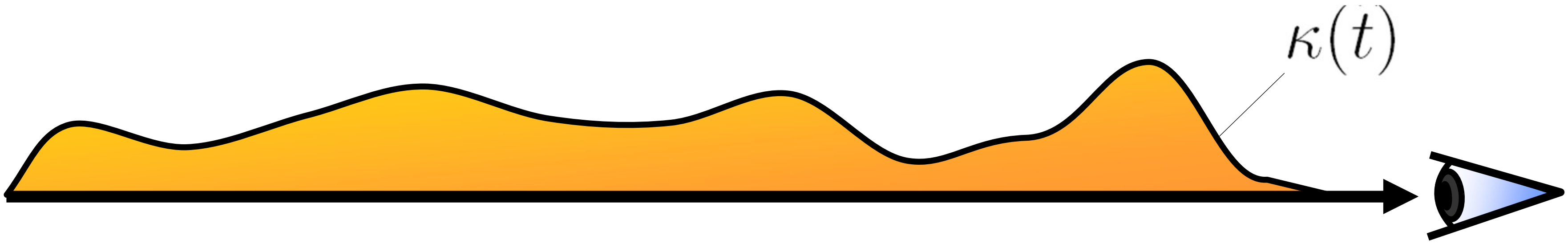
Physical model: emission and absorption, no scattering



Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

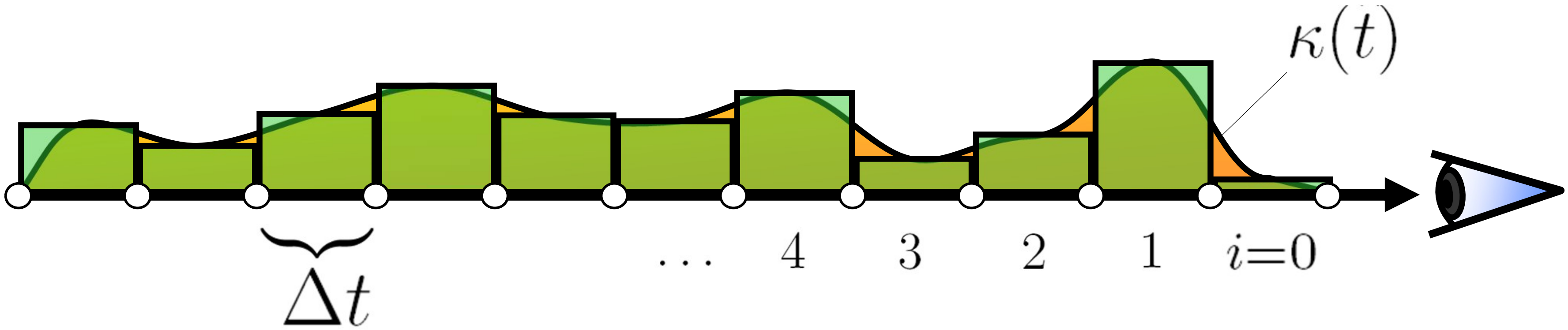
$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0,t) \;=\; \int_0^t \kappa(\hat{t})\, d\hat{t}$

Volume Rendering Integral: Numerical Solution

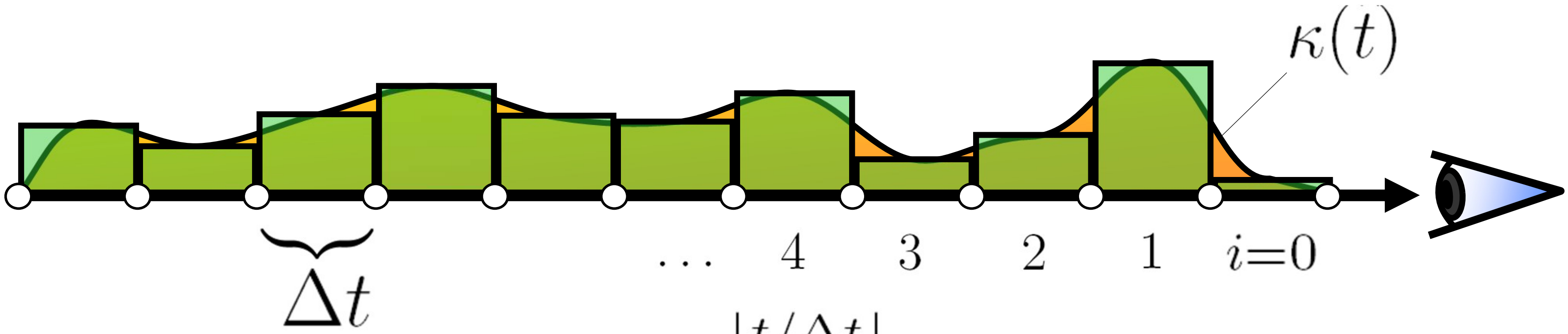


Optical depth: $\tau(0,t) = \int_0^t \kappa(\hat{t})\, d\hat{t}$

Approximate Riemann integral by Riemann sum:

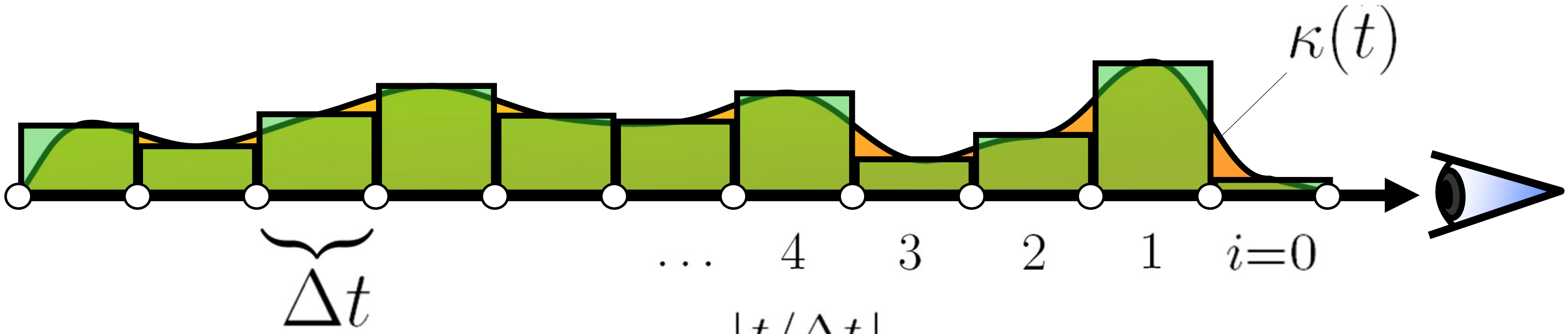
$$\tau(0,t) \approx \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

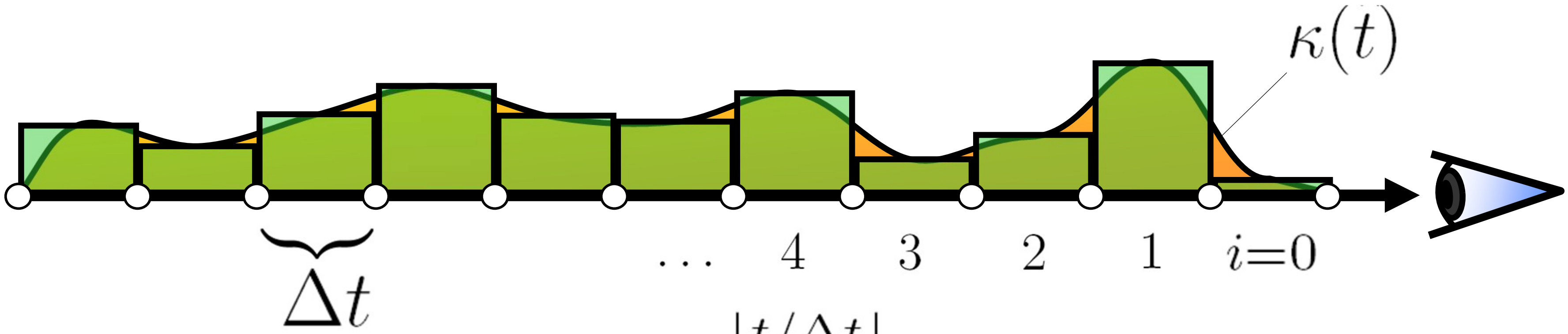
Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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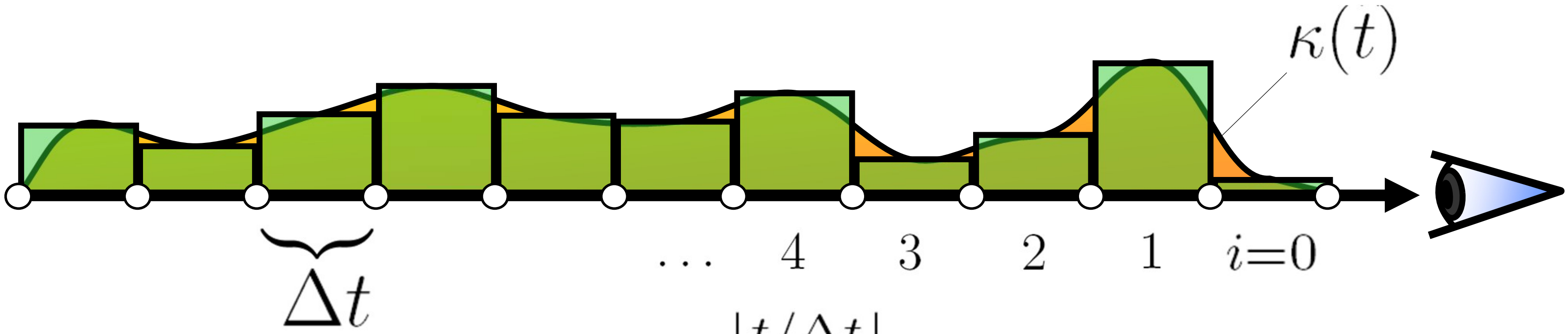
Volume Rendering Integral: Numerical Solution



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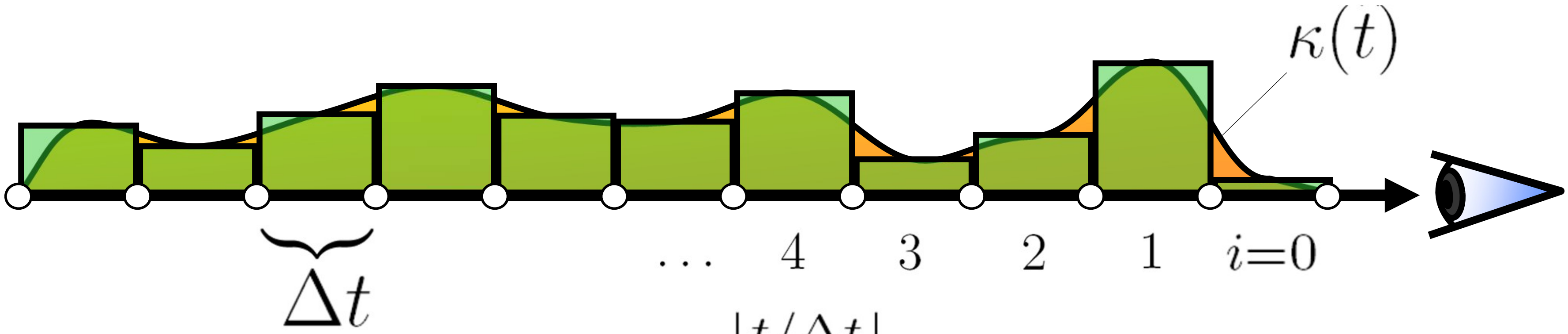
$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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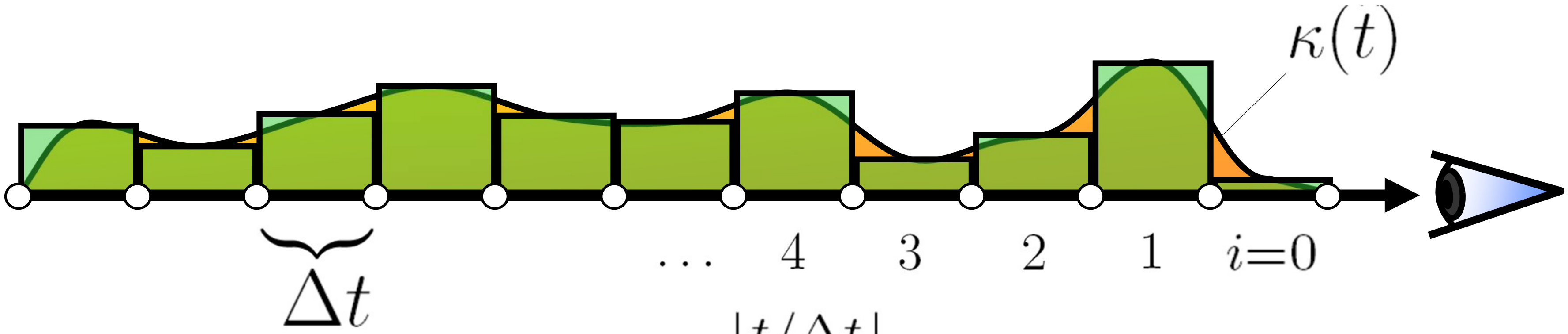
$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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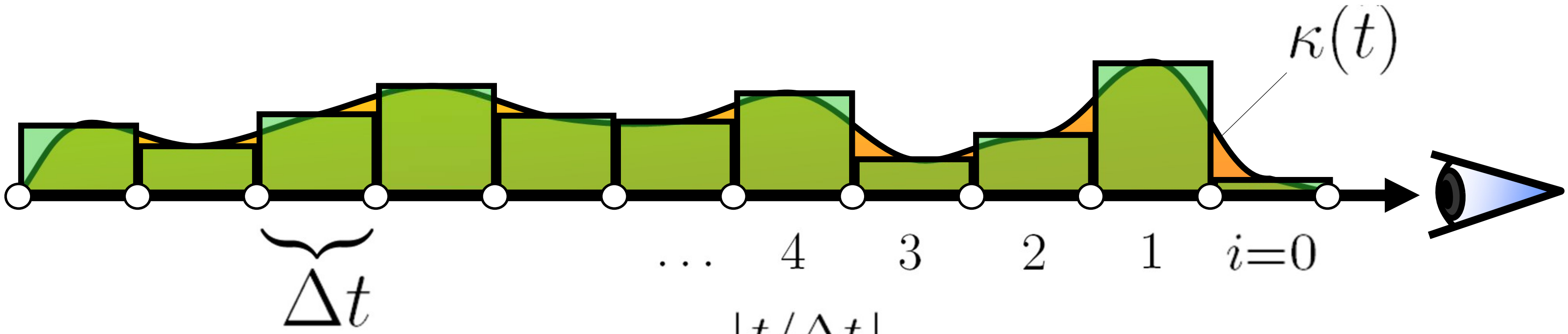
Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

Now we introduce *opacity*:

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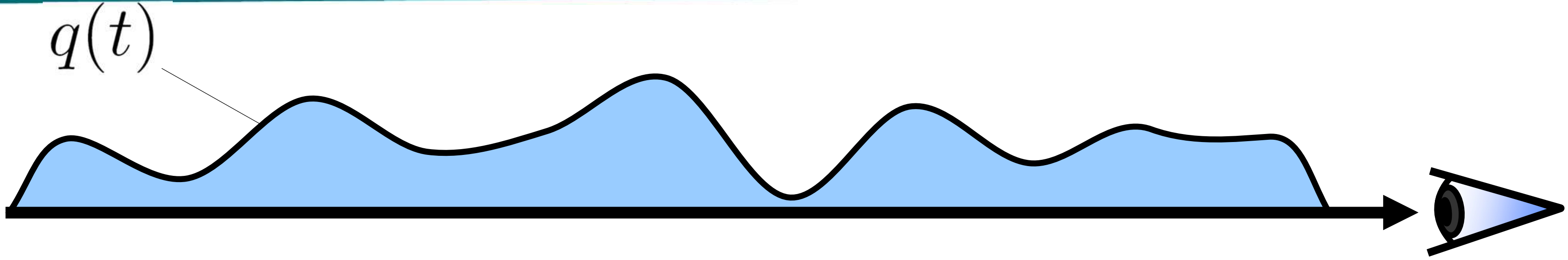
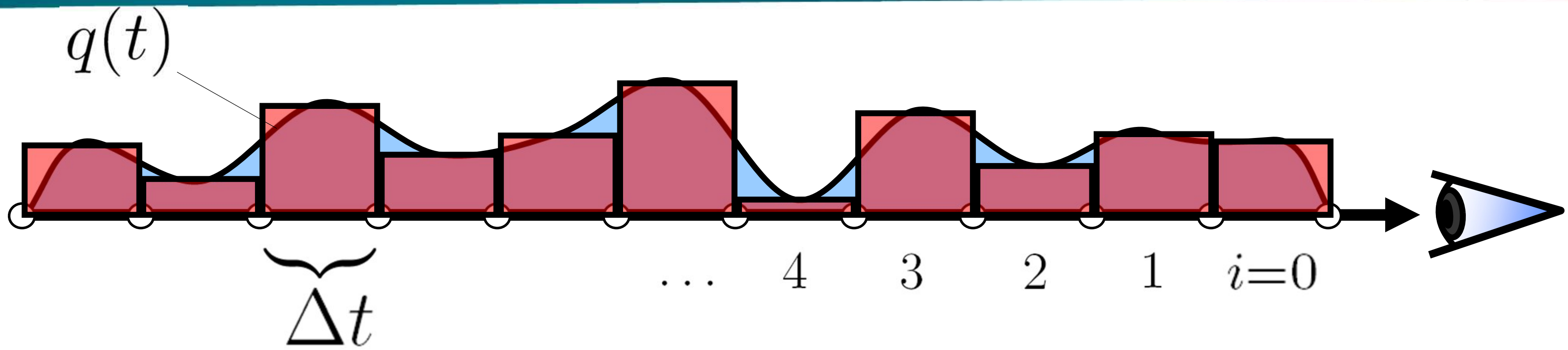
Volume Rendering Integral: Numerical Solution



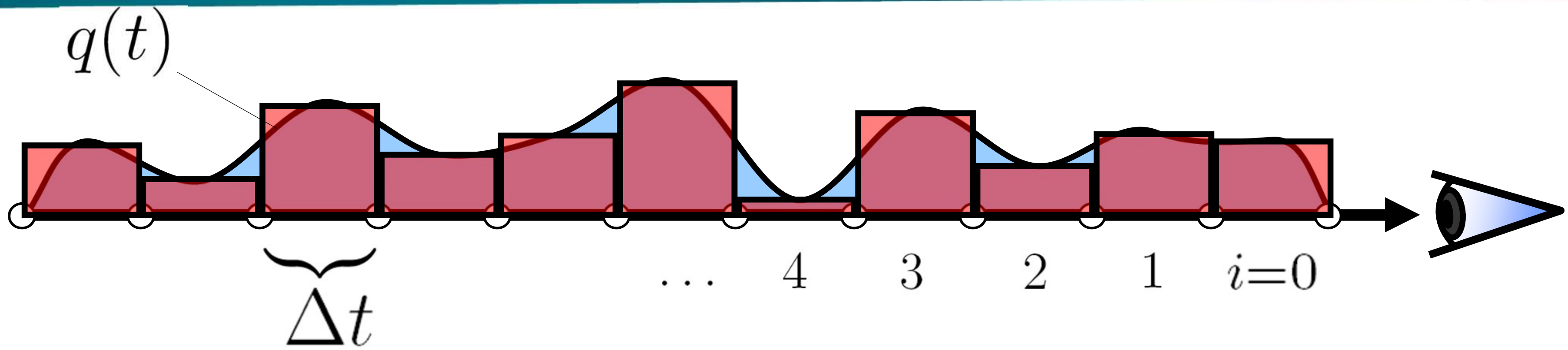
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

Volume Rendering Integral: Numerical Solution

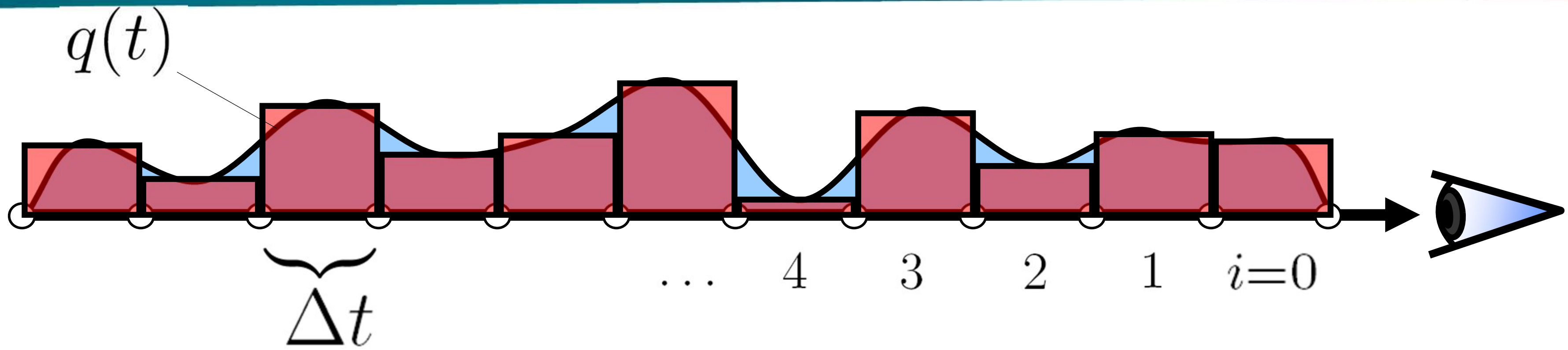


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution

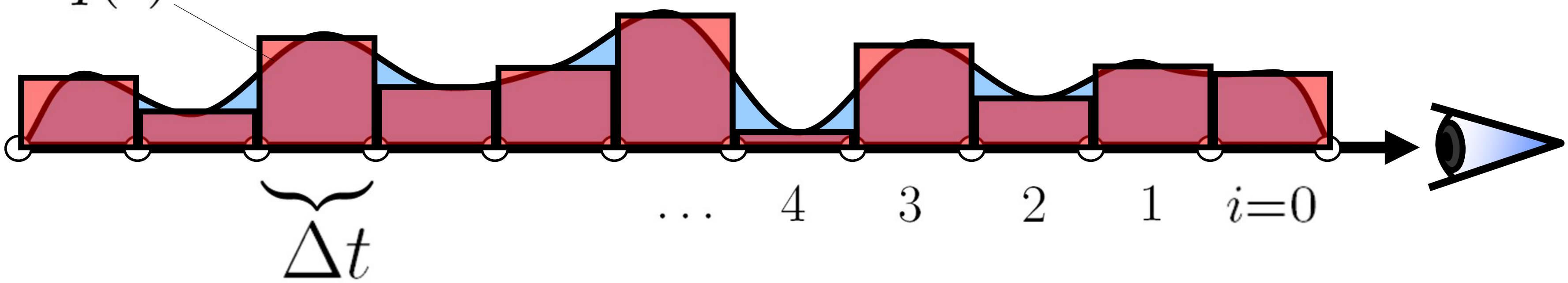


$$\boxed{e^{-\tilde{\tau}(0,t)}} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \boxed{e^{-\tilde{\tau}(0,t)}}$$

Volume Rendering Integral: Numerical Solution

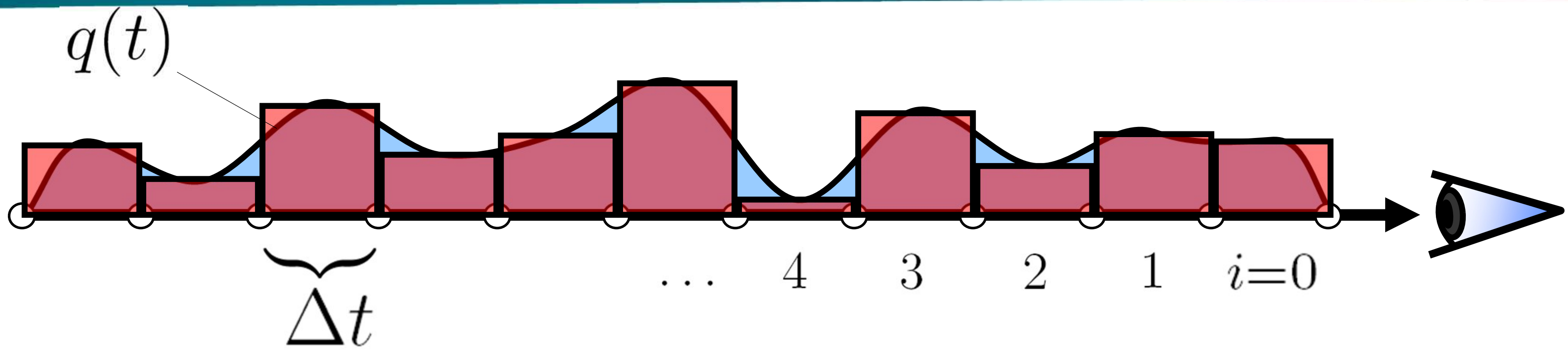


$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\, \Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

Volume Rendering Integral: Numerical Solution



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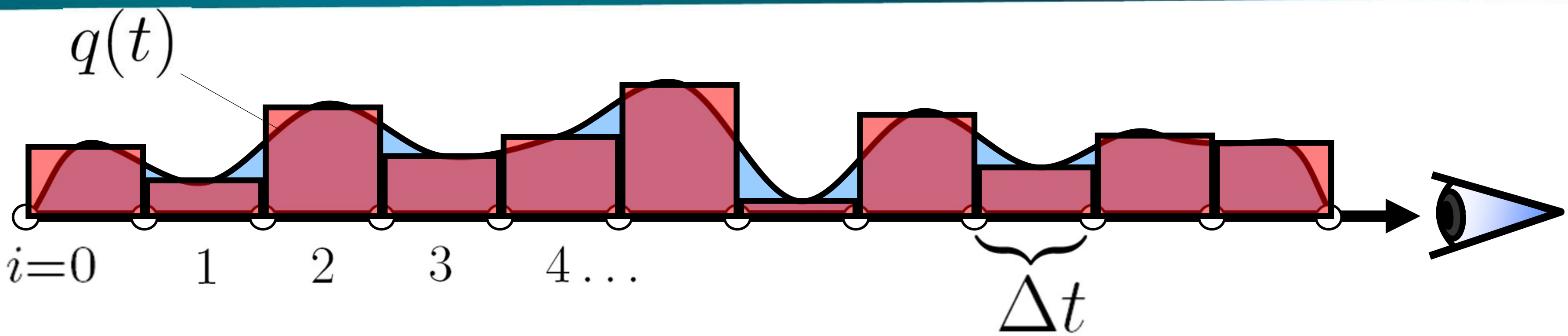
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed iteratively/recursively!

Volume Rendering Integral: Numerical Solution



Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !

can be computed iteratively/recursively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

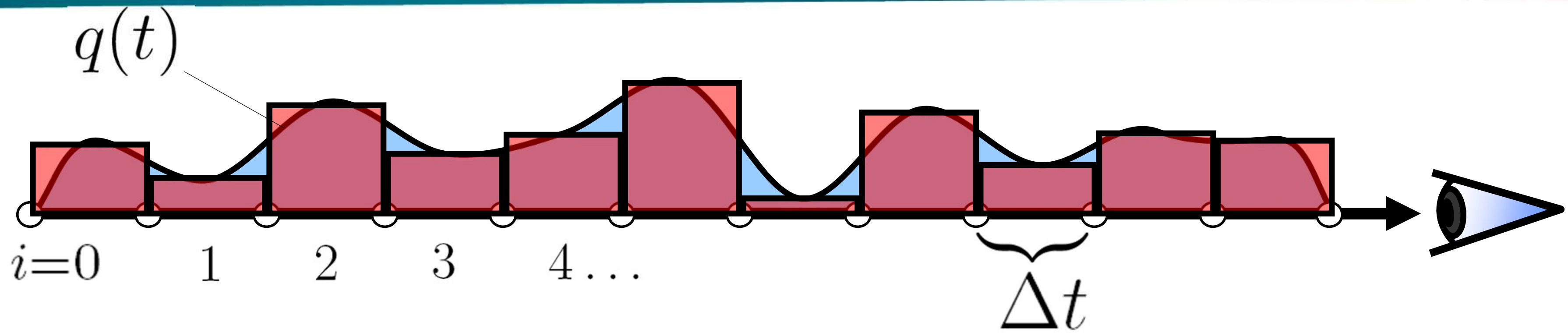
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

iterate from *i*=0 (back) to *i*=max (front): *i* increases

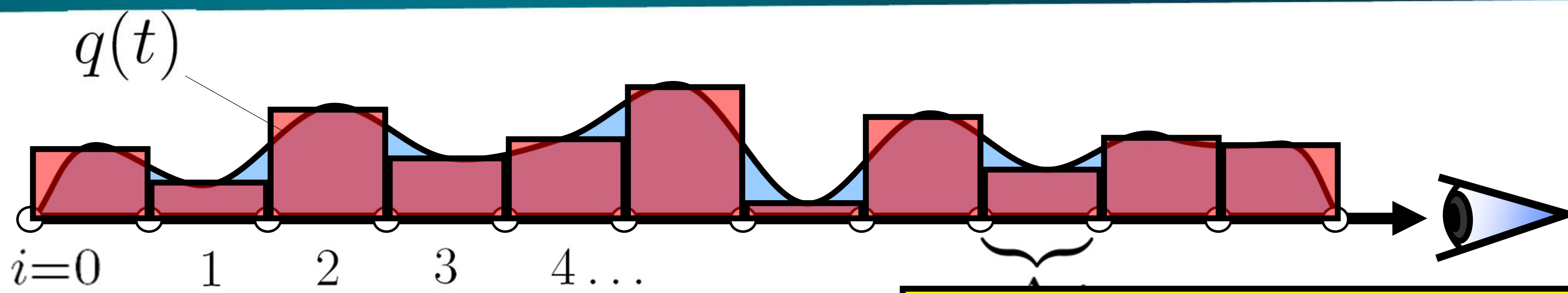
Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1$$

iterate from *i*=0 (back)

Early Ray Termination:
Stop the calculation when

$$A'_i \approx 1$$

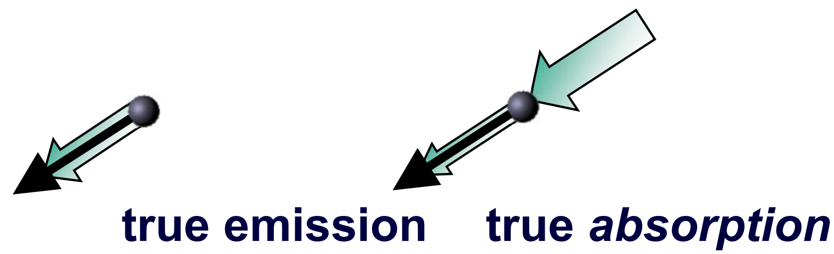
Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1}) C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1}) A_i$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model



$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) \;=\; \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

Back-to-front compositing

$$C'_i \;=\; C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1})A_i$$

here, all colors are *associated* colors!

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama