



CS 247 – Scientific Visualization

Lecture 8: Scalar Field Visualization, Pt. 2

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Reading Assignment #4 (until Feb 22)



Read (required):

- Real-Time Volume Graphics book, Chapter 5 until 5.4 inclusive
(*Terminology, Types of Light Sources, Gradient-Based Illumination, Local Illumination Models*)
- Paper:
Marching Cubes: A high resolution 3D surface construction algorithm,
Bill Lorensen and Harvey Cline, ACM SIGGRAPH 1987
[> 22,100 citations and counting...]

<https://dl.acm.org/doi/10.1145/37402.37422>

Contours



Set of points where the scalar field s has a given value c :

$$S(c) := f^{-1}(c) \quad S(c) := \{x \in \mathbb{R}^2 : f(x) = c\}$$

Common contouring algorithms

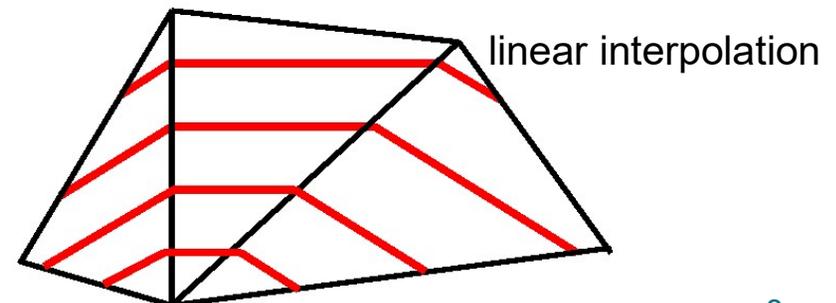
- 2D: marching squares, marching triangles
- 3D: marching cubes, marching tetrahedra

bilinear interpolation



Implicit methods

- Point-on-contour test
- Isosurface ray-casting



What are contours?

Set of points where the scalar field s has a given value c :

$$S(c) := \{x \in \mathbb{R}^n : f(x) = c\}$$

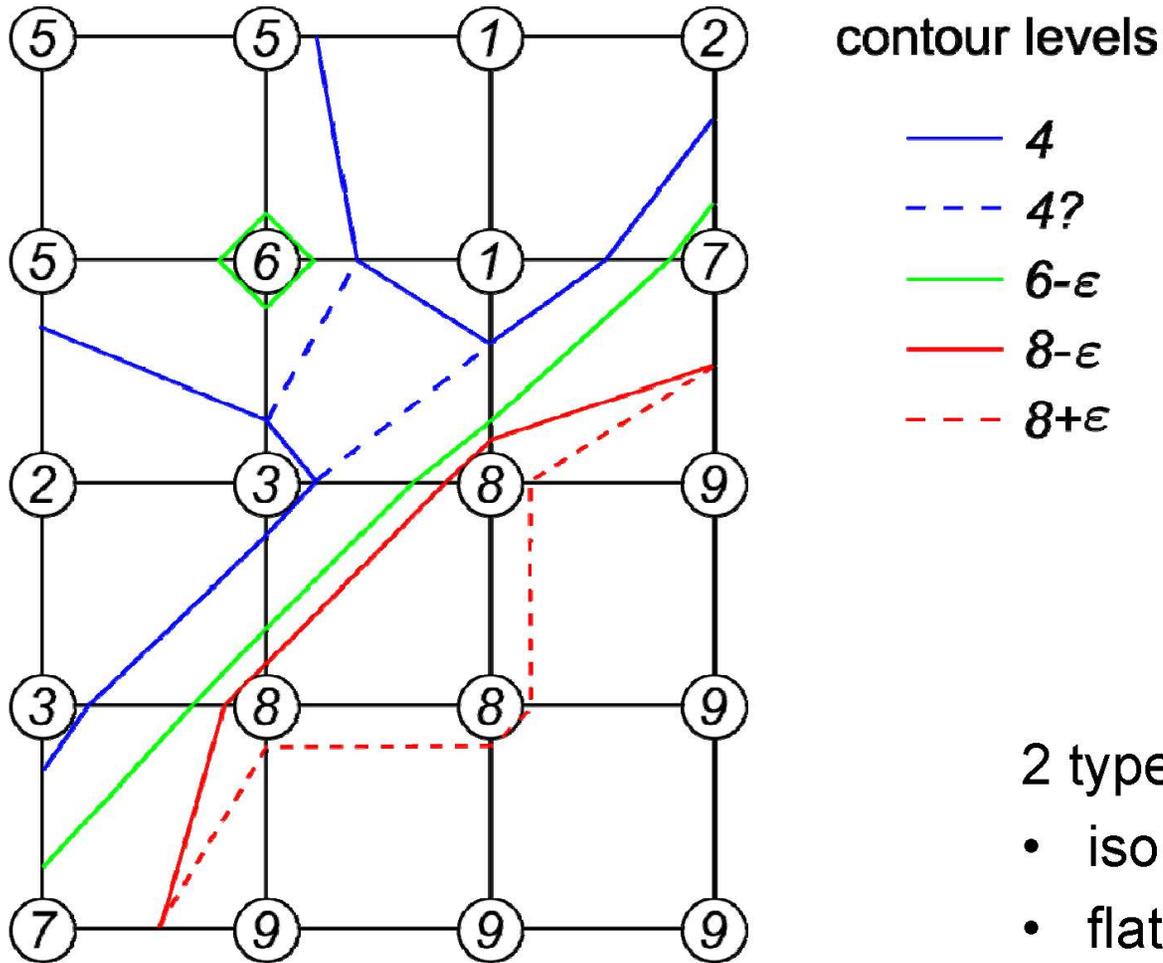
Examples in 2D:

- height contours on maps
- isobars on weather maps

Contouring algorithm:

- find intersection with grid edges
- connect points in each cell

Example



Contours in a quadrangle cell

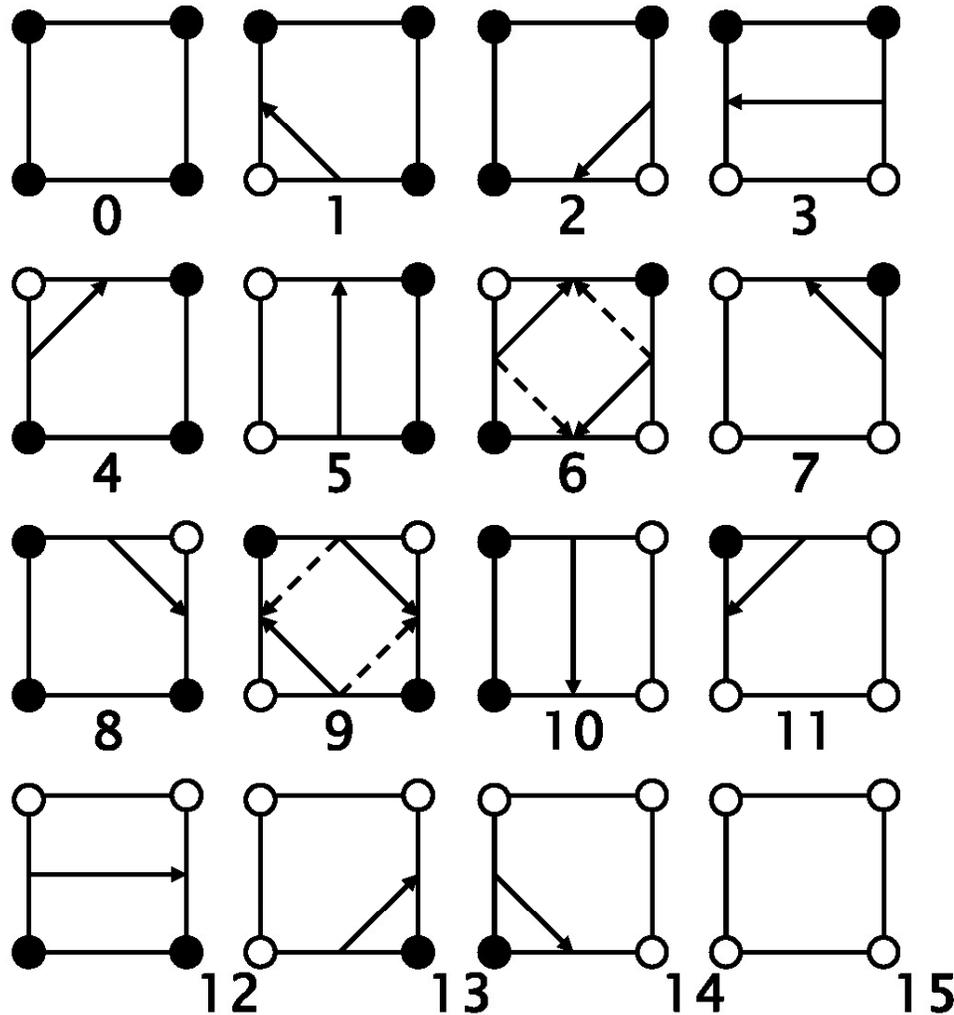
Basic contouring algorithms:

- **cell-by-cell** algorithms: simple structure, but generate disconnected segments, require post-processing
- **contour propagation** methods: more complicated, but generate connected contours

"Marching squares" algorithm (systematic cell-by-cell):

- process nodes in ccw order, denoted here as x_0, x_1, x_2, x_3
- compute at each node x_i the reduced field $\tilde{f}(x_i) = f(x_i) - (c - \varepsilon)$ (which is forced to be nonzero)
- take its sign as the i^{th} bit of a 4-bit integer
- use this as an index for lookup table containing the connectivity information:

Contours in a quadrangle cell



- $\tilde{f}(x_i) < 0$
- $\tilde{f}(x_i) > 0$

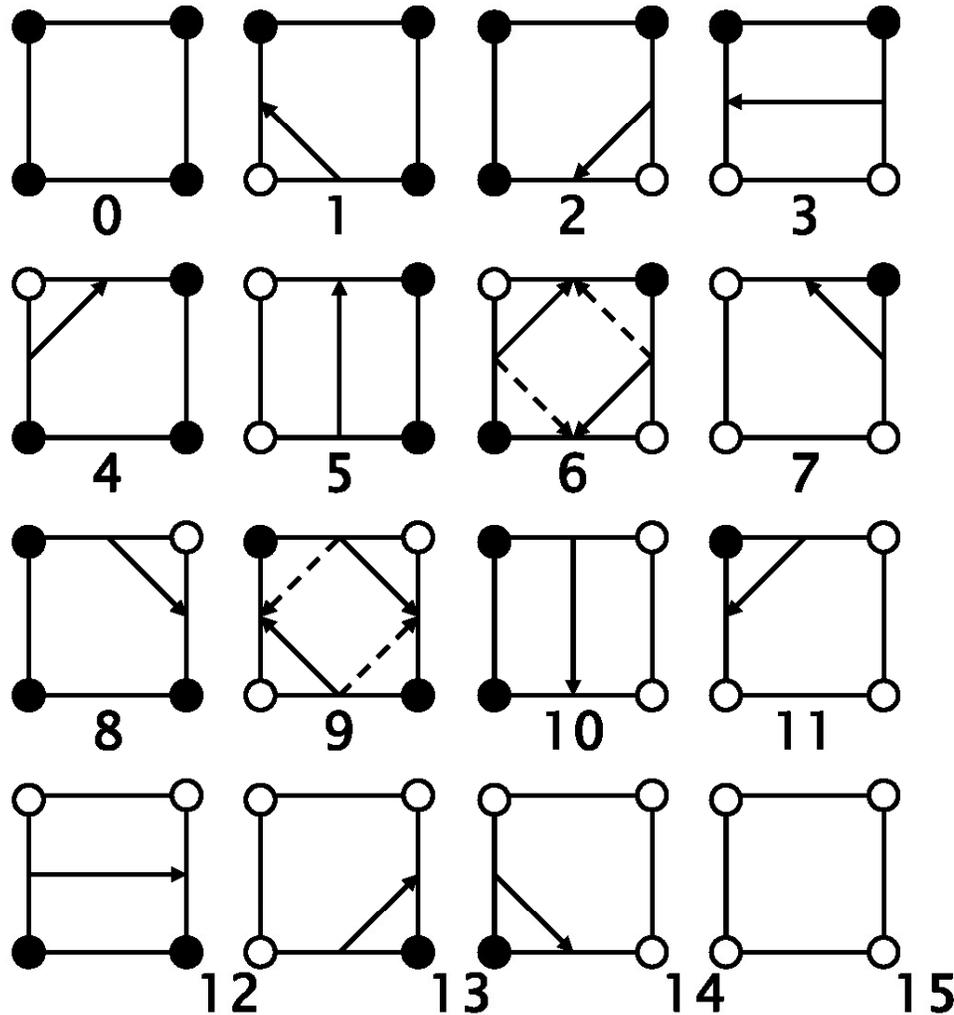
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



- $f(x_i) < c$
- $f(x_i) \geq c$

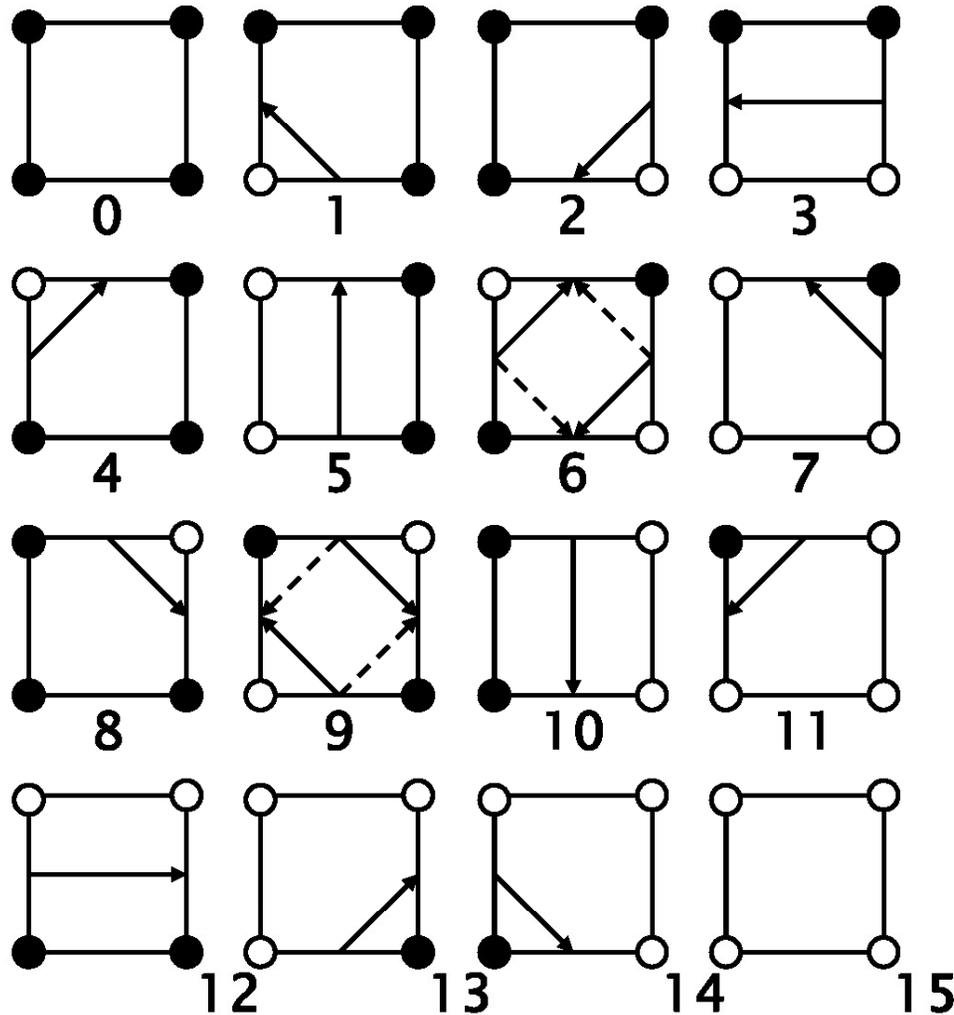
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Contours in a quadrangle cell



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Orientability (1-manifold embedded in 2D)

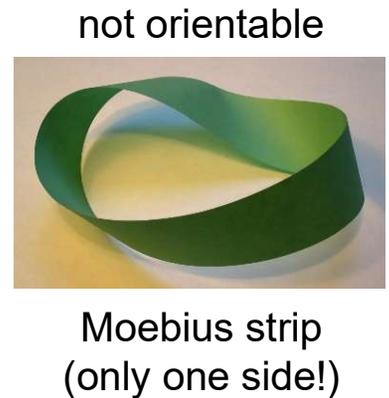
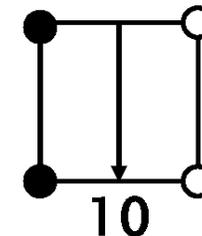
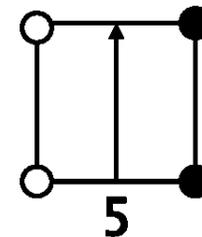


Orientability of 1-manifold:

Possible to assign consistent left/right orientation

Iso-contours

- Consistent side for scalar values...
 - greater than iso-value (e.g, *left* side)
 - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices (e.g., larger vertex index is “tip” of arrow; if (0,1) points “up”, “left” is left, ...)



- $\tilde{f}(x_i) < 0$
- $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

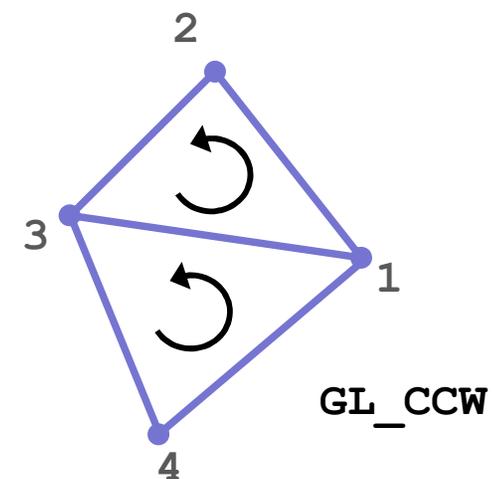


Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

Triangle meshes

- Edges
 - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
 - Consistent front side vs. back side
 - Normal vector; or ordering of vertices (CCW/CW)
 - See also: “right-hand rule”



not orientable



Moebius strip
(only one side!)

Topological consistency

To avoid degeneracies, use **symbolic perturbations**:

If level c is found as a node value, set the level to $c-\varepsilon$ where ε is a symbolic infinitesimal.

Then:

- contours intersect edges at some (possibly infinitesimal) distance from end points
- flat regions can be visualized by pair of contours at $c-\varepsilon$ and $c+\varepsilon$
- contours are **topologically consistent**, meaning:

Contours are **closed, orientable, nonintersecting lines**.

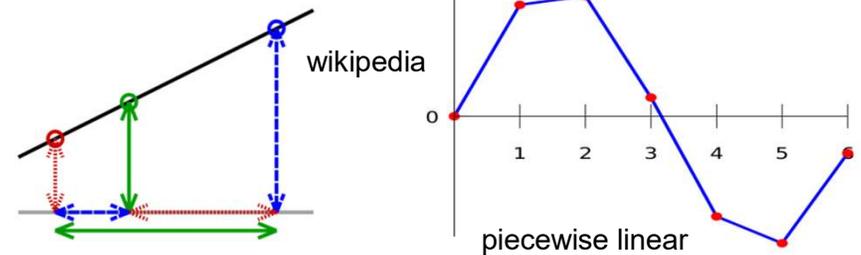
(except where the
boundary is hit)

Linear Interpolation / Convex Combinations



Linear interpolation in 1D:

$$f(\alpha) = (1 - \alpha)v_1 + \alpha v_2$$



Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2$$

$$\alpha_1 + \alpha_2 = 1$$

$$f(\alpha) = v_1 + \alpha(v_2 - v_1)$$

$$\alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (\rightarrow convex combination)

Compare to line parameterization
with parameter t :

$$v(t) = v_1 + t(v_2 - v_1)$$

Linear Interpolation / Convex Combinations



Linear combination (n -dim. space):

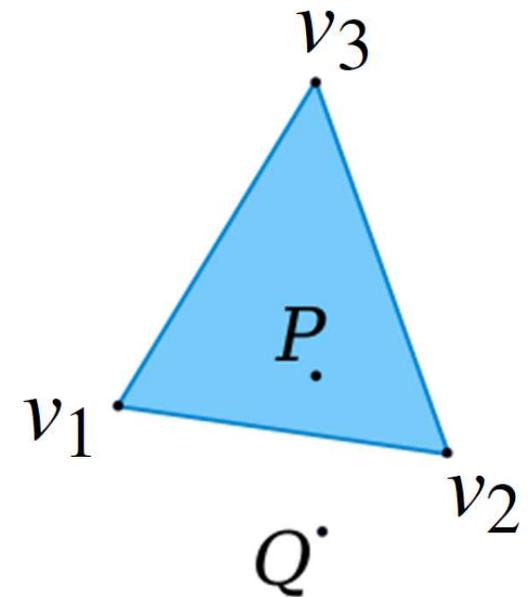
$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \sum_{i=1}^n \alpha_i v_i$$

Affine combination: Restrict to $(n - 1)$ -dim. subspace:

$$\alpha_1 + \alpha_2 + \dots + \alpha_n = \sum_{i=1}^n \alpha_i = 1$$

Convex combination: $\alpha_i \geq 0$

(restrict to simplex in subspace)



Linear Interpolation / Convex Combinations



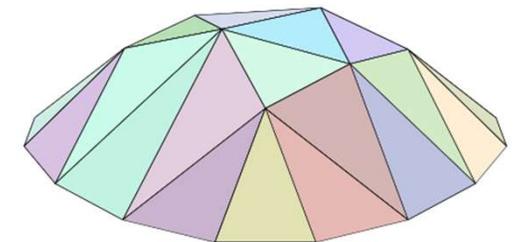
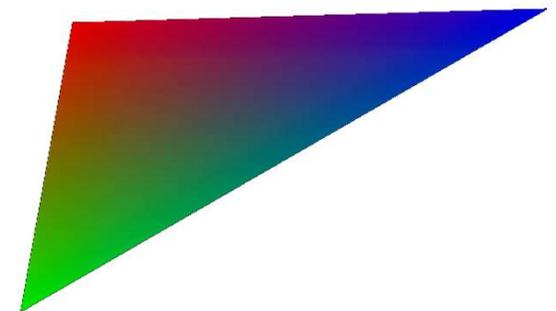
The weights α_i are the n normalized **barycentric** coordinates

→ linear attribute interpolation in simplex

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \sum_{i=1}^n \alpha_i v_i$$

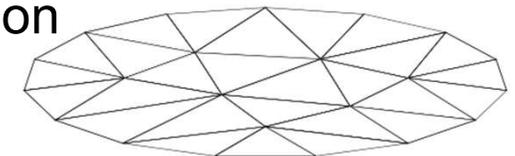
$$\alpha_1 + \alpha_2 + \dots + \alpha_n = \sum_{i=1}^n \alpha_i = 1$$
$$\alpha_i \geq 0$$

attribute interpolation



spatial position
interpolation

wikipedia



Linear Interpolation / Convex Combinations

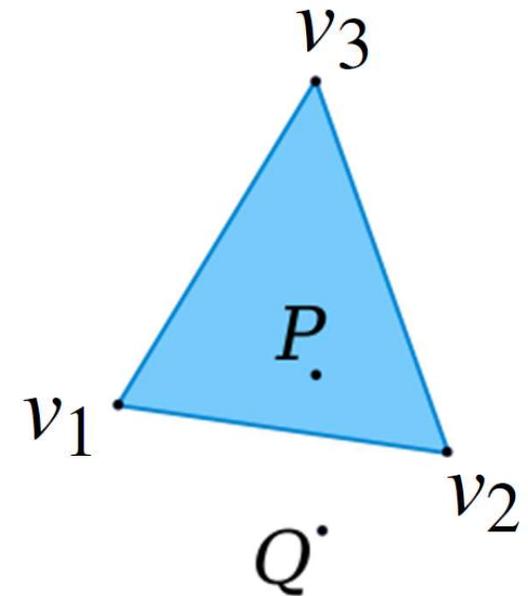


$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n = \sum_{i=1}^n \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \dots + \alpha_n = \sum_{i=1}^n \alpha_i = 1$$

Can re-parameterize to get $(n - 1)$ **affine** coordinates:

$$\begin{aligned} \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 &= \\ \tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1 & \\ \tilde{\alpha}_1 &= \alpha_2 \\ \tilde{\alpha}_2 &= \alpha_3 \end{aligned}$$



Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama