King Abdullah University of Science and Technology
KAUST

CS 247 – Scientific Visualization
Lecture 4: Data Representation, Pt. 2

Markus Hadwiger, KAUST

Reading Assignment #2 (until Feb 8)

Read (required):

- Data Visualization book, finish Chapter 2
- Data Visualization book, Chapter 3 until 3.5 (inclusive)
- Data Visualization book, Chapter 4 until 4.1 (inclusive)

- Continue familiarizing yourself with OpenGL if you do not know it !

Programming Assignments Schedule (tentative)

Assignment 0:	Lab sign-up: join discord, setup github account + get repo	until	**Feb 1**
	Basic OpenGL example		
Assignment 1:	Volume slice viewer	until	**Feb 15**
Assignment 2:	Iso-contours (marching squares)	until	**Mar 1**
Assignment 3:	Iso-surface rendering (marching cubes)	until	**Mar 15**
Assignment 4:	Volume ray-casting, part 1	until	**Apr 12**
	Volume ray-casting, part 2	until	**Apr 19**
Assignment 5:	Flow vis, part 1 (hedgehog plots, streamlines, pathlines)	until	**May 3**
Assignment 6:	Flow vis, part 2 (LIC with color coding)	until	**May 13**

Programming Assignment #1: Slice Viewer

Basic tasks

- Download data into 3D volume texture
- Display three different axis-aligned slices using OpenGL texture mapping using the 3D volume texture

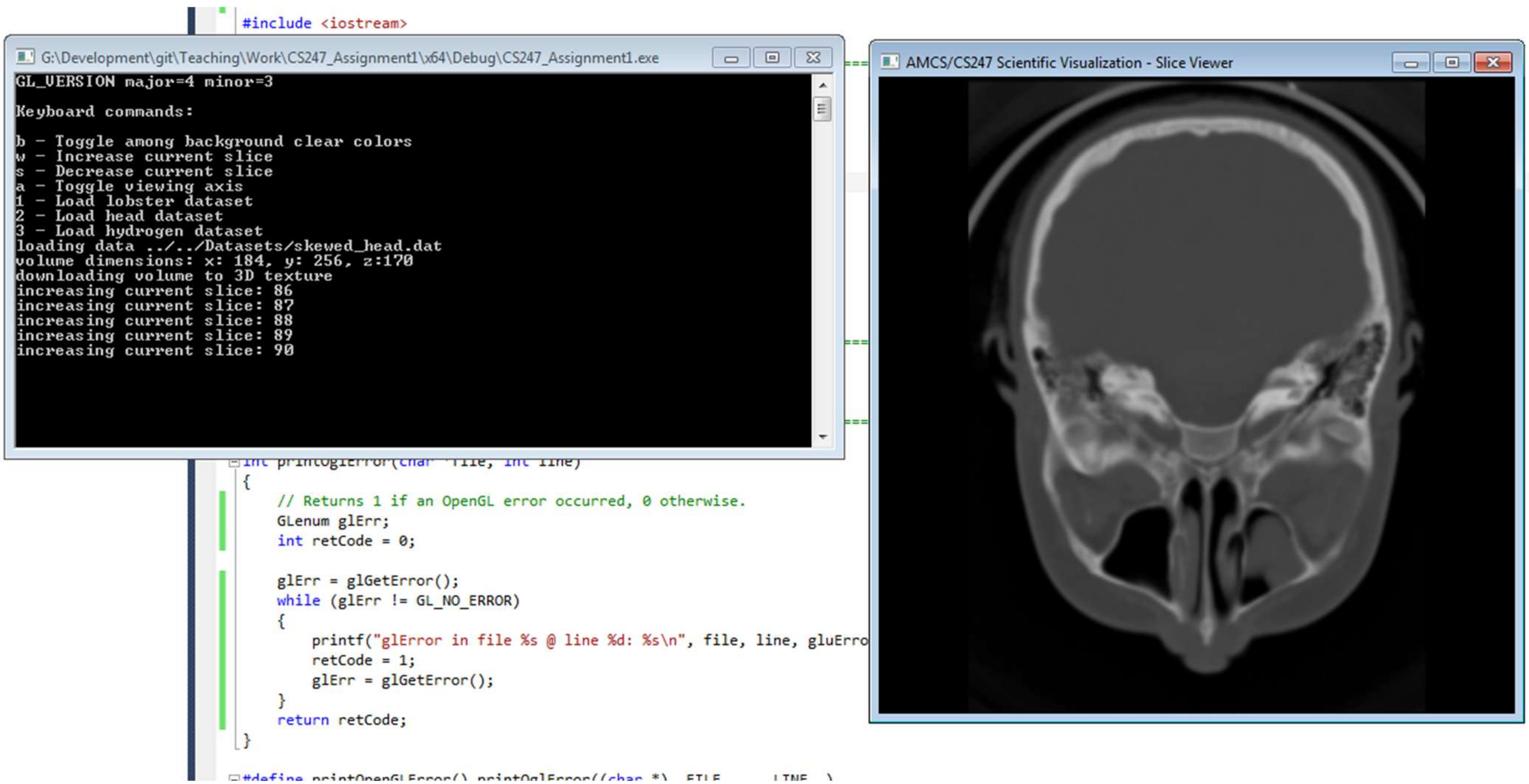
Minimum

- The slice position should be adjustable for each slice view.
- Make sure the aspect ratio of the shown slices is correct.
- If the window is resized, the slice is resized with the correct aspect ratio (no distortions)

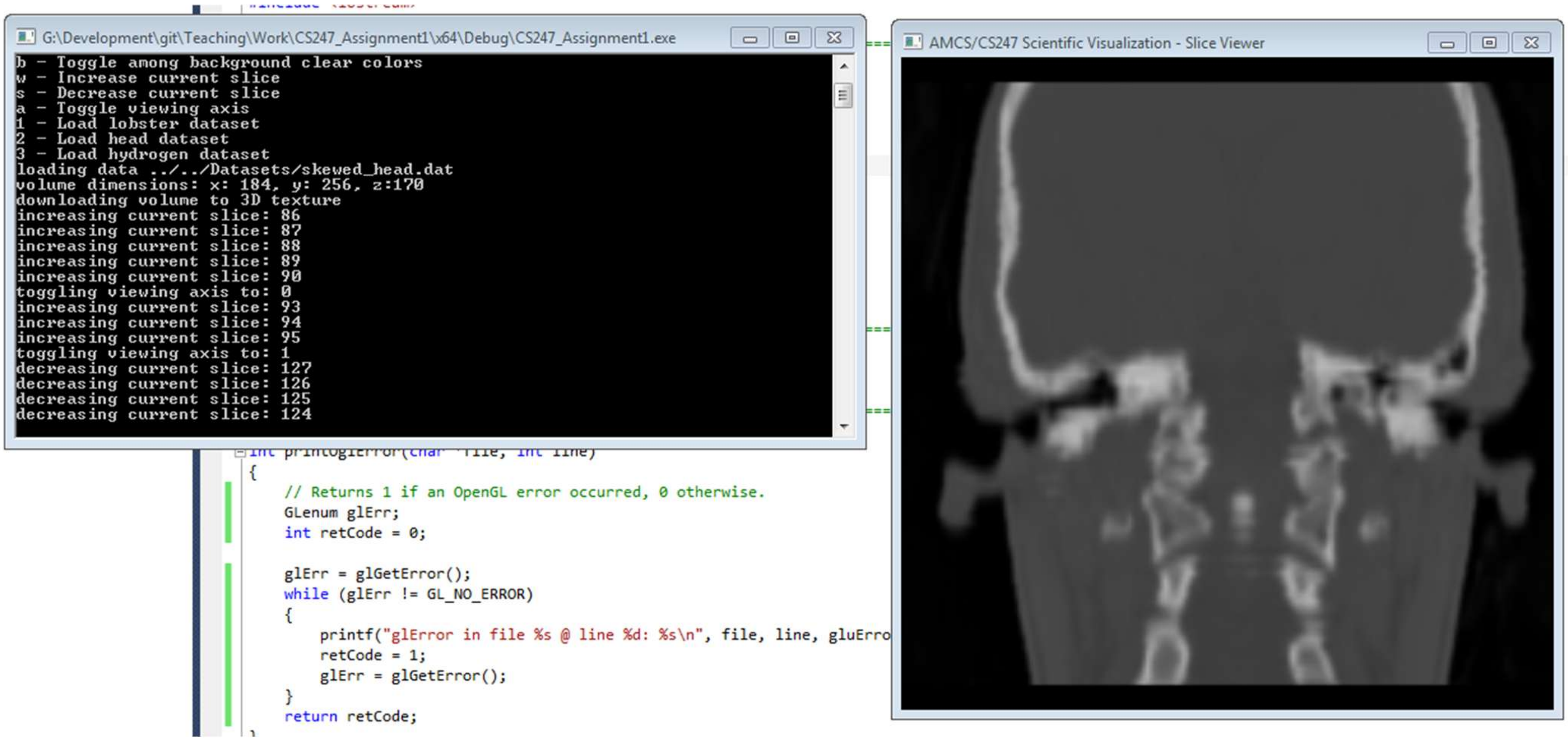
Bonus

- Show all three axis aligned slices at once
- Show arbitrarily aligned slices with an interface to change the arbitrary slice

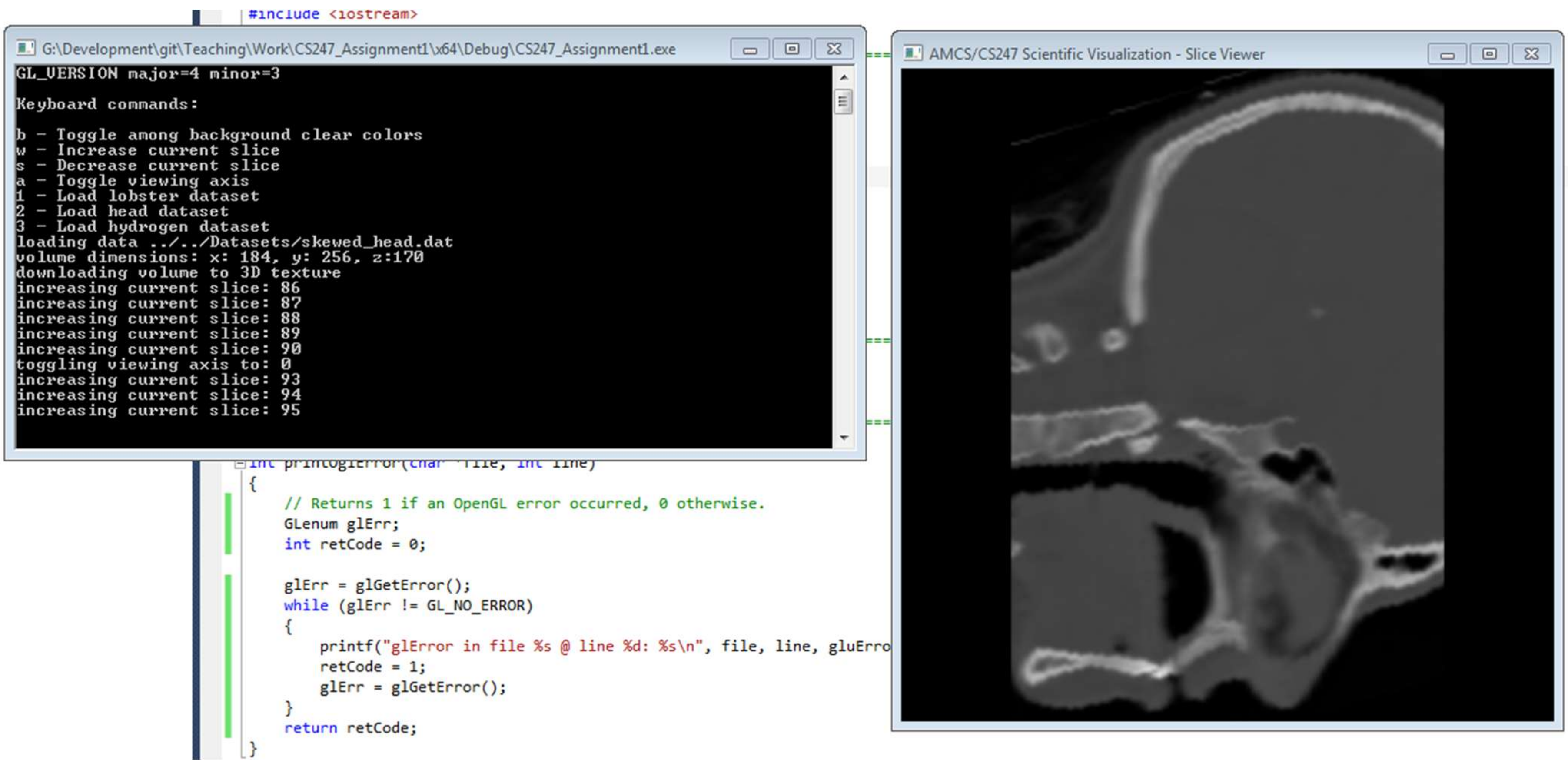
Programming Assignment #1 Example



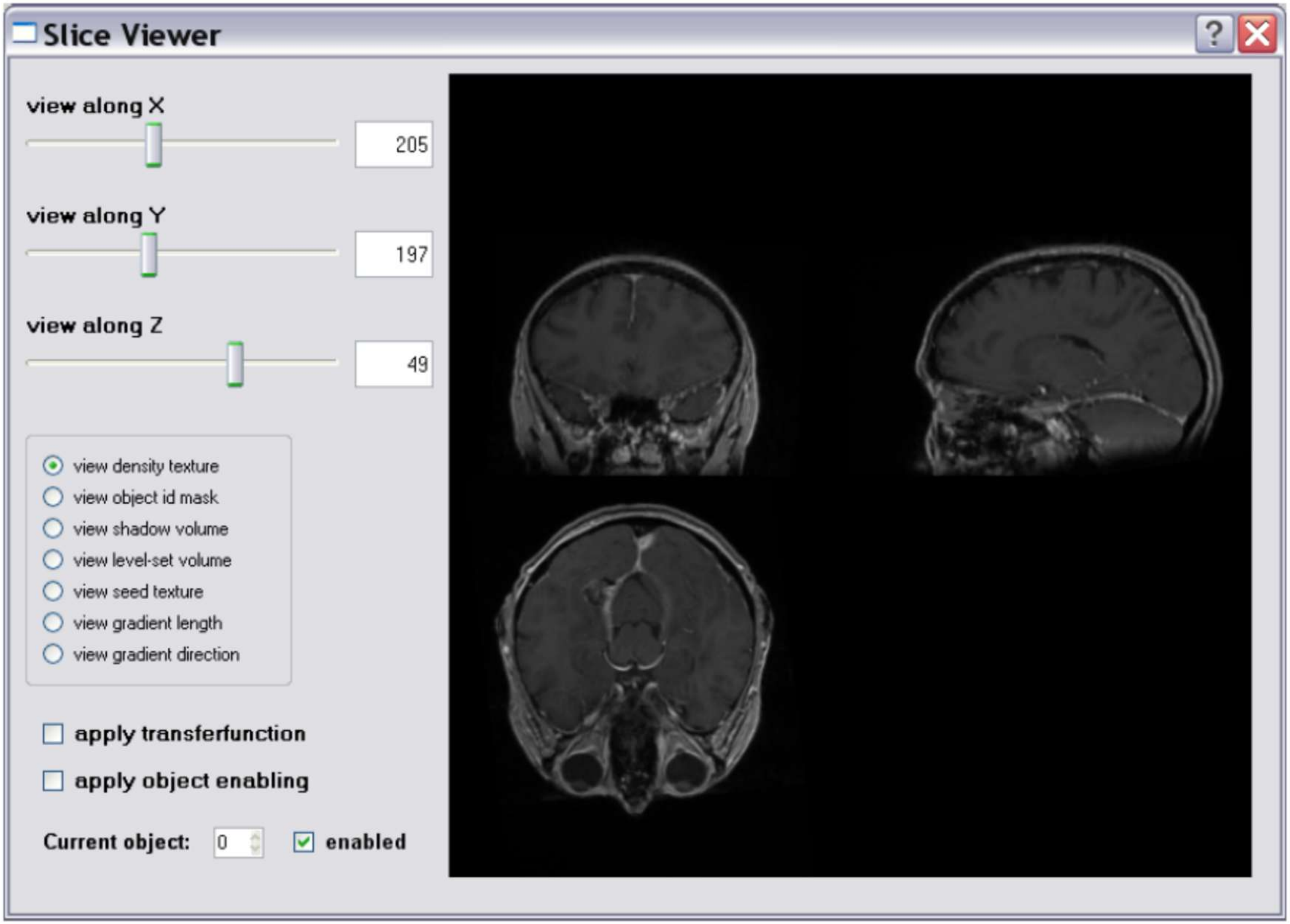
Programming Assignment #1 Example



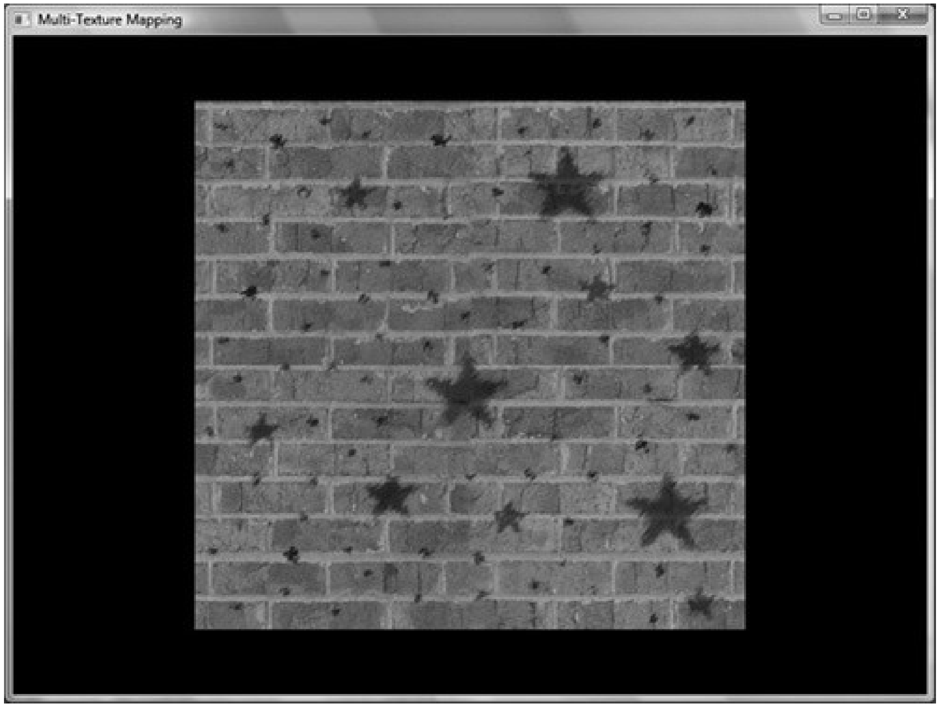
Programming Assignment #1 Example

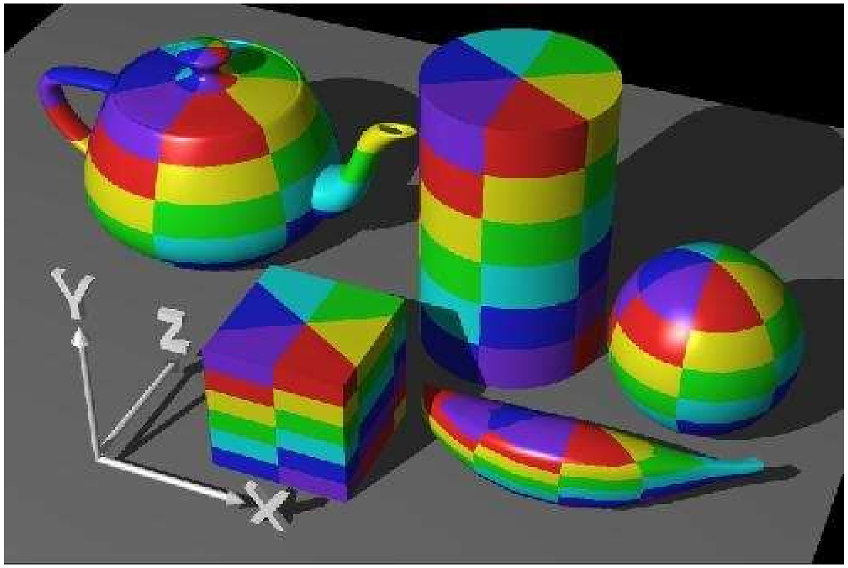


Programming Assignment #1 Example

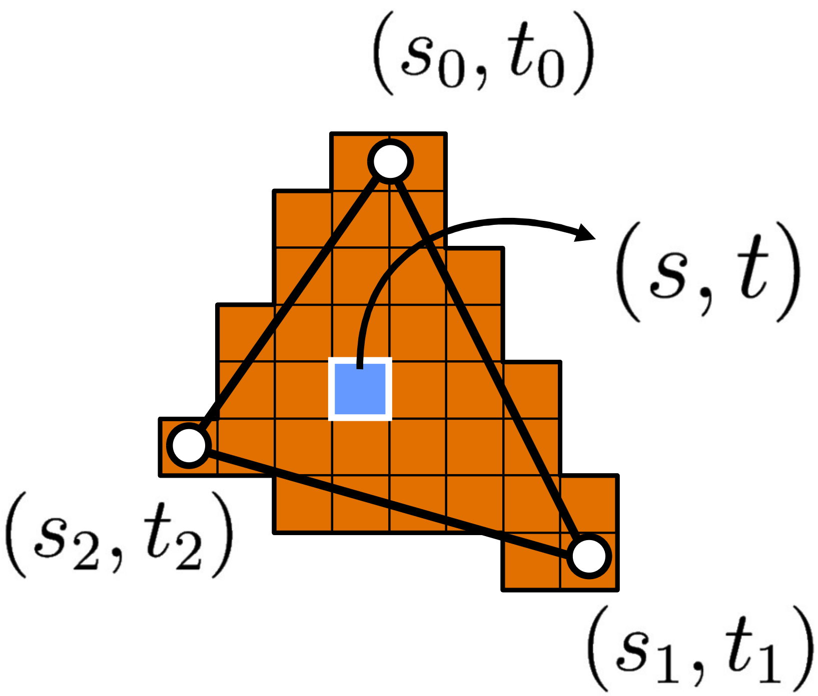


Texture Mapping

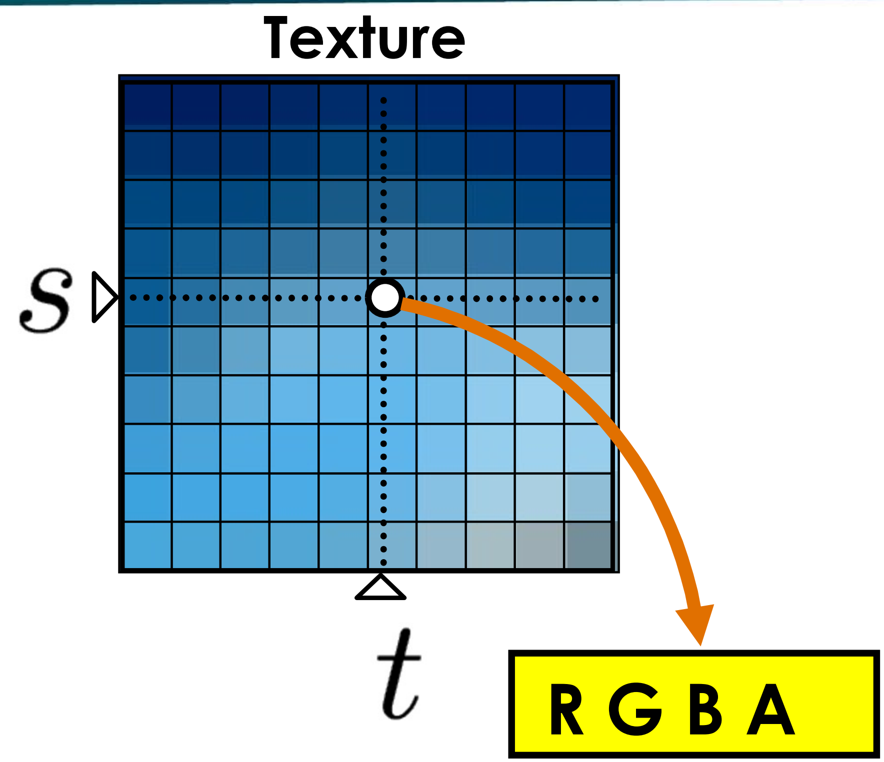




2D Texture Mapping

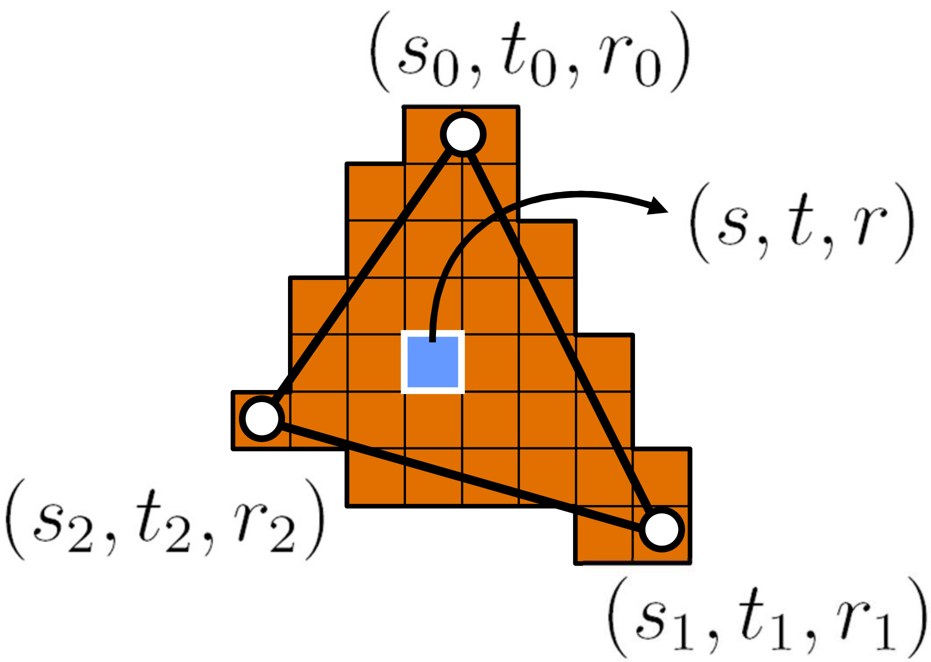


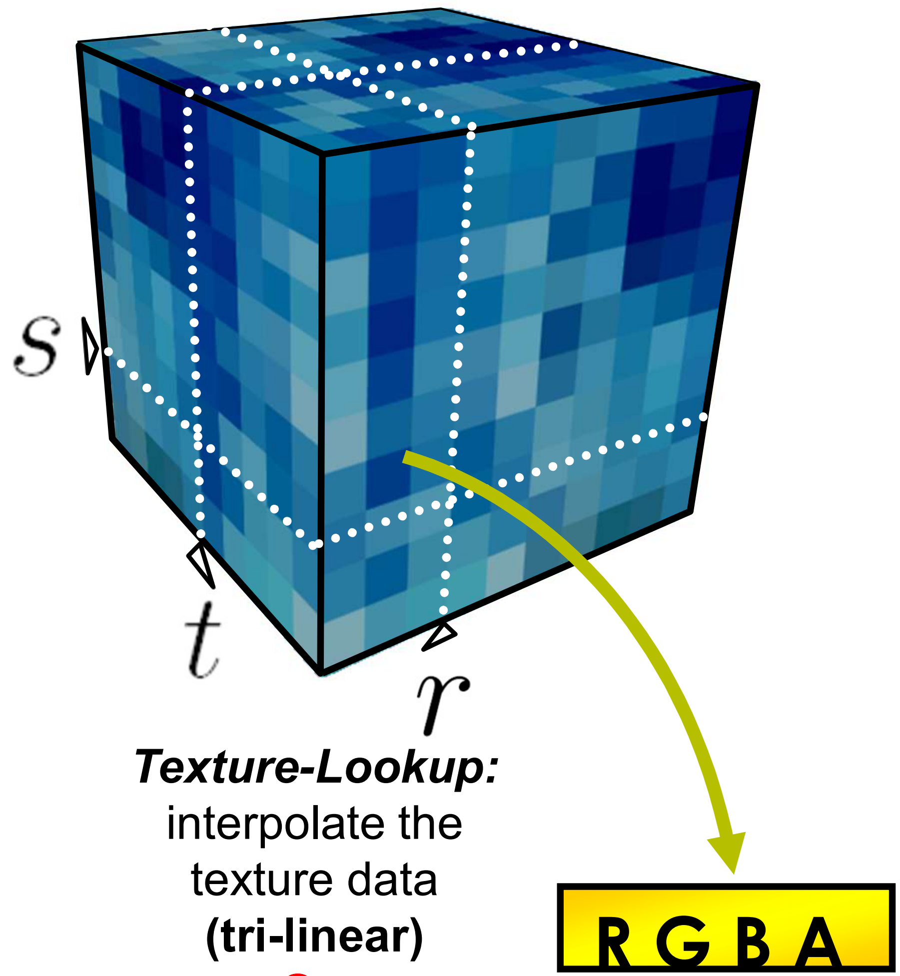
For each fragment:
interpolate the
texture coordinates
(barycentric)
Or:
Use arbitrary, computed coordinates



Texture-Lookup:
interpolate the
texture data
(bi-linear)
Or:
Nearest-neighbor for “array lookup”

3D Texture Mapping





For each fragment:
interpolate the
texture coordinates
(barycentric)
Or:
Use arbitrary, computed coordinates

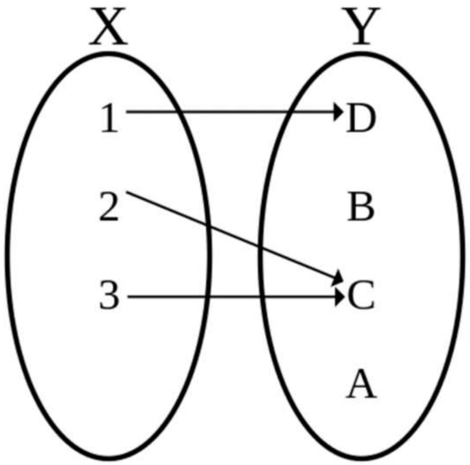
Texture-Lookup:
interpolate the
texture data
(tri-linear)
Or:
Nearest-neighbor for "array lookup"

Data == Functions

Mathematical Functions

Associates every element of a set (e.g., X) with *exactly one* element of another set (e.g., Y)

Maps from *domain* (X) to *codomain* (Y)



$$f : X \to Y$$
$$x \mapsto f(x)$$

Also important: *range/image*; *preimage*; continuity, differentiability, dimensionality, ...

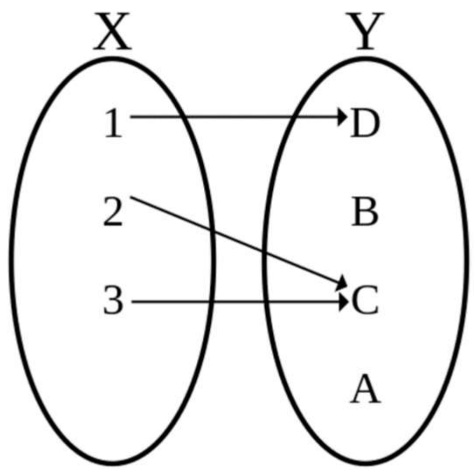
Graph of a function (mathematical definition):

$$G(f) := \{(x, f(x)) | x \in X\} \subset X \times Y$$

Mathematical Functions

Associates every element of a set (e.g., X) with *exactly one* element of another set (e.g., Y)

Maps from *domain* (X) to *codomain* (Y)



$$\begin{aligned} f: \mathbb{R}^n &\to \mathbb{R}^m \\ x &\mapsto f(x) \end{aligned}$$

Also important: *range/image*; *preimage*; continuity, differentiability, dimensionality, ...

Graph of a function (mathematical definition):

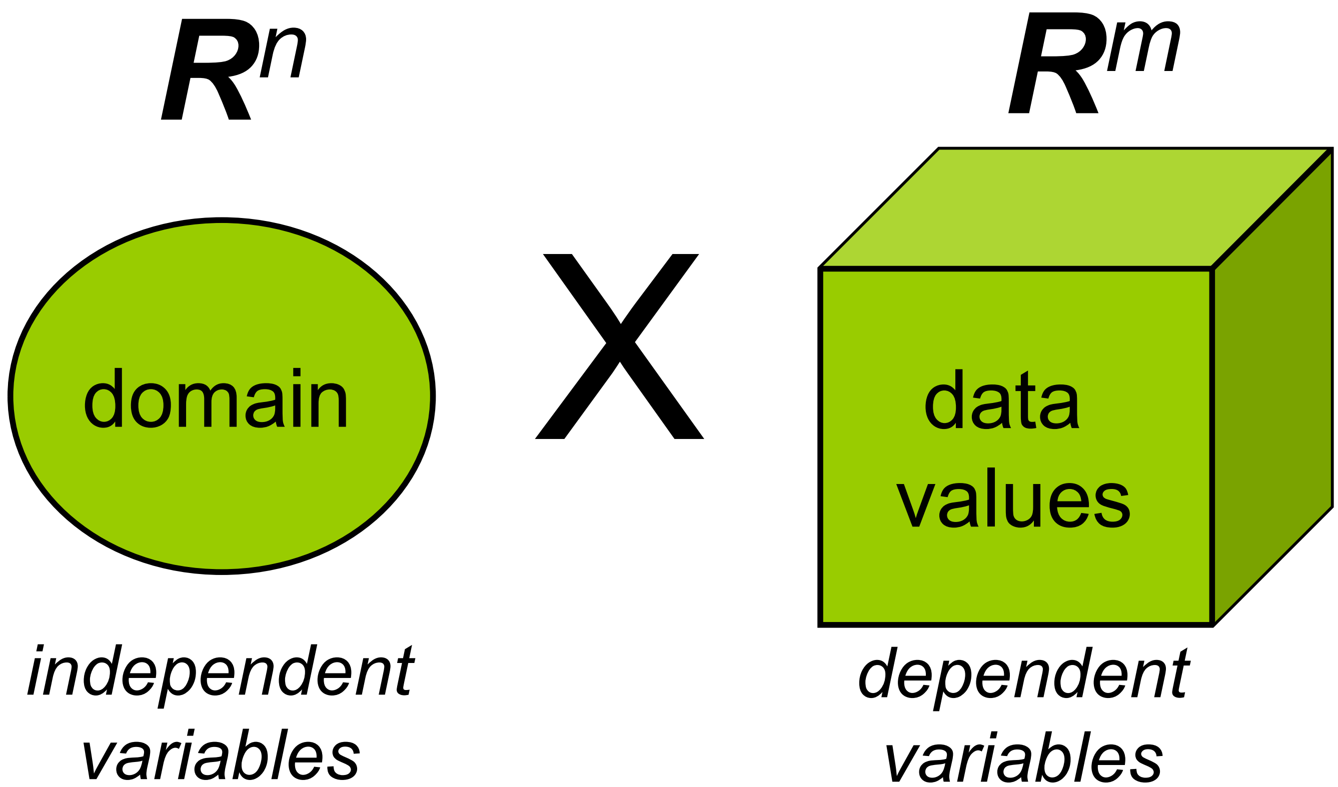
$$G(f) := \{(x, f(x)) | x \in \mathbb{R}^n\} \subset \mathbb{R}^n \times \mathbb{R}^m \simeq \mathbb{R}^{n+m}$$

Data Space (Domain) vs. Data Type (Codomain)

	1D	2D	3D
1D	y=f(x)		spatial curve **x**(t)
2D		2D flow **v**(**x**)	
3D	CT data s(**x**)		3D flow **v**(**x**)

examples

Data Representation



scientific data $\subseteq \boldsymbol{R}^{n+m}$

Example: Scalar Fields

2D scalar field

$$f : \mathbb{R}^2 \to \mathbb{R}$$
$$x \mapsto f(x)$$

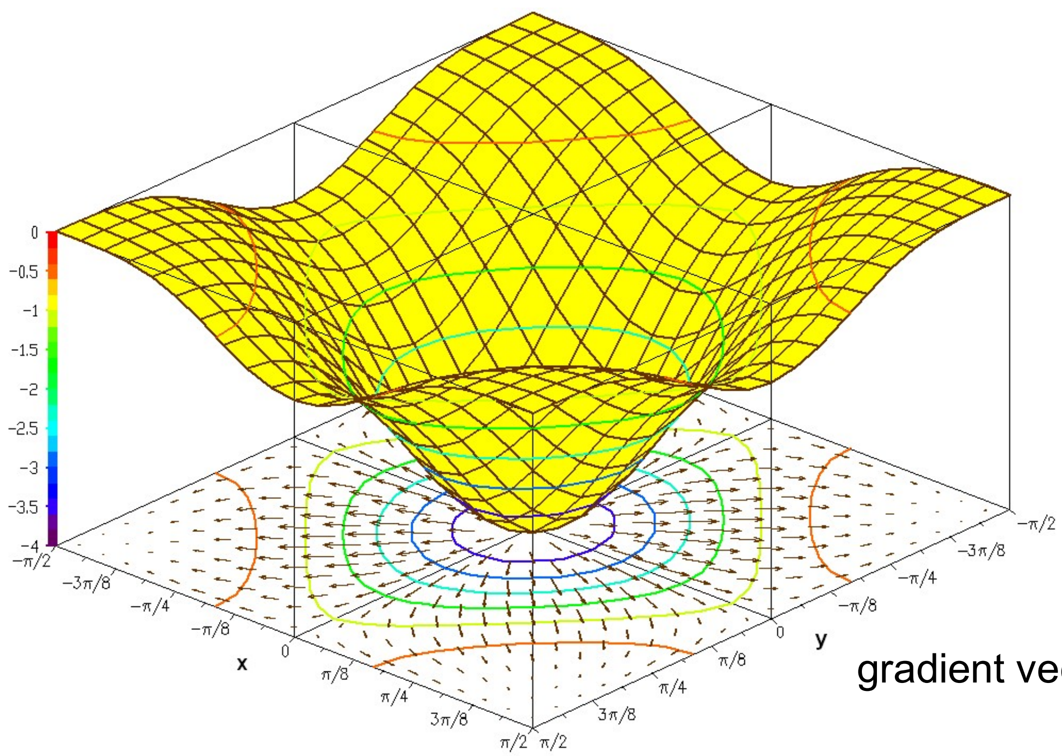
Graph: $G(f) := \{(x, f(x)) | x \in \mathbb{R}^2\} \subset \mathbb{R}^2 \times \mathbb{R} \simeq \mathbb{R}^3$

pre-image

$$S(c) := f^{-1}(c)$$

iso-contour

$$(\nabla f \neq 0)$$



gradient vector field ∇f

Example: Scalar Fields

3D scalar field

$$f : \mathbb{R}^3 \to \mathbb{R}$$
$$x \mapsto f(x)$$

Graph: $G(f) := \{(x, f(x)) | x \in \mathbb{R}^3\} \subset \mathbb{R}^3 \times \mathbb{R} \simeq \mathbb{R}^4$

?

pre-image

$$S(c) := f^{-1}(c)$$

iso-surface

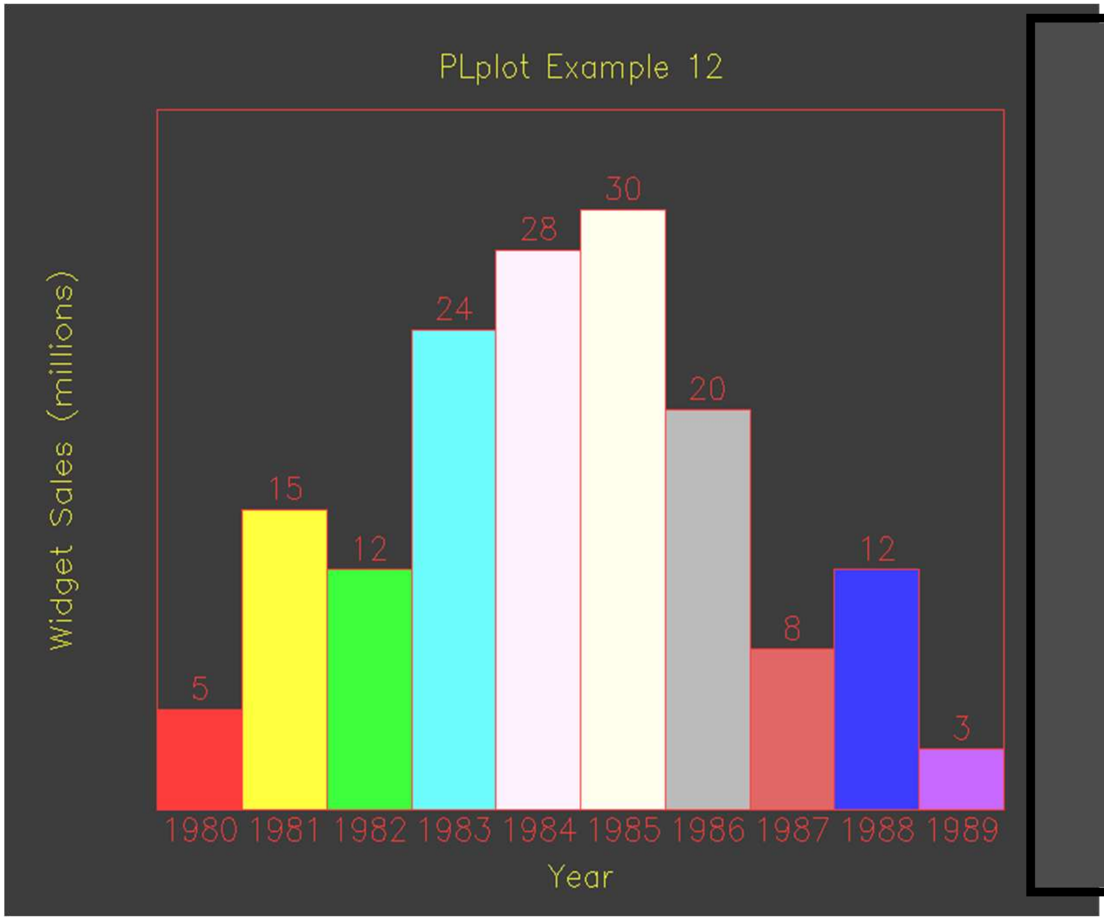
$$(\nabla f \neq 0)$$

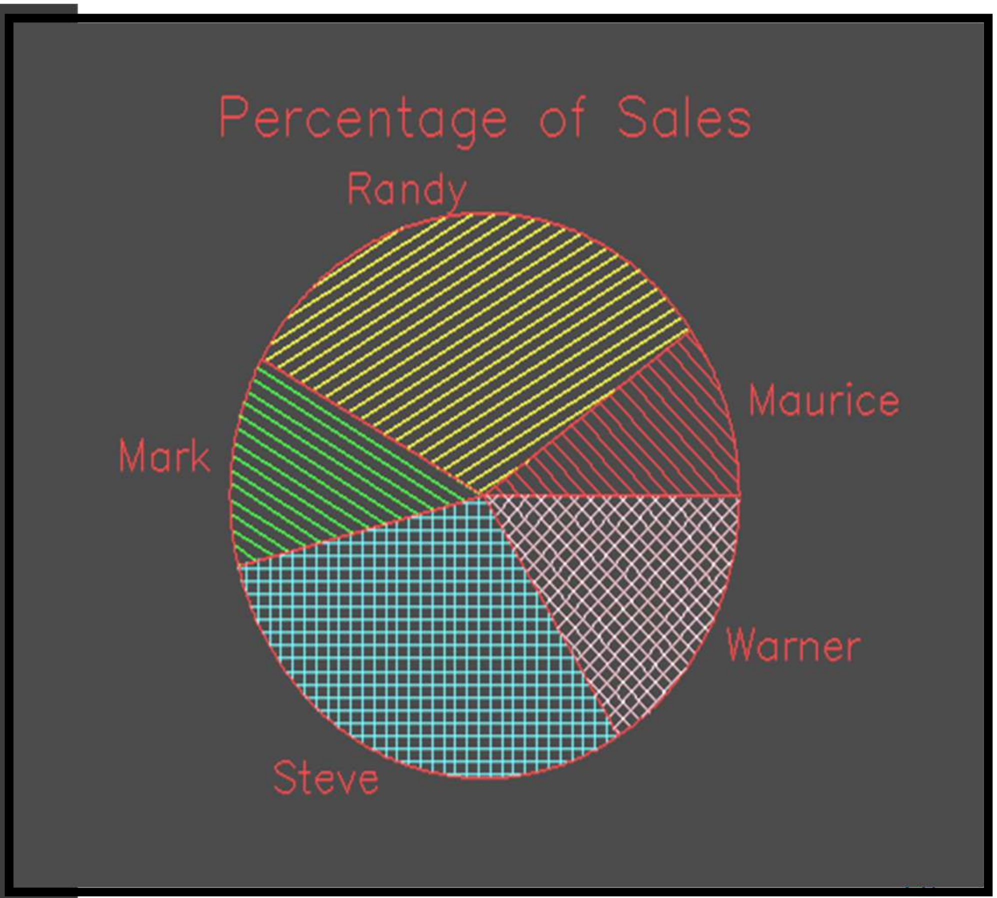
Visualization Examples

data	description	visualization example
$N^1 \rightarrow R^1$	value series	bar chart, pie chart, etc.
$R^1 \rightarrow R^1$	scalar function over R	(line) graph
$R^2 \rightarrow R^1$	scalar function over R^2	2D-height map in 3D, contour lines in 2D, false color map
$R^2 \rightarrow R^2$	2D vector field	hedgehog plot, LIC, streamlets, etc.
$R^3 \rightarrow R^1$	scalar function over R^3 (3D densities)	iso-surfaces in 3D, volume rendering
$R^3 \rightarrow R^3$	3D vector field	streamlines/pathlines in 3D

Visualization Examples

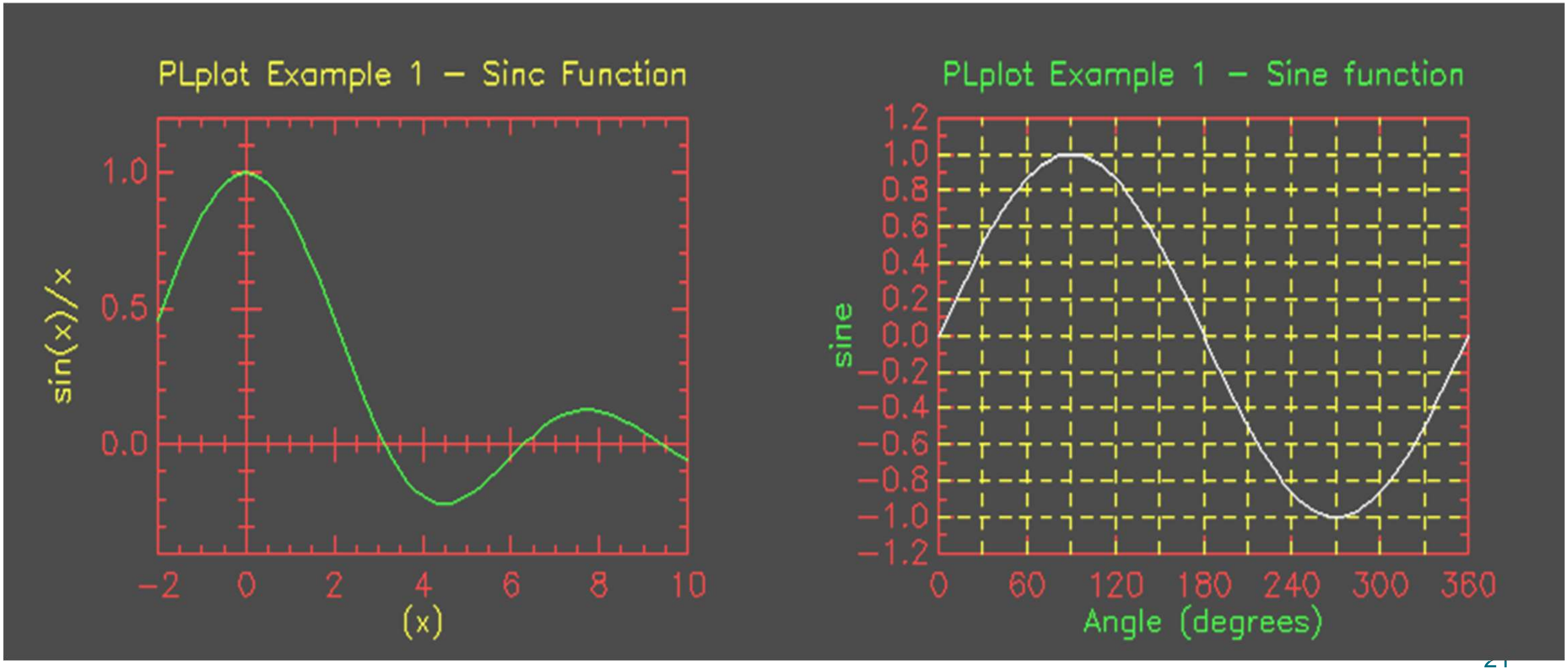
data	description	visualization example
$N^1 \rightarrow R^1$	value series	bar chart, pie chart, etc.





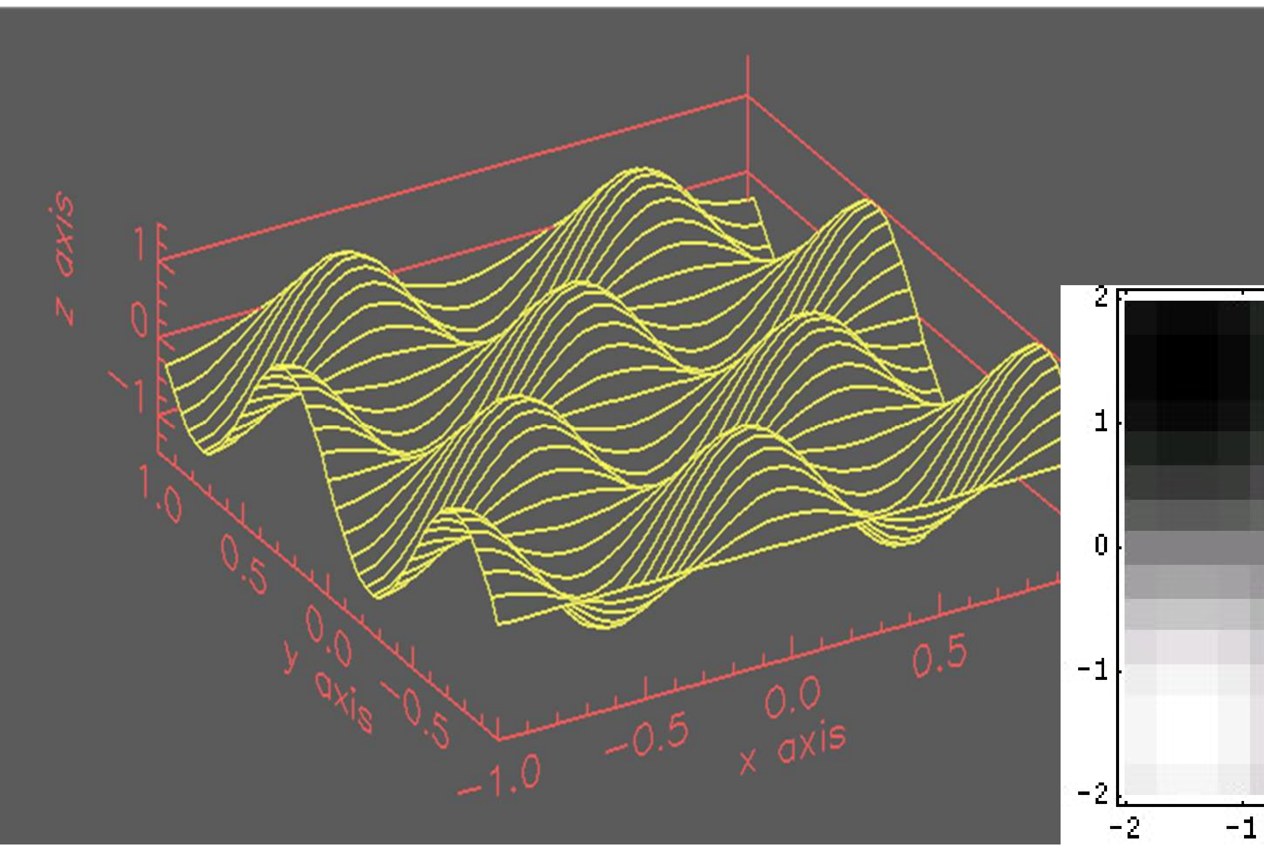
Visualization Examples

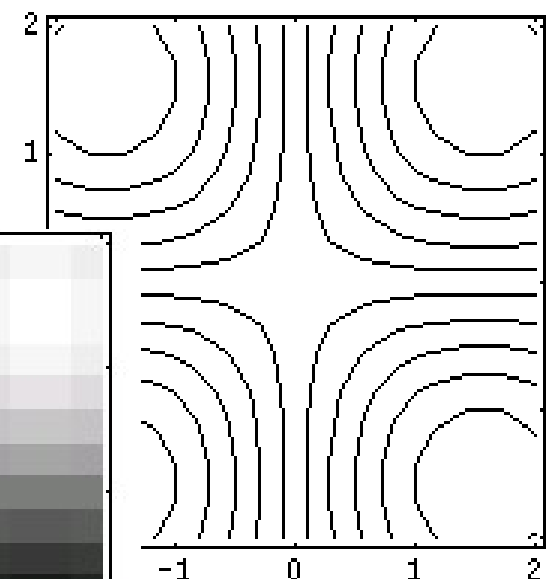
data	description	visualization example
$R^1 \rightarrow R^1$	function over R	(line) graph

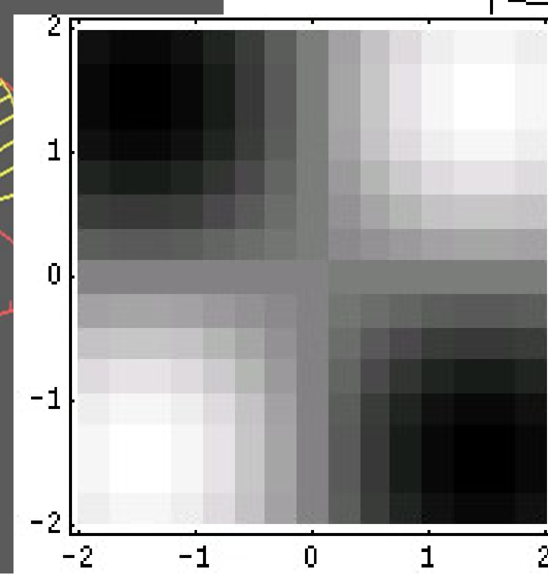


Visualization Examples

data	description	visualization example
$R^2 \rightarrow R^1$	function over R^2	2D-height map in 3D, contour lines in 2D, false colors (heat map)

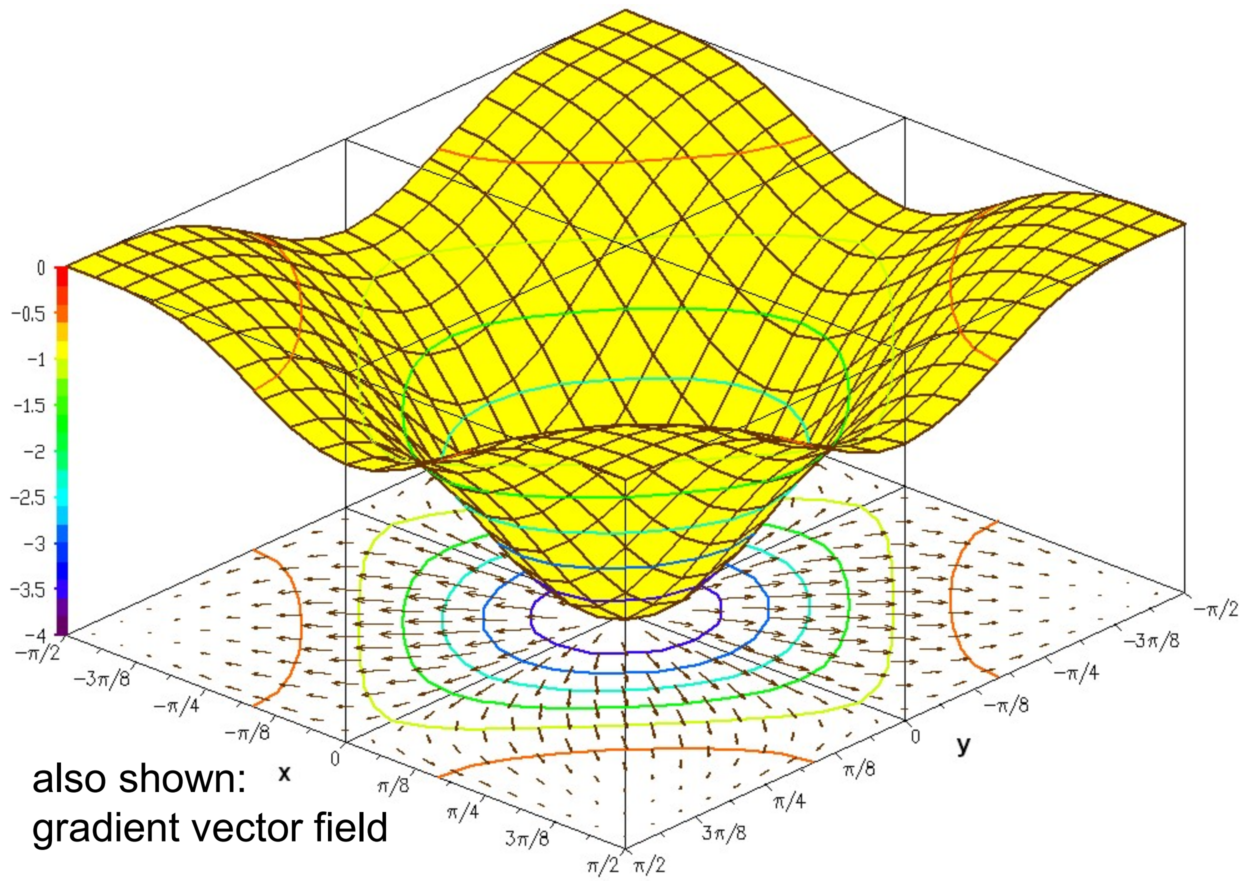




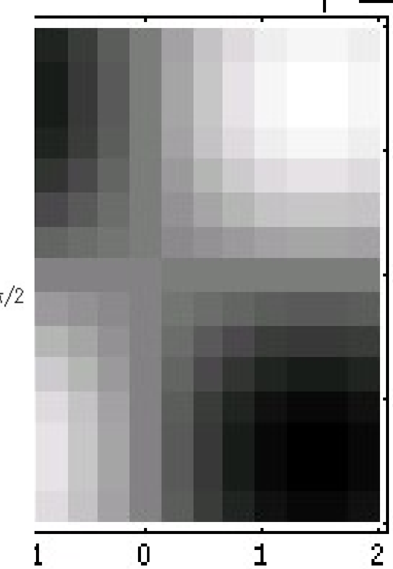


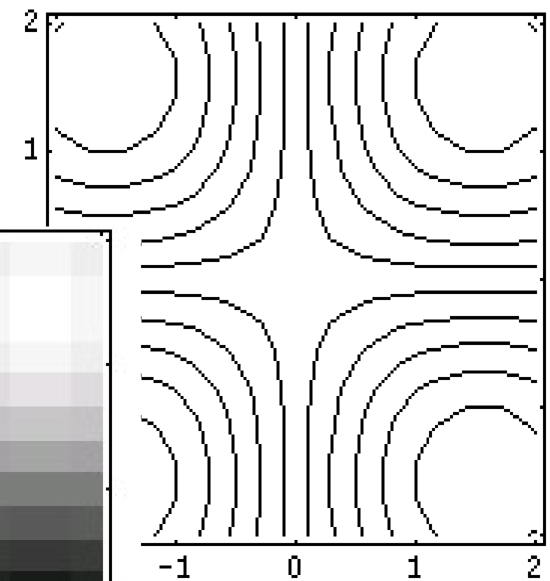
Visualization Examples

data	description	visualization example
$R^2 \rightarrow R^1$	function over R^2	2D-height map in 3D, contour lines in 2D, false colors (heat map)



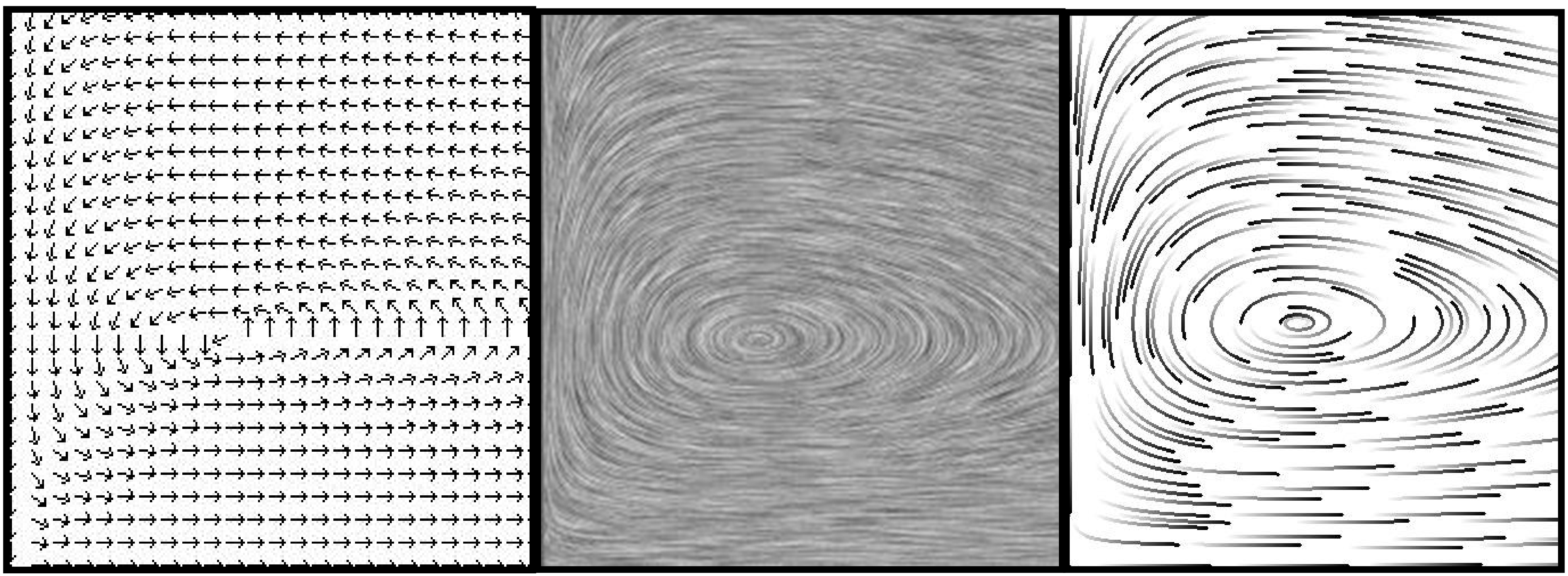
also shown:
gradient vector field





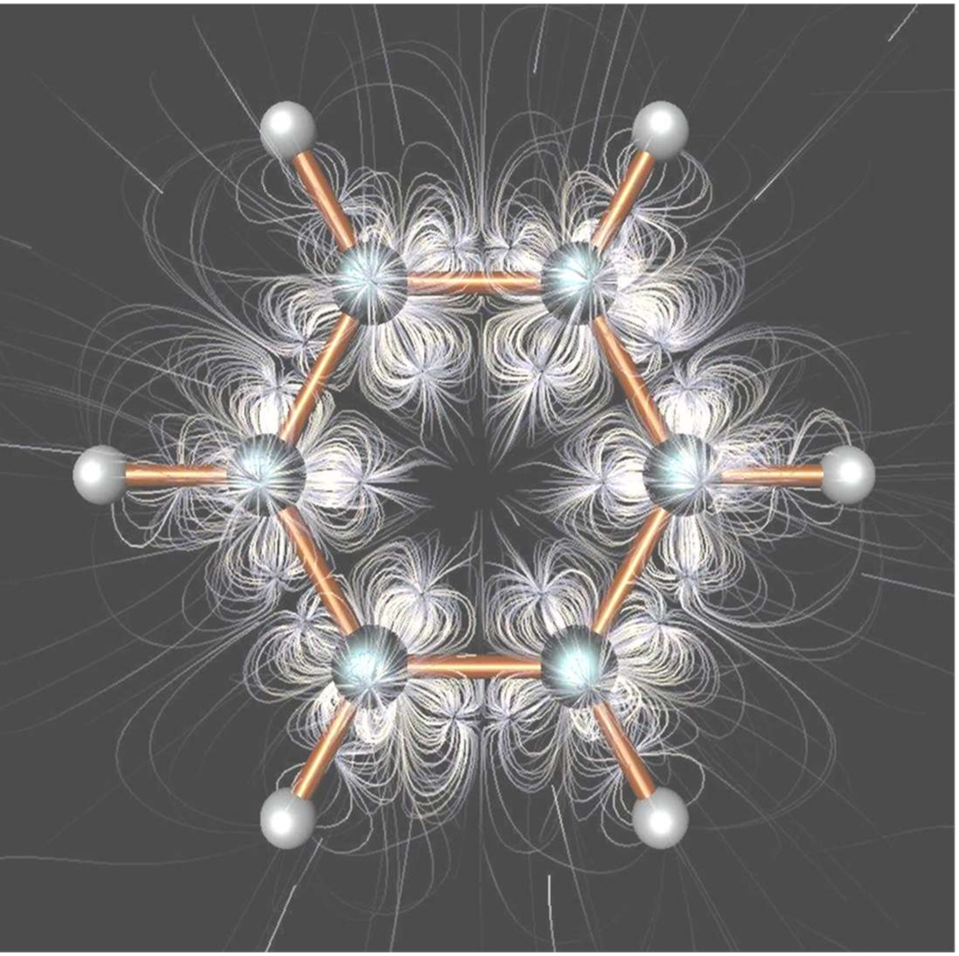
Visualization Examples

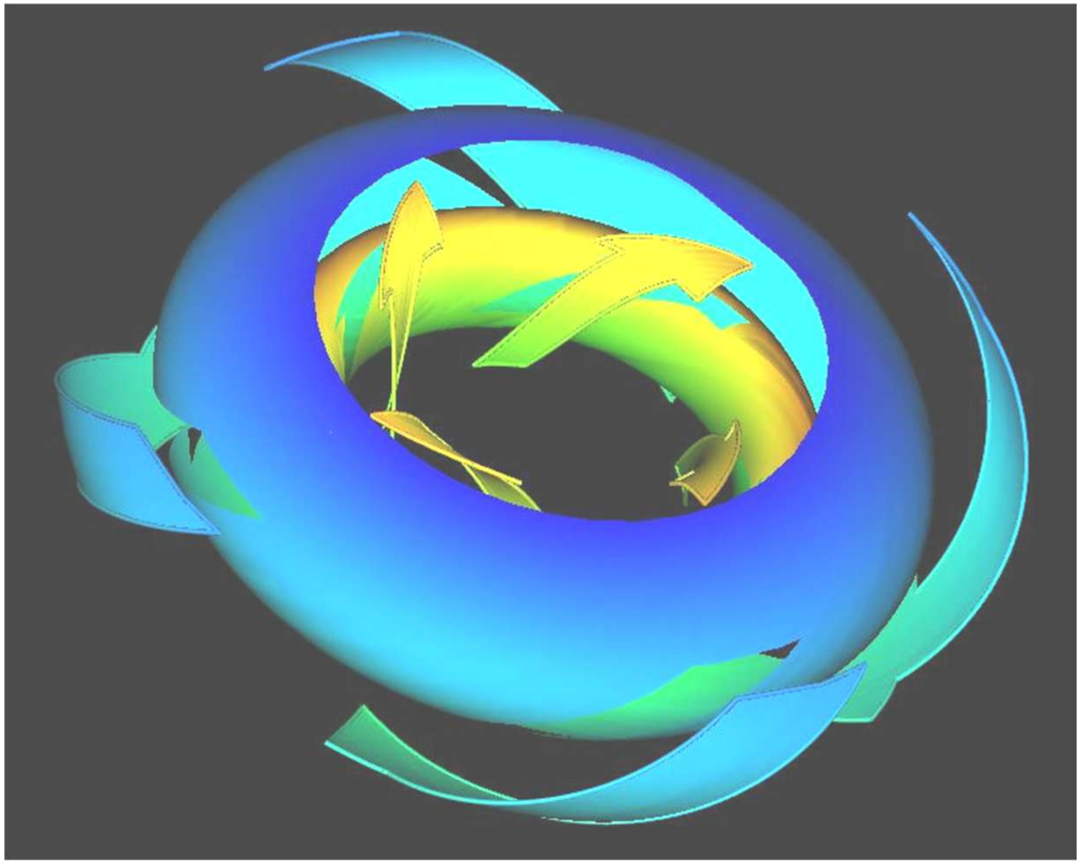
data	description	visualization example
$R^2 \rightarrow R^2$	2D-vector field	hedgehog plot, LIC, streamlets, etc



Visualization Examples

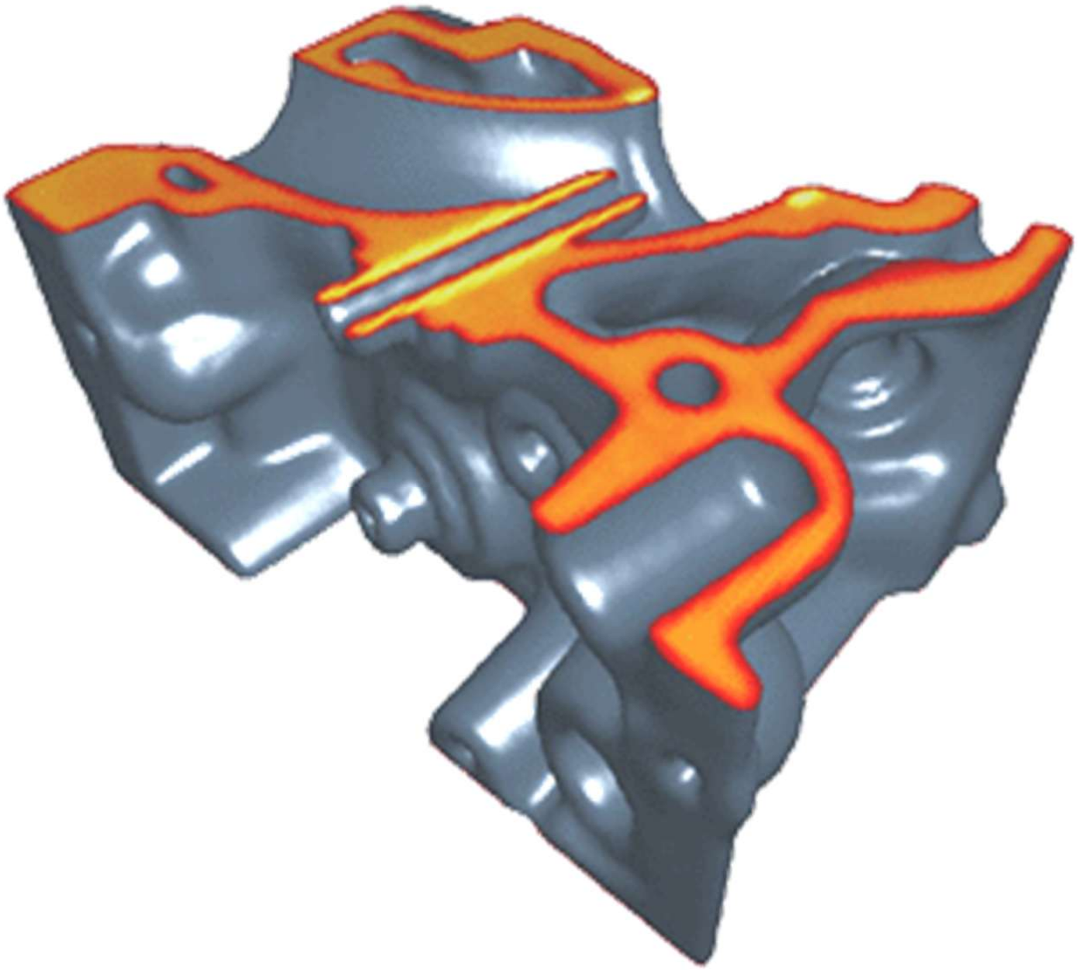
data	description	visualization example
$R^3 \rightarrow R^3$	3D-flow	streamlines, streamsurfaces

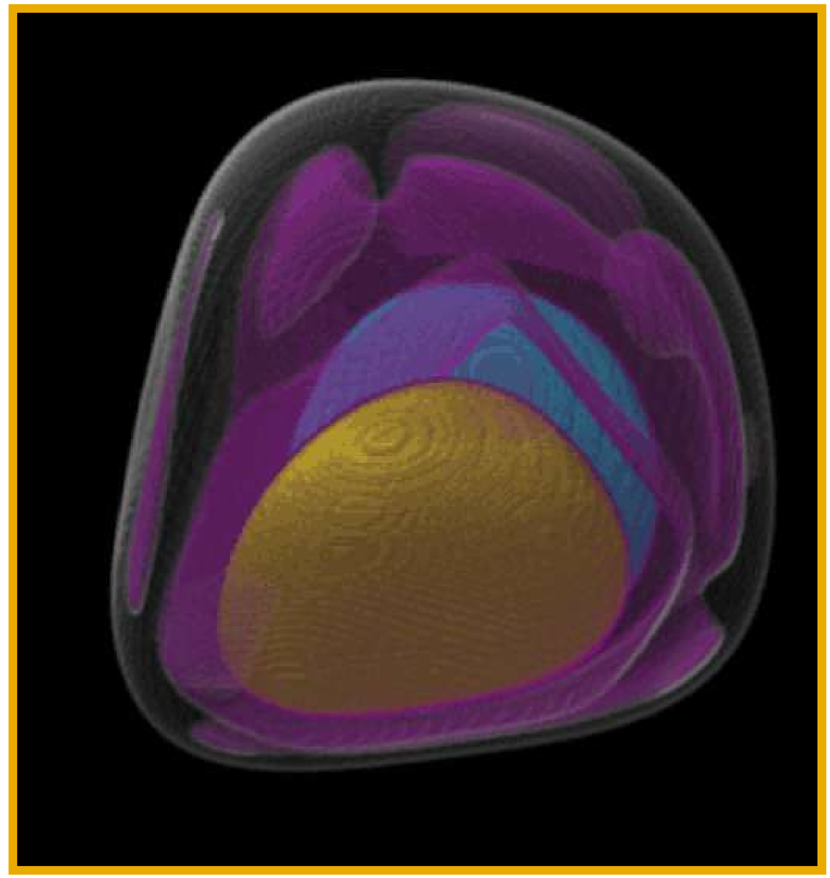




Visualization Examples

data	description	visualization example
$R^3 \rightarrow R^1$	3D-densities	iso-surfaces in 3D, volume rendering



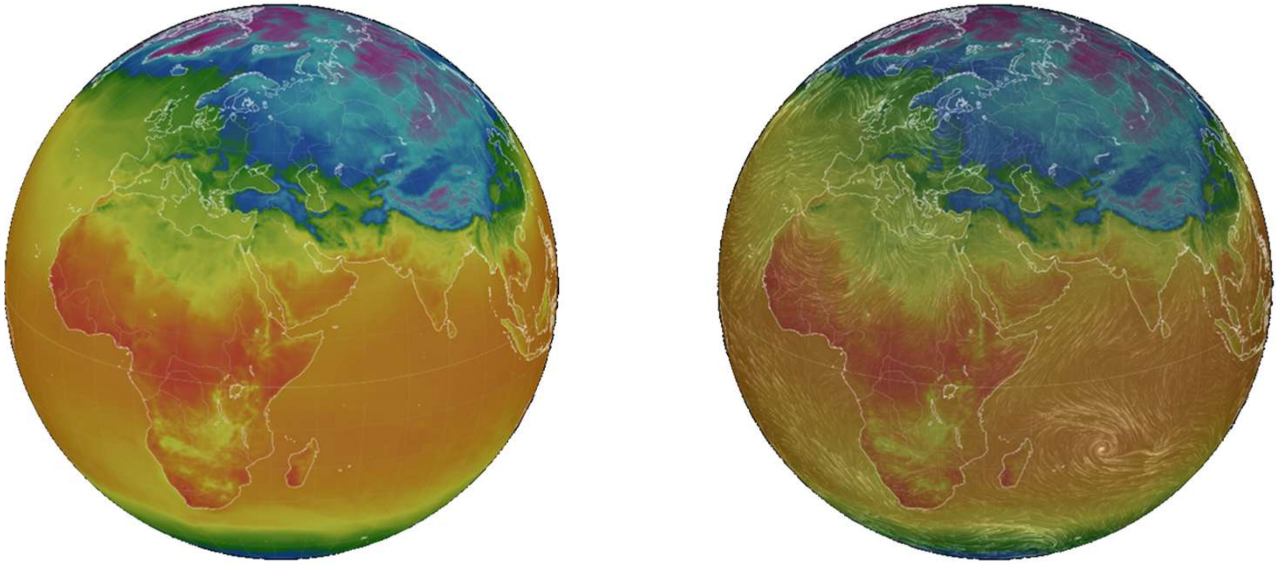


Domain is Not Always Euclidean

Manifolds

...

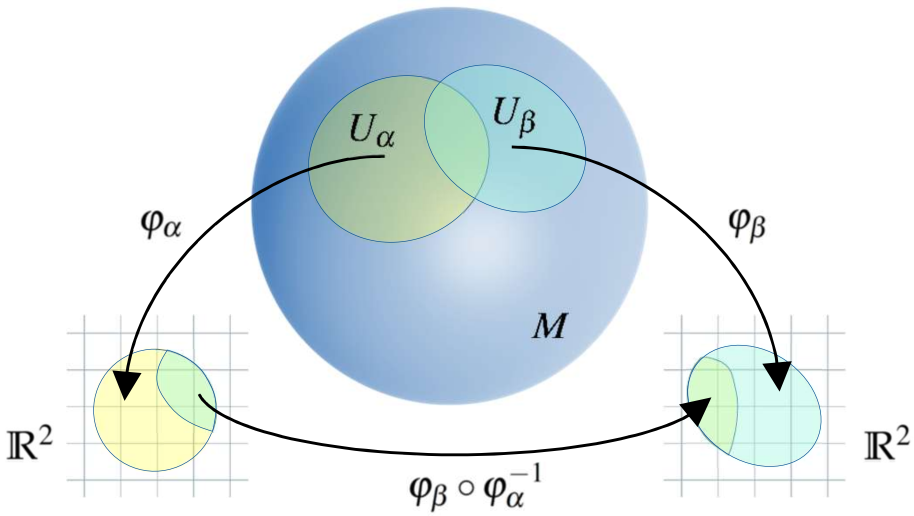
- Scalar, vector, tensor fields on manifolds

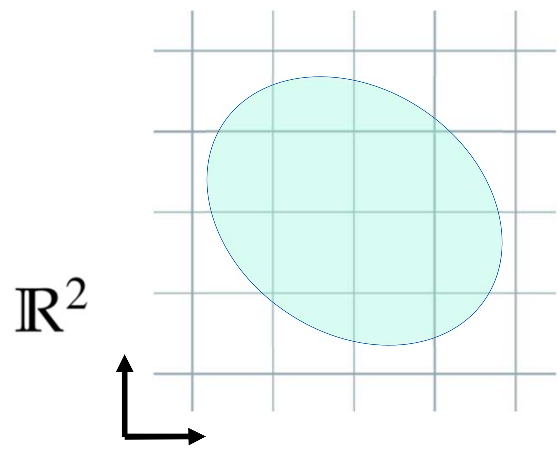


Topological Manifolds

Every point of an *n*-manifold is homeomorphic
(topologically equivalent) to a region of $\mathbb{R}^n$

Think about being able to assign coordinates to a region:
coordinate chart; (collection of charts: atlas)



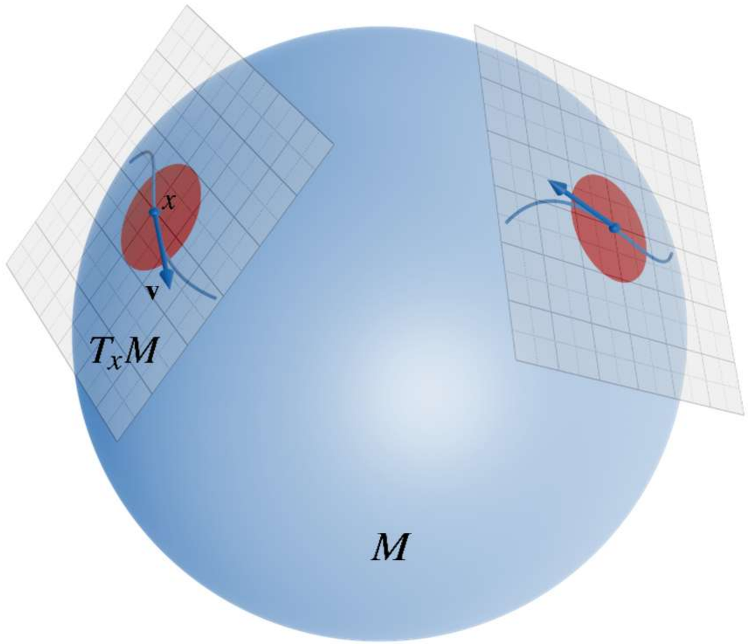


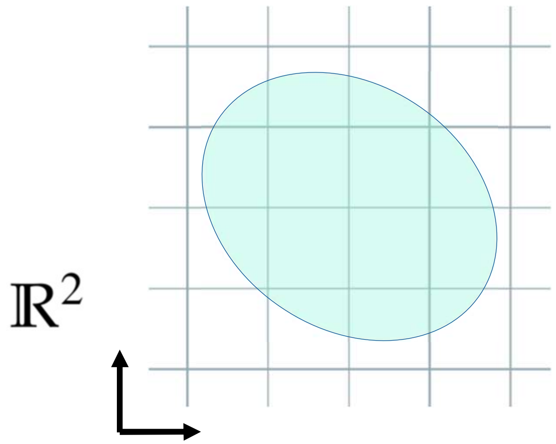
Smooth Manifolds

Well-defined tangent space at every point

- Dimensionality of each tangent space is the same as that of manifold

Enables calculus on manifolds (and vector fields, tensor fields, …)





Sampled Functions and Data Structures

Data Representation

- Discrete (sampled) representations
  - The objects we want to visualize are often ‘continuous’
  - But in most cases, the visualization data is given only at discrete locations in space and/or time
  - Discrete structures consist of samples, from which grids/meshes consisting of cells are generated
- Primitives in different dimensions

dimension	cell	mesh
0D	points	
1D	lines (edges)	polyline(–gon)
2D	triangles, quadrilaterals (rectangles)	2D mesh
3D	tetrahedra, prisms, hexahedra	3D mesh

Domain

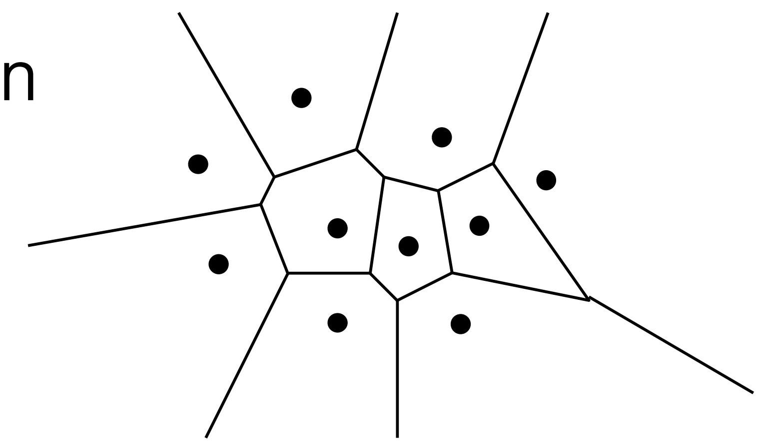
- The (geometric) shape of the domain is determined by the positions of sample points
- Domain is characterized by
  - Dimensionality: 0D, 1D, 2D, 3D, 4D, …
  - Influence: How does a data point influence its neighborhood?
  - Structure: Are data points connected? How? (Topology)

Domain

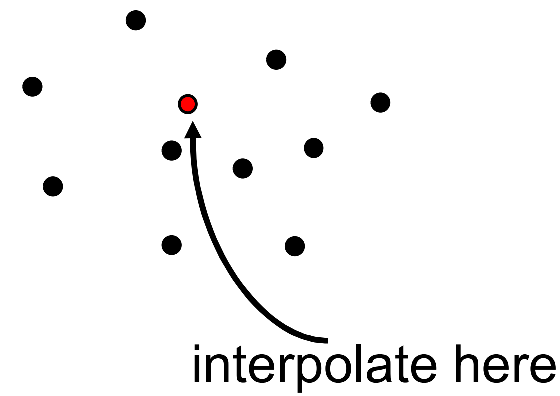
- Influence of data points
  - Values at sample points influence the data distribution in a certain region around these samples
  - To reconstruct the data at arbitrary points within the domain, the distribution of all samples has to be calculated
- Point influence
  - Only influence on point itself
- Local influence
  - Only within a certain region
    - Voronoi diagram
    - Cell-wise interpolation (see later in course)
- Global influence
  - Each sample might influence any other point within the domain
    - Material properties for whole object
    - Scattered data interpolation

Domain

- Voronoi diagram
  - Construct a region around each sample point that covers all points that are closer to that sample than to every other sample
  - Each point within a certain region gets assigned the value of the sample point
  - Nearest-neighbor interpolation



Domain



- Scattered data interpolation
  - At each point the weighted average of all sample points in the domain is computed
  - Weighting functions determine the support of each sample point
    - Radial basis functions simulate decreasing influence with increasing distance from samples
  - Schemes might be non-interpolating and expensive in terms of numerical operations

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama