King Abdullah University of
Science and Technology

KAUST

CS 247 – Scientific Visualization
Lecture 17: Volume Visualization, Pt. 4

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Reading Assignment #10 (until Apr 13)

Read (required):

- Real-Time Volume Graphics, Chapter 7 (GPU-Based Ray Casting)
- Real-Time Volume Graphics, Chapter 4.5 – 4.8

Quiz #2: Apr 9

Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

Content of questions

- Lectures (both actual lectures and slides)
- Reading assignments (except optional ones)
- Programming assignments (algorithms, methods)
- Solve short practical examples

Volume Rendering

Theory

Optical Models: Physical Model gives ODE

Optical Models for Direct Volume Rendering, Nelson Max

Emission-Absorption optical model

$$\boxed{\frac{dI}{ds}(s) = q(s) - \kappa(s)\, I(s)}$$

Right-hand side: *Rates of change* (derivatives) of light intensity along ray

Absorption rate is proportional to light intensity: Solution is exponential

Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model

true emission **true *absorption***

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

Back-to-front compositing

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

here, all colors are *associated* colors!

Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

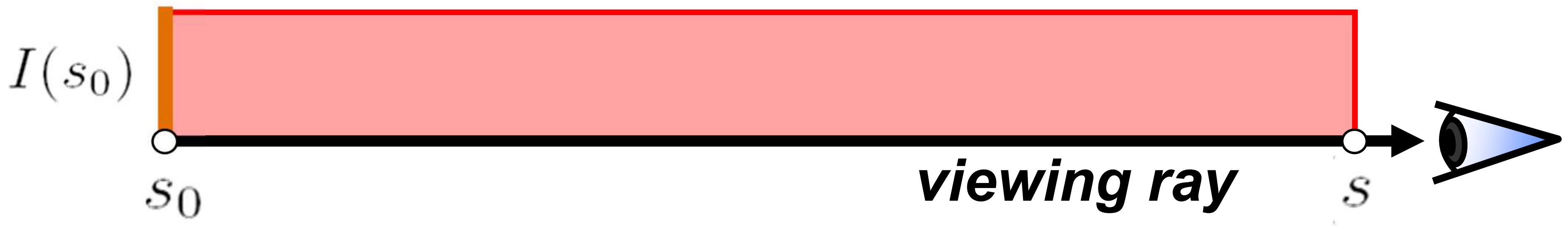
Initial intensity at s_0

$$I(s) = \boxed{I(s_0)}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Initial intensity at s_0

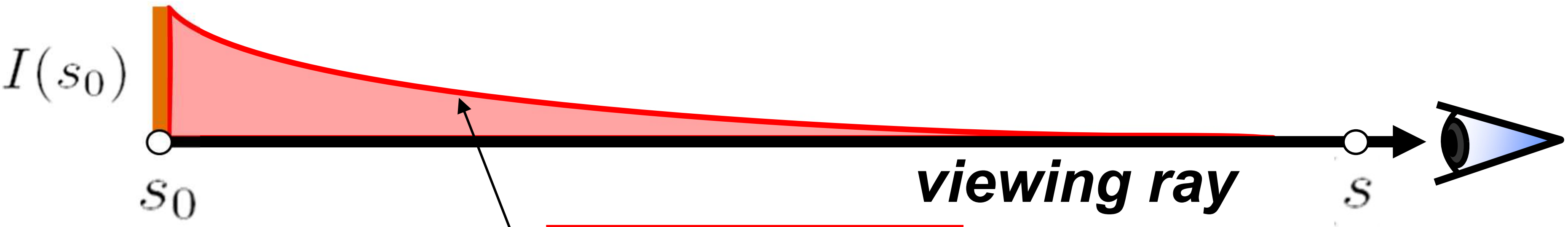
$$I(s) = \boxed{I(s_0)}$$

Without absorption all the initial radiant energy would reach the point ***s.***

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



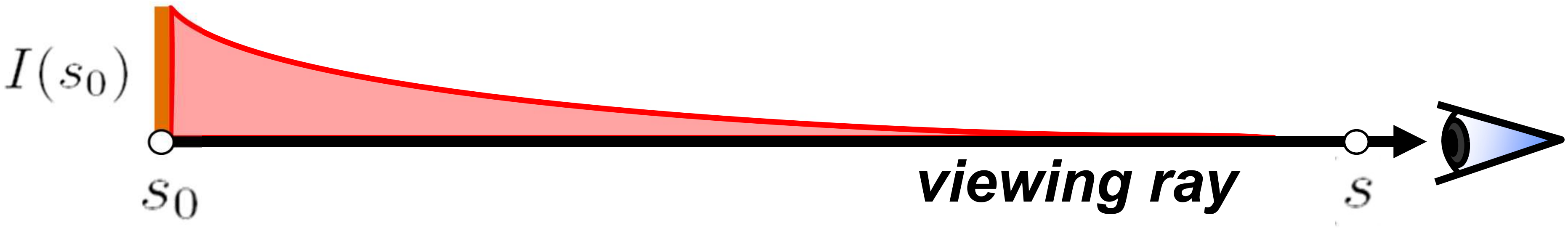
Absorption along the ray segment s_0 - s

$$I(s) = I(s_0)\boxed{e^{-\tau(s_0,s)}}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

Optical depth τ

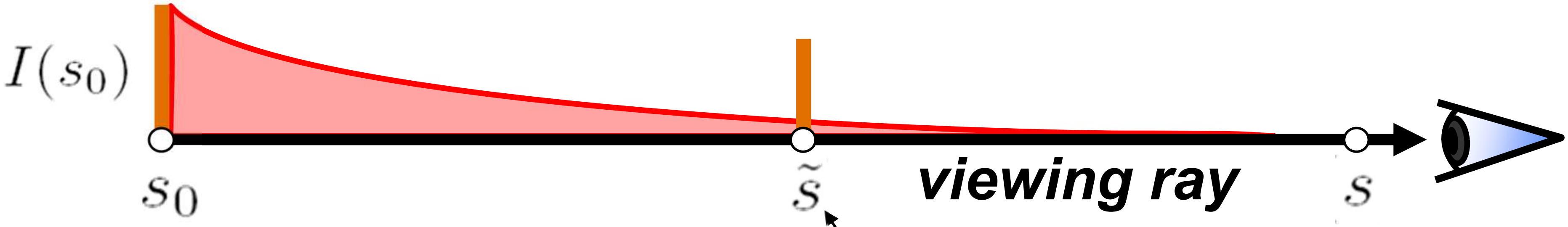
Absorption κ

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

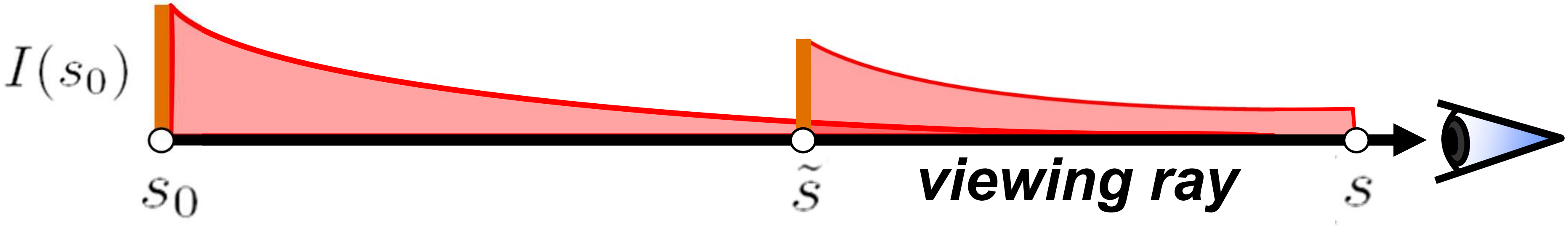
Active emission
at point $\tilde{s}$

$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \boxed{q(\tilde{s})}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

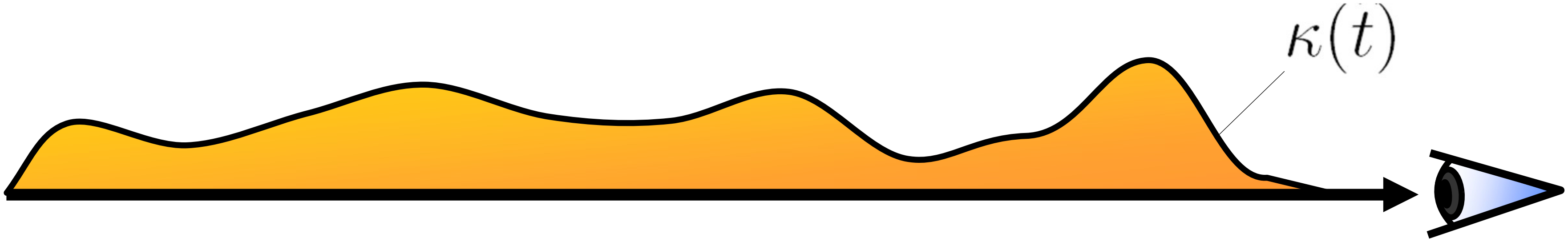
Physical model: emission and absorption, no scattering



Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

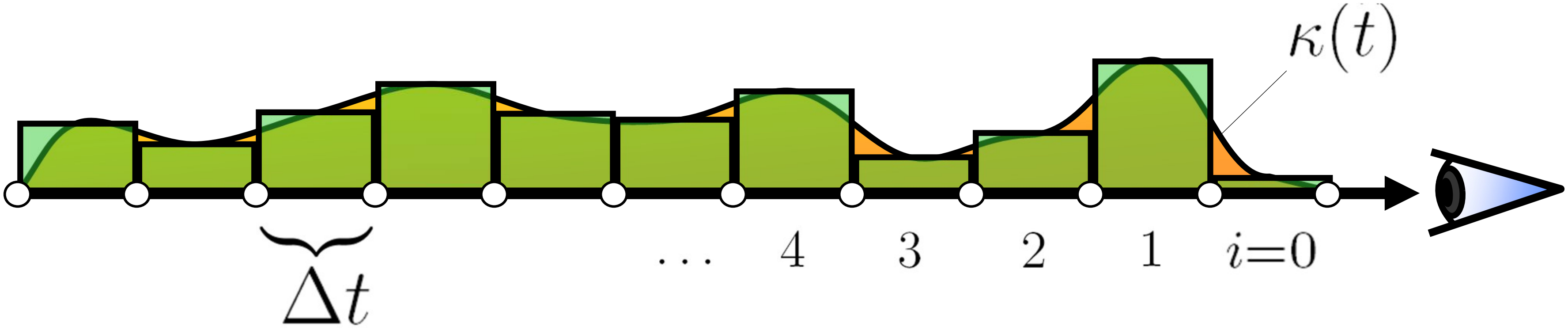
$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0,t) = \int_0^t \kappa(\hat{t})\, d\hat{t}$

Volume Rendering Integral: Numerical Solution

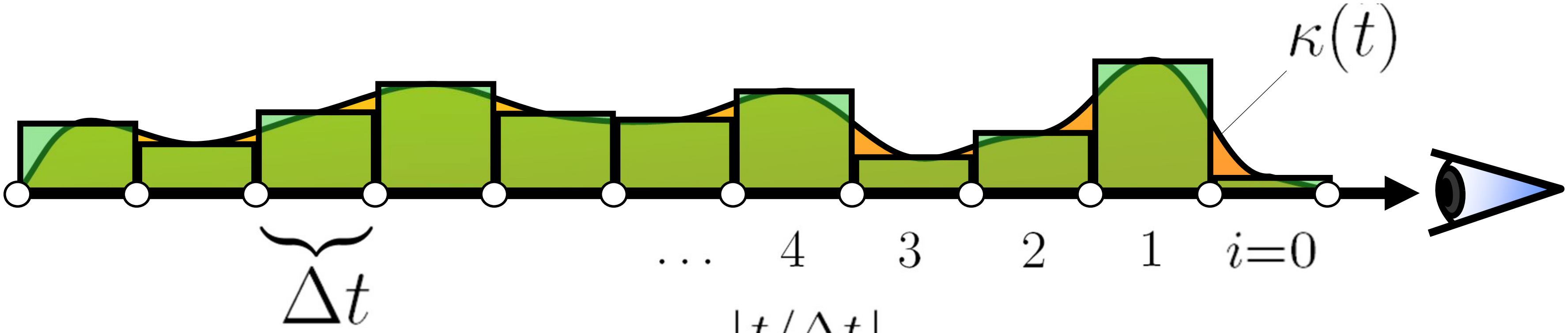


Optical depth: $\tau(0,t) \;=\; \int_0^t \kappa(\hat{t})\,d\hat{t}$

Approximate Riemann integral by Riemann sum:

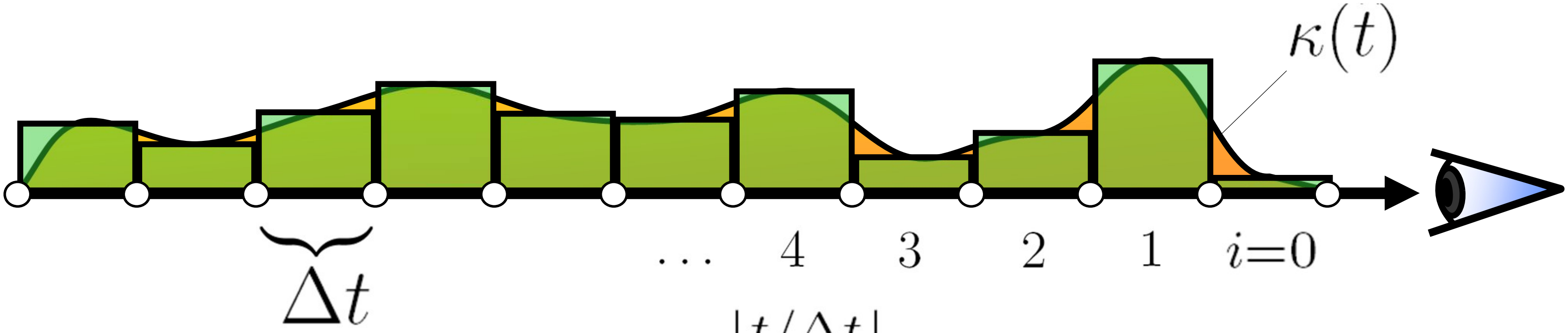
$$\tau(0,t) \;\approx\; \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Volume Rendering Integral: Numerical Solution



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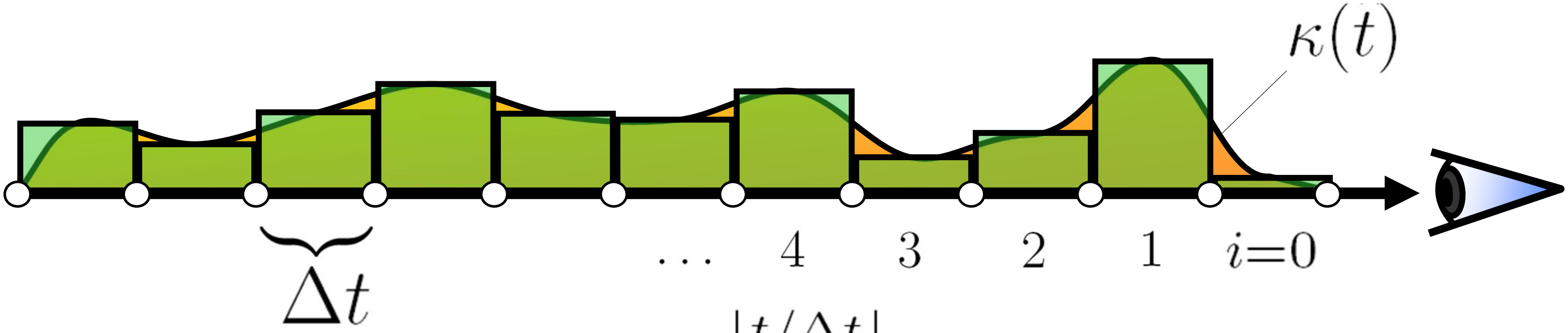
$$e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\,\Delta t}$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

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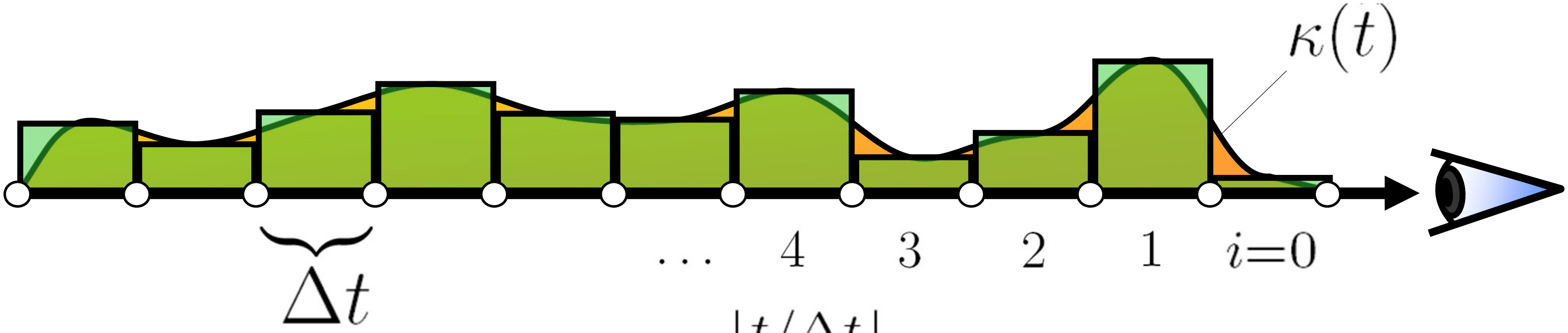
Volume Rendering Integral: Numerical Solution



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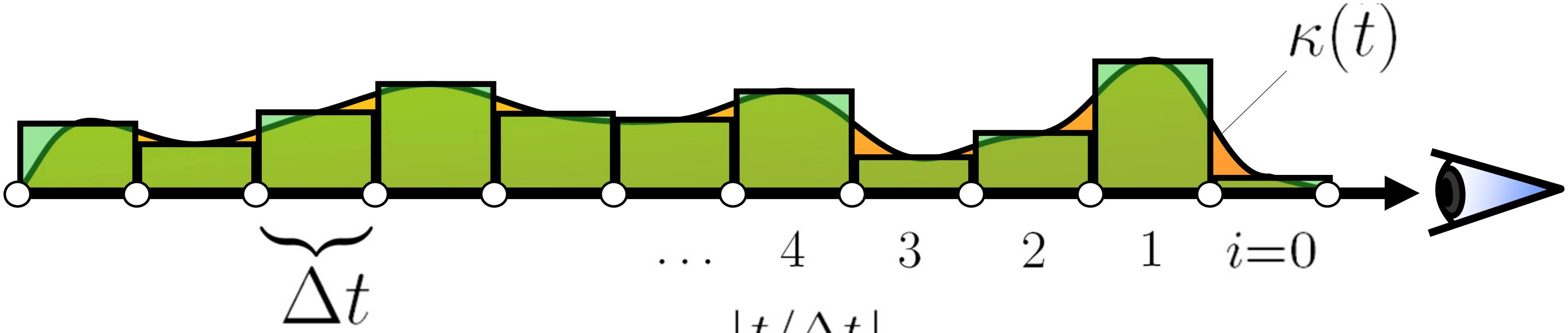
$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

Volume Rendering Integral: Numerical Solution



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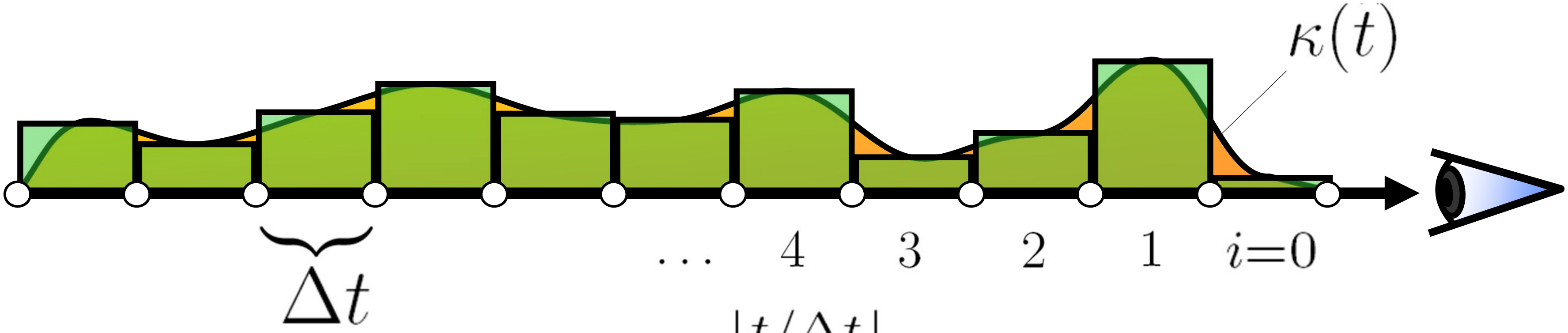
Now we introduce *opacity*:

$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\,\Delta t}$$

Volume Rendering Integral: Numerical Solution



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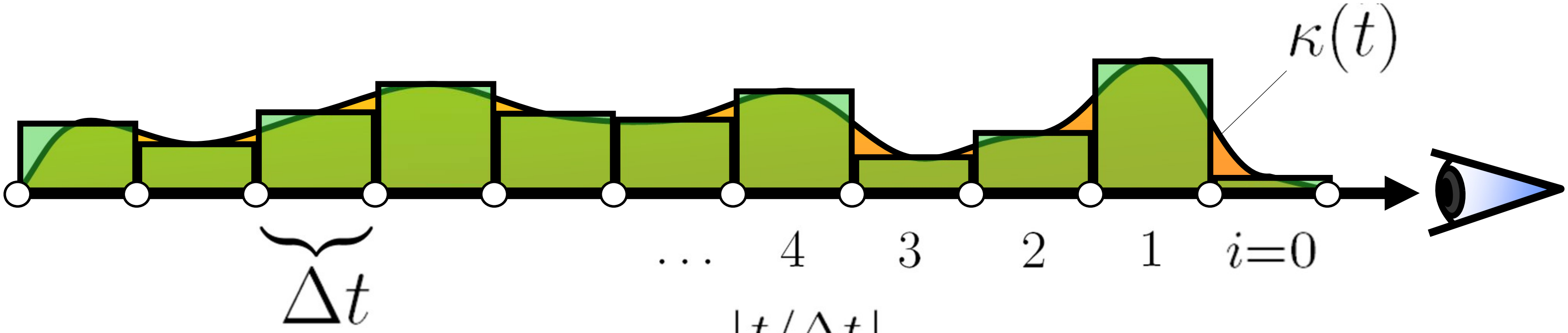

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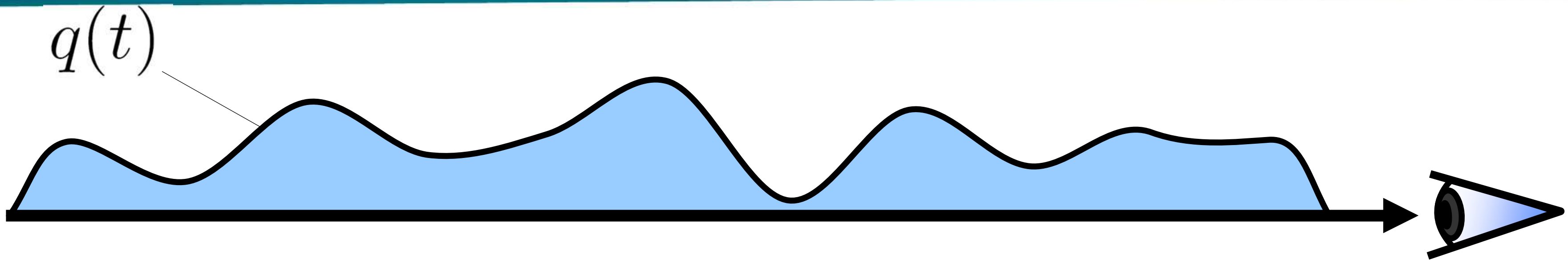
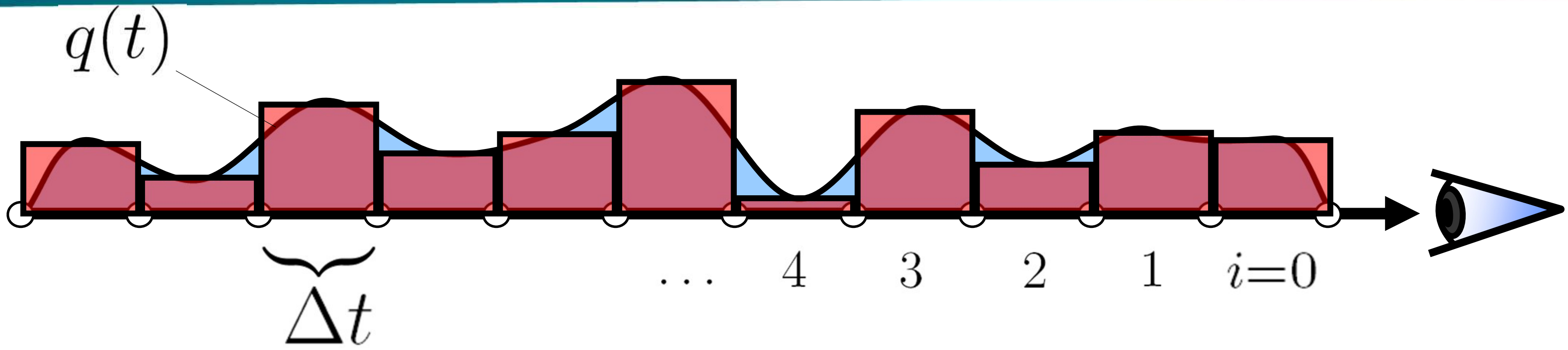


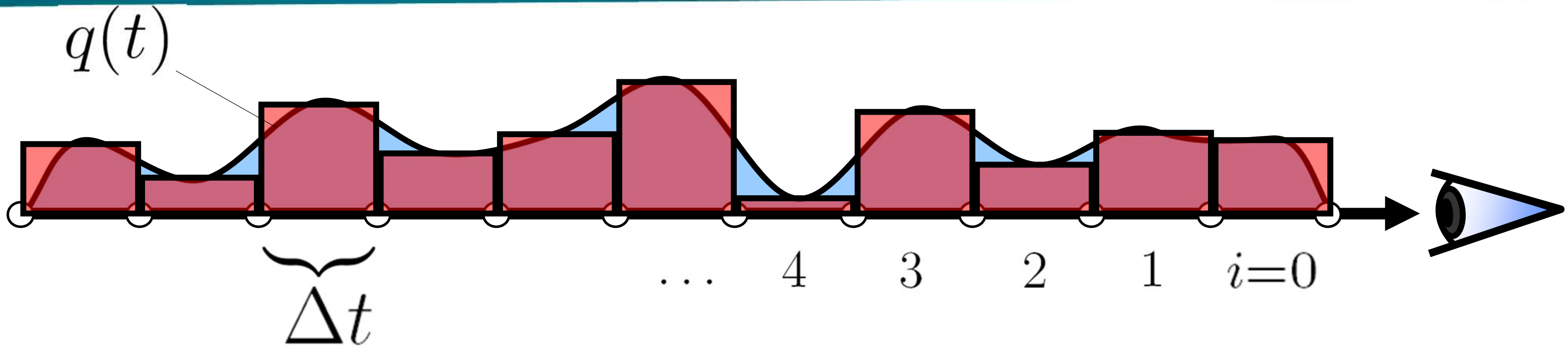
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

Volume Rendering Integral: Numerical Solution



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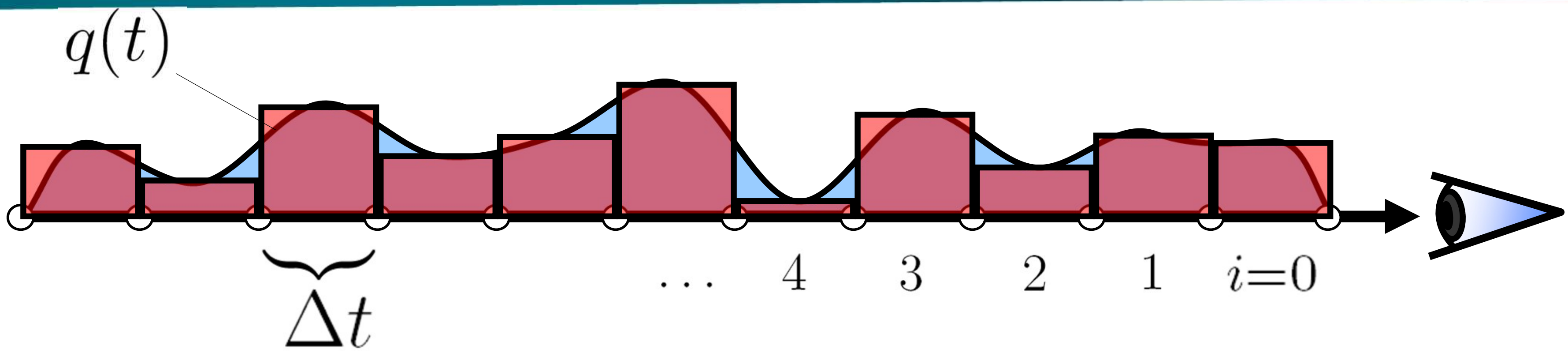
$$q(t) \approx C_i = c(i \cdot \Delta t)\, \Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i\, e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution



$$\boxed{e^{-\tilde{\tau}(0,t)}} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

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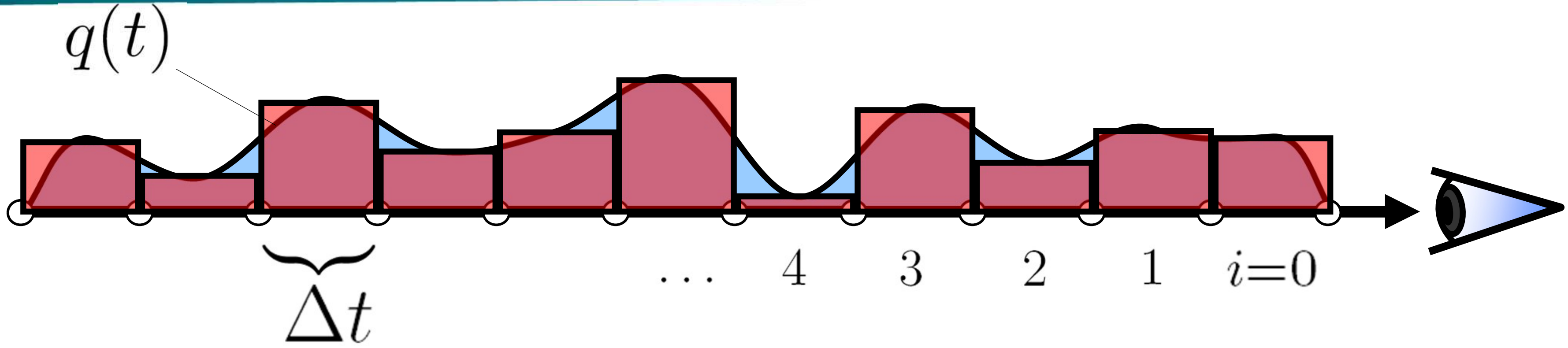
Volume Rendering Integral: Numerical Solution



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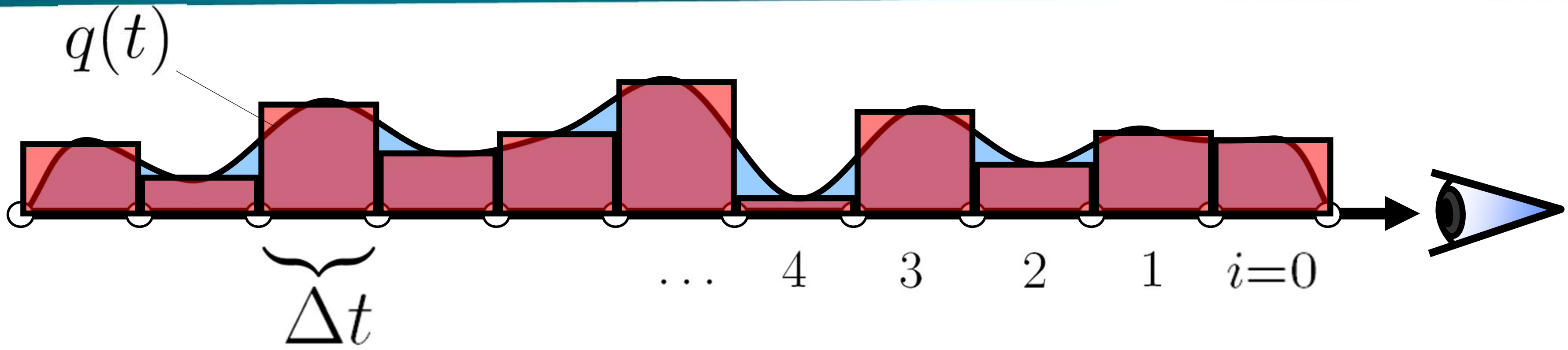
Volume Rendering Integral: Numerical Solution



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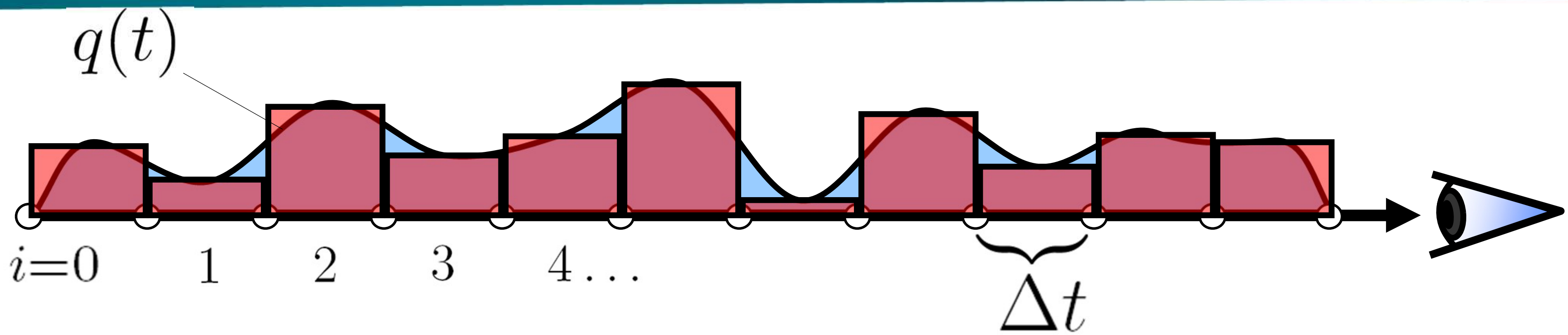
can be computed iteratively/recursively!

Volume Rendering Integral: Numerical Solution



Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !

can be computed iteratively/recursively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

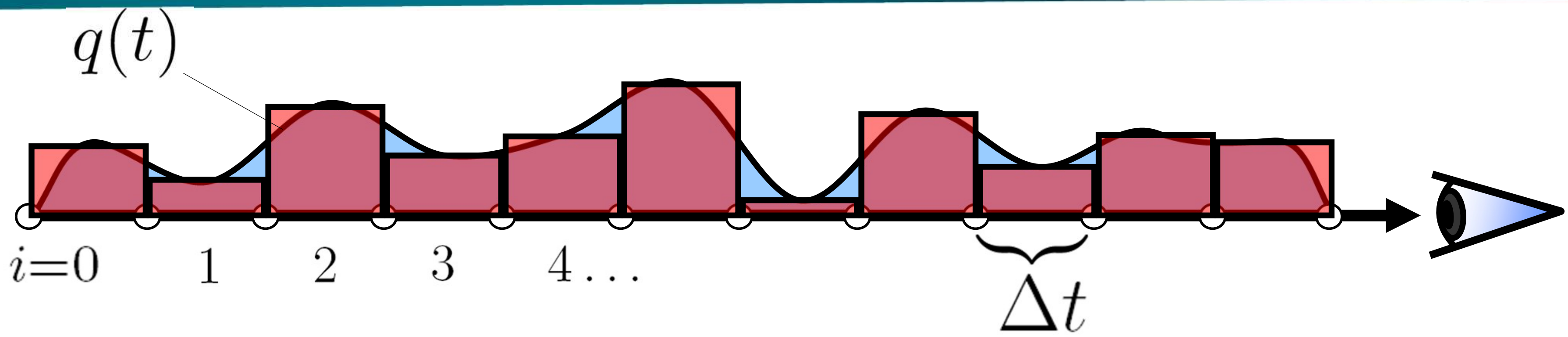
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C_i' = C_i + (1 - A_i)C_{i-1}'$$

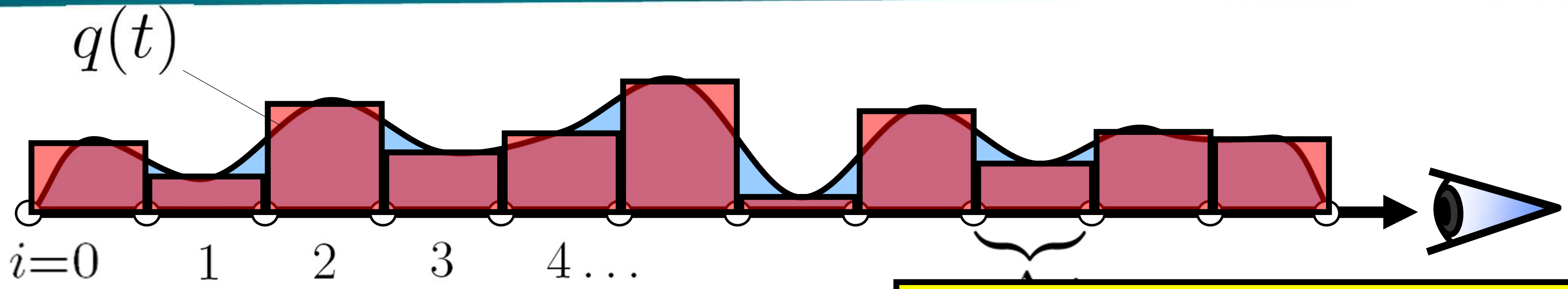
iterate from *i*=0 (back) to *i*=max (front): *i* increases

Front-to-back compositing

$$C_i' = C_{i+1}' + (1 - A_{i+1}')C_i$$
$$A_i' = A_{i+1}' + (1 - A_{i+1}')A_i$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1$$

iterate from *i*=0 (back)

Early Ray Termination:
Stop the calculation when

$$A'_i \approx 1$$

Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + \boxed{(1 - A'_{i+1})} C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model

true emission **true *absorption***

$$I(s) \quad = \quad I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

$$\tau(s_1, s_2) \quad = \quad \int_{s_1}^{s_2} \kappa(s)\, ds.$$

Iterative/recursive numerical solutions:

Back-to-front compositing

$$C'_i \quad = \quad C_i + (1 - A_i) C'_{i-1}$$

here, all colors are *associated* colors!

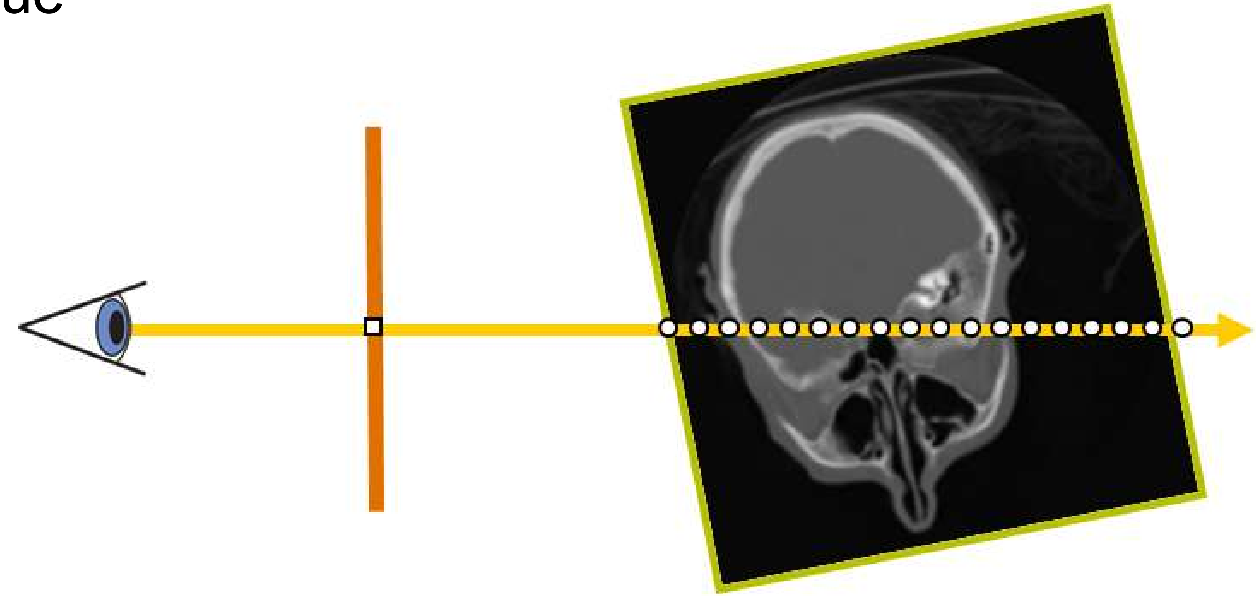
Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

Implementation

Implementation

- Ray setup
- Loop over ray
    - Resample scalar value
    - Classification
    - Shading
    - Compositing



Implementation

Ray setup

Loop over ray

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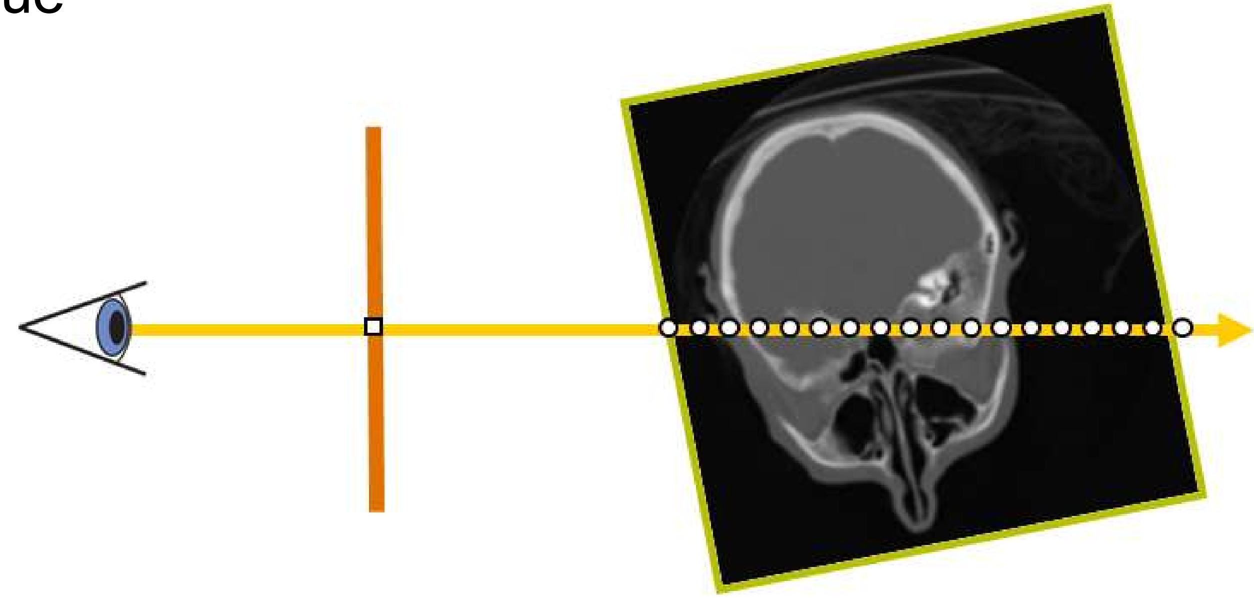
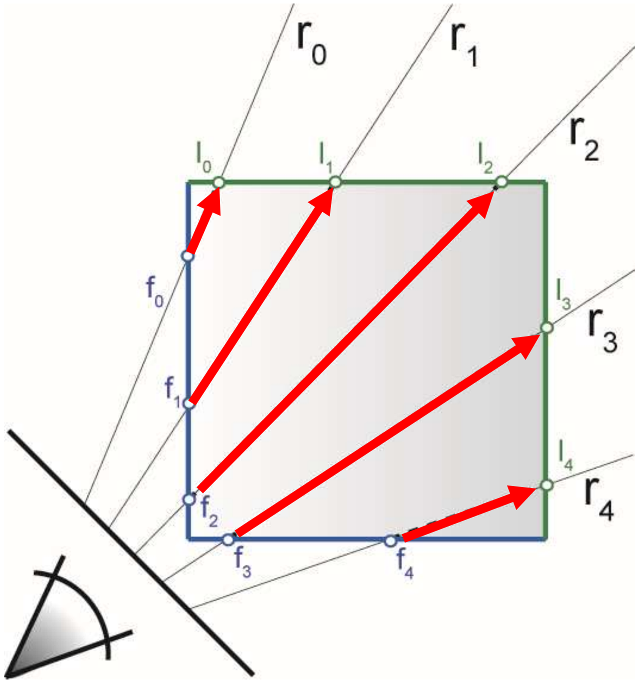


Ray Setup

Two main approaches:

- Procedural ray/box intersection
  [Röttger et al., 2003], [Green, 2004]
- Rasterize bounding box
  [Krüger and Westermann, 2003]



Some possibilities

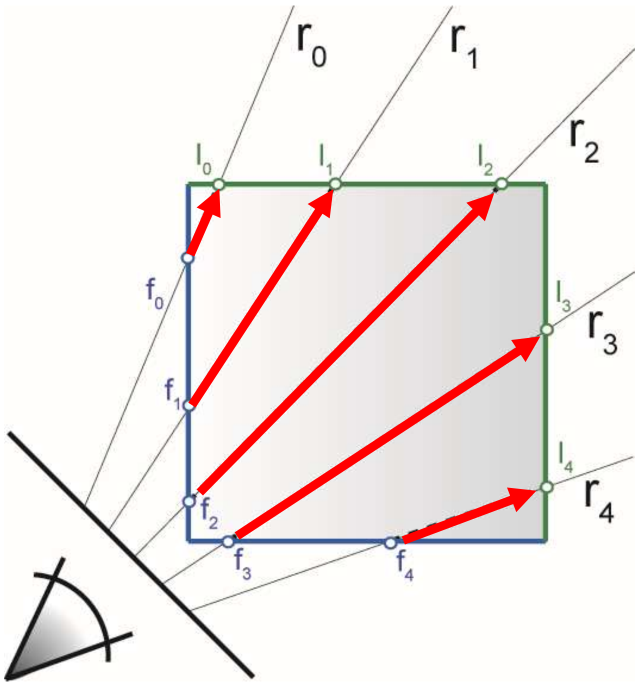
- Ray start position and exit check
- Ray start position and exit position
- Ray start position and direction vector

Procedural Ray Setup/Termination

- Everything handled in the fragment shader / CUDA kernel
- Procedural ray / bounding box intersection

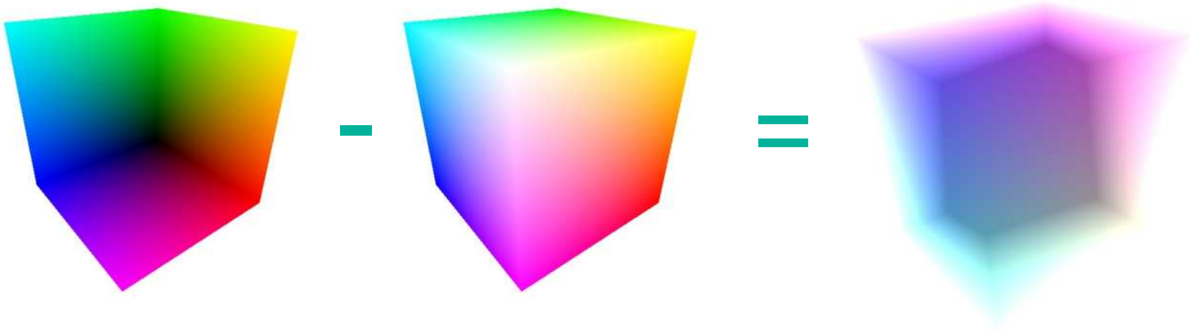
- Ray is given by camera position and volume entry position
- Exit criterion needed

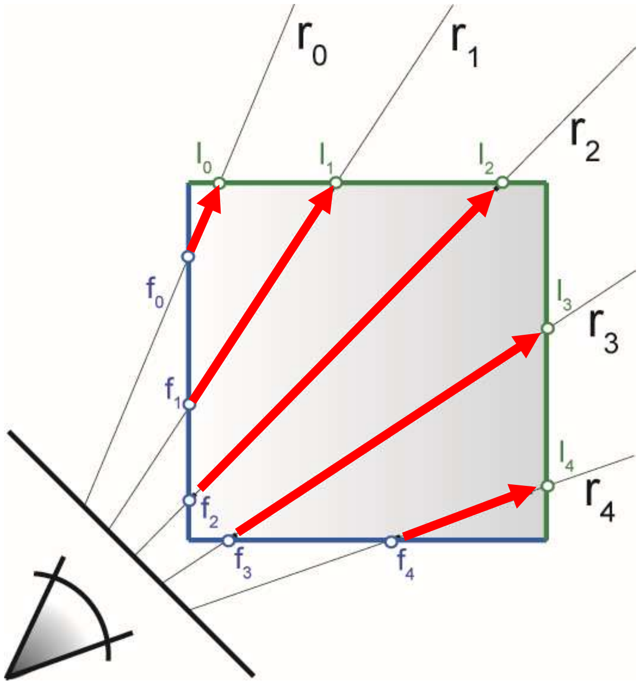
- Pro: simple and self-contained
- Con: full computational load per-pixel/fragment



Rasterization-Based Ray Setup

- Fragment == ray
- Need ray start pos, direction vector
- Rasterize bounding box

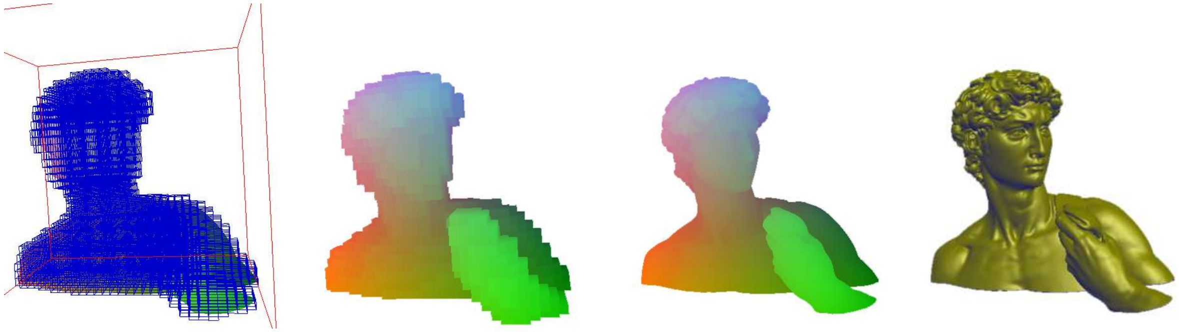


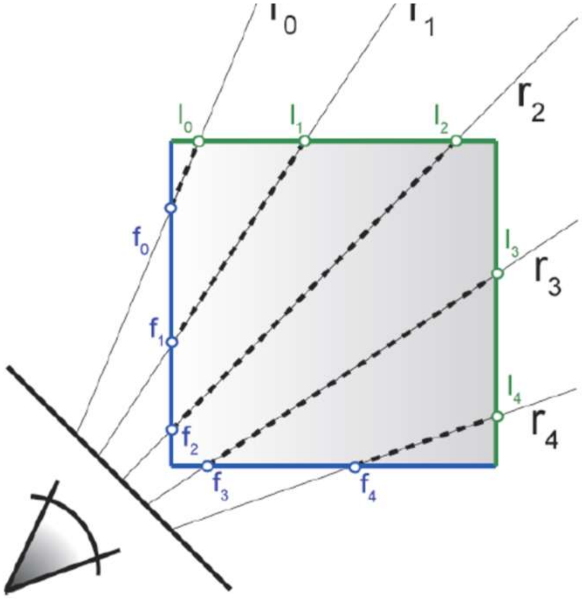


- Identical for orthogonal and perspective projection!

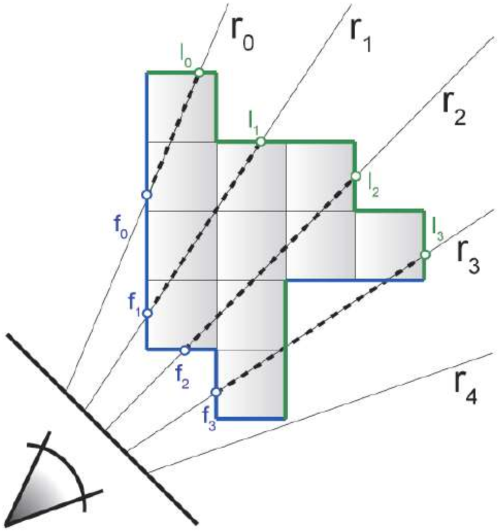
Object-Order Empty Space Skipping

Modify initial rasterization step





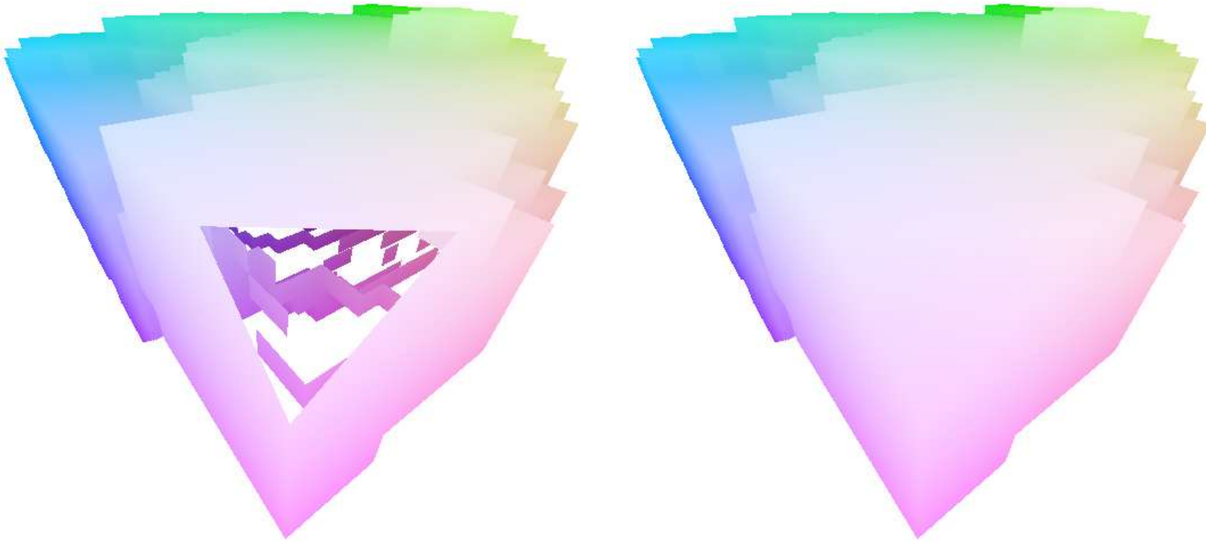
rasterize bounding box

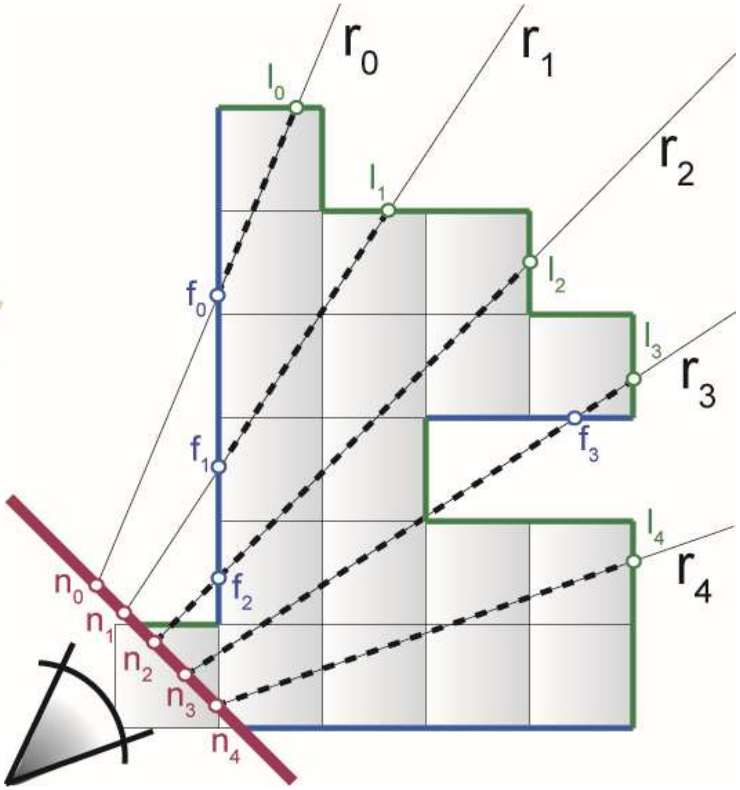


rasterize “tight" bounding geometry

Moving Into The Volume

Near clipping plane clips into front faces





Fill in holes with near clipping plane

Can use depth buffer [Scharsach et al., 2006]

Implementation

Ray setup

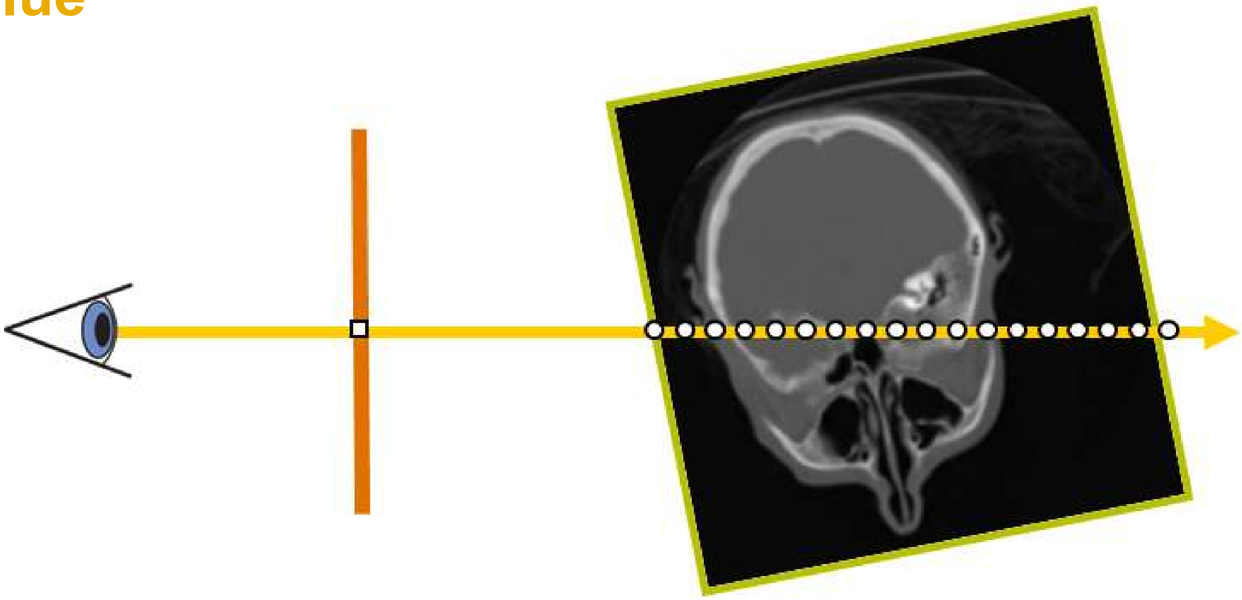
Loop over ray

  Resample scalar value

  Classification

  Shading

  Compositing



Classification – Transfer Functions

During Classification the user defines the ***"look"*** of the data.

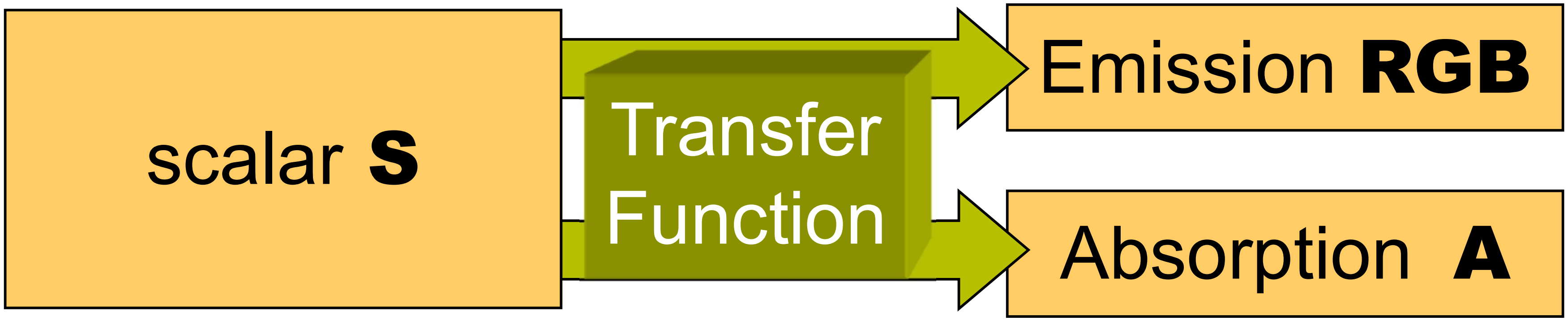
- Which parts are transparent?
- Which parts have what color?

Classification – Transfer Functions

During Classification the user defines the ***“look“*** of the data.

- Which parts are transparent?
- Which parts have what color?

The user defines a ***transfer function.***



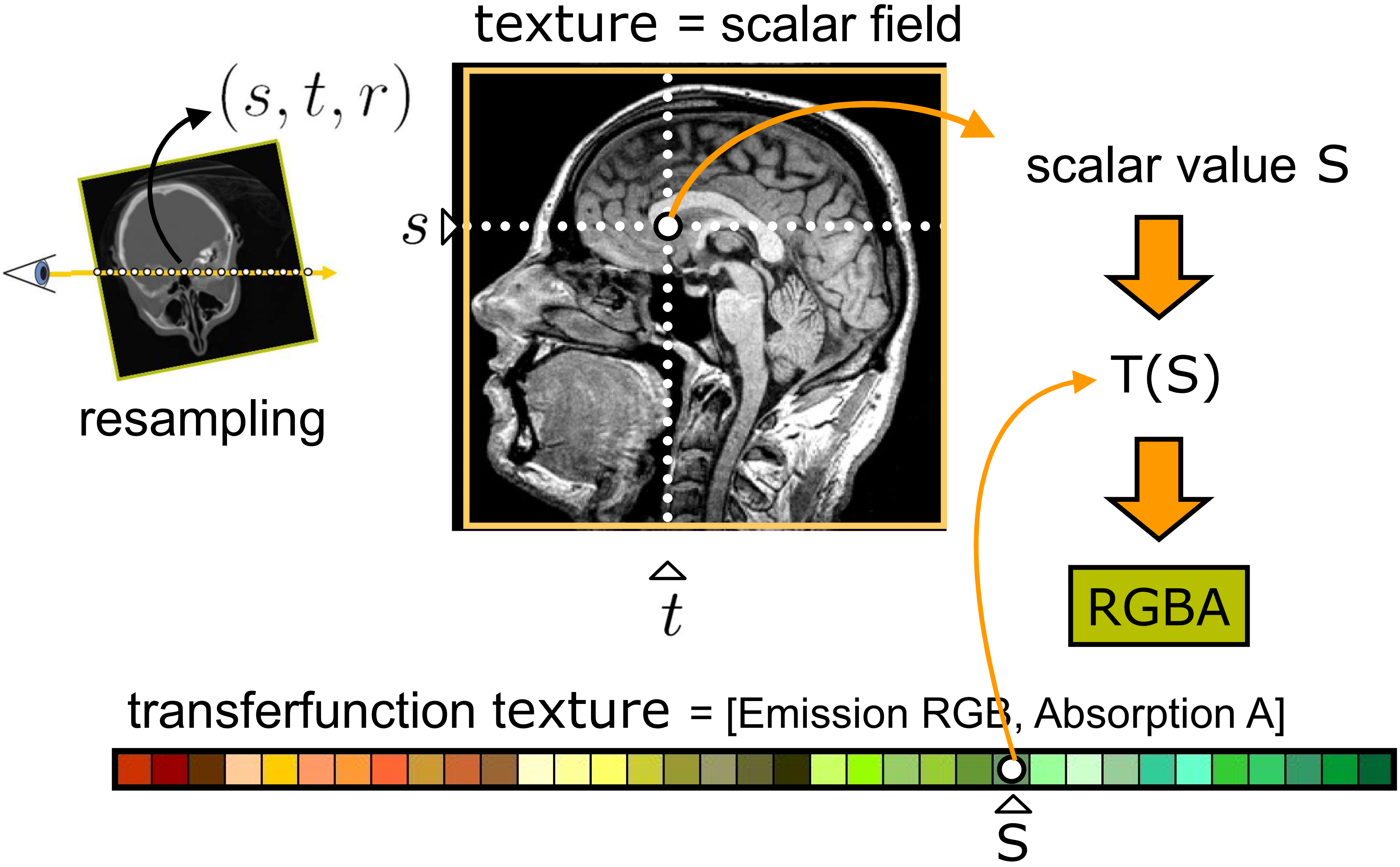
scalar **s**

Transfer Function

Emission **RGB**

Absorption **A**

1D Transfer Functions



texture = scalar field

(s, t, r)

resampling

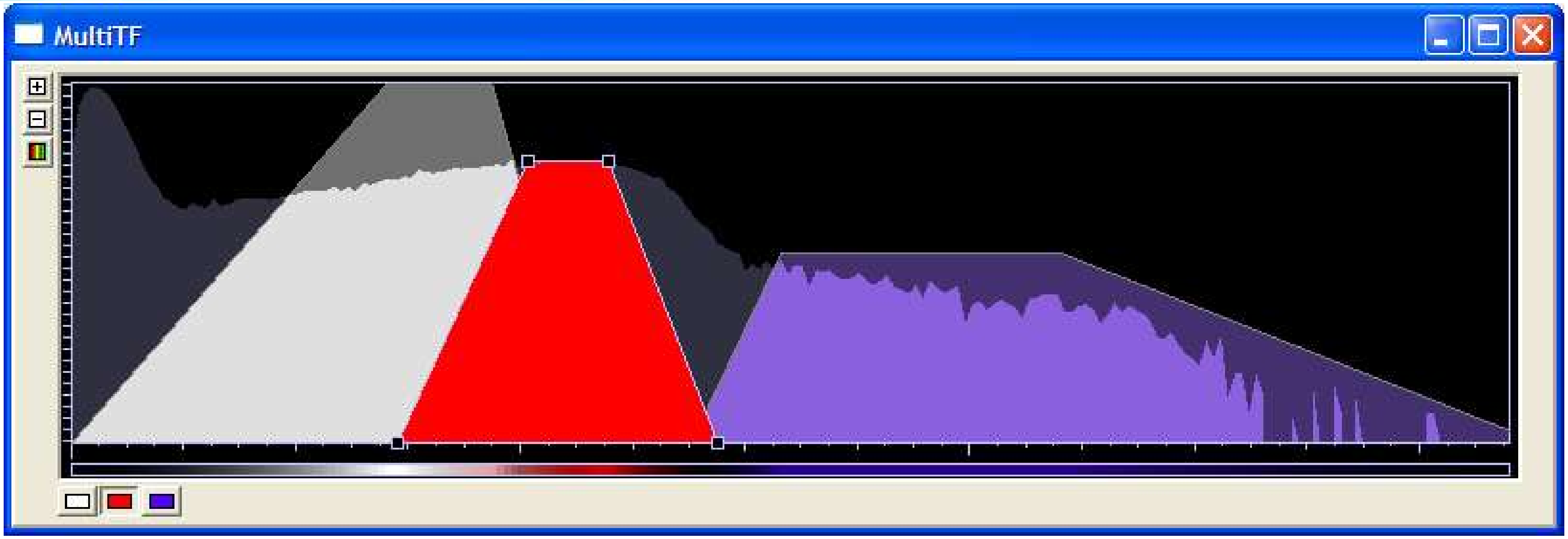
scalar value S

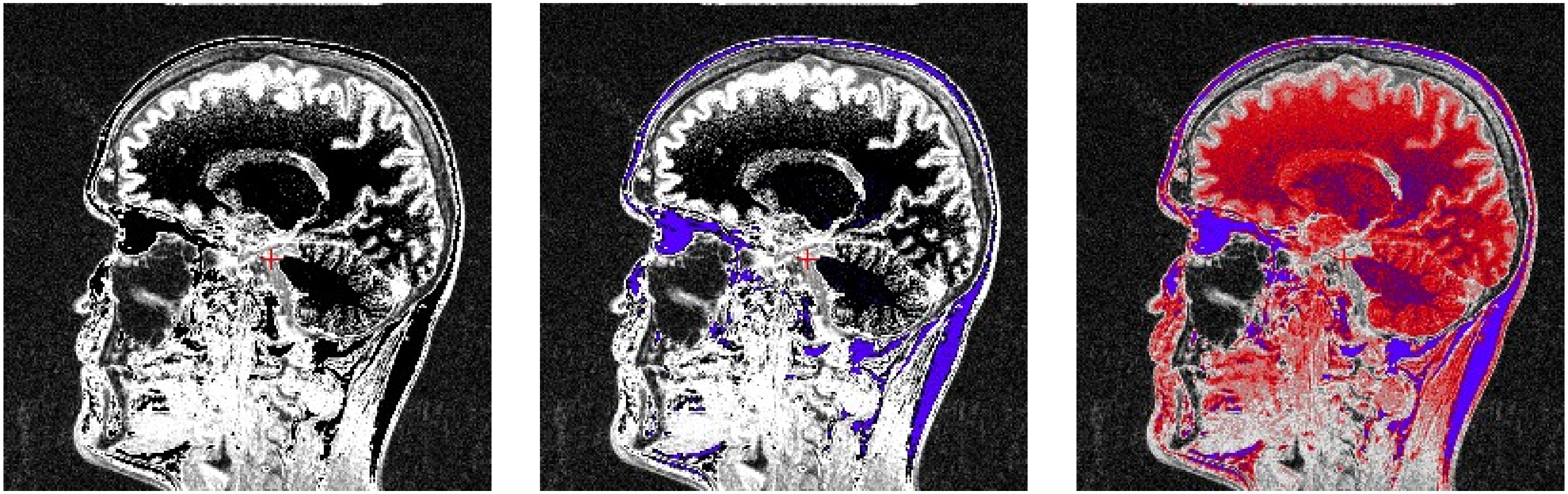
T(S)

RGBA

transferfunction texture = [Emission RGB, Absorption A]

1D Transfer Functions





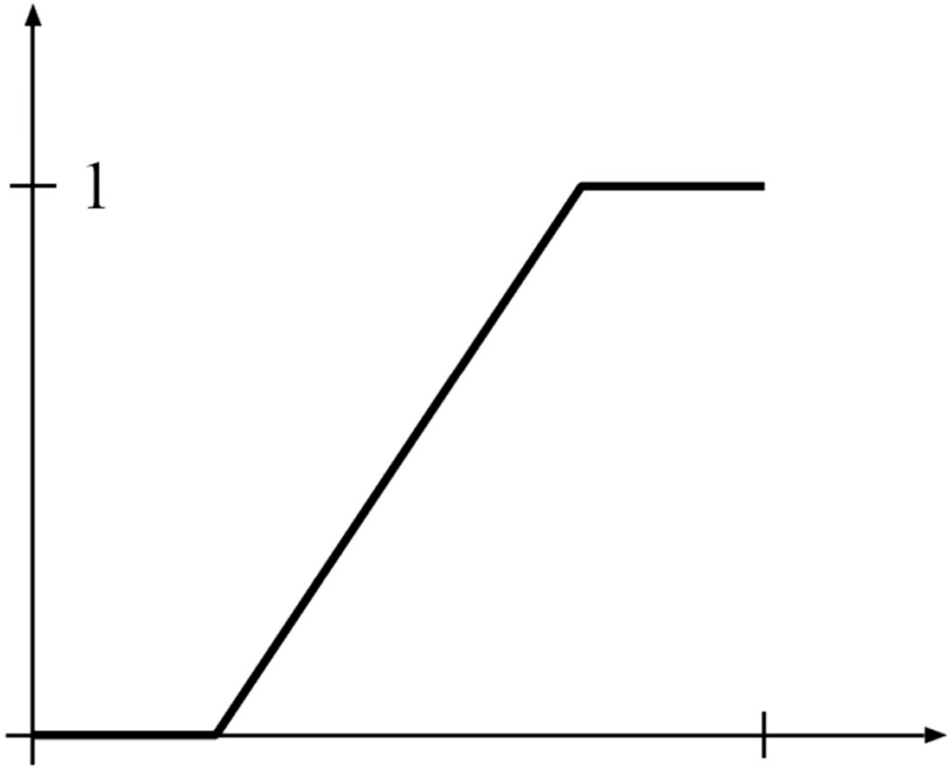
Applying Transfer Function: Cg Example

```
// Cg fragment program for post-classification
// using 3D textures
float4 main (float3 texUV : TEXCOORD0,
             uniform sampler3D volume_texture,
             uniform sampler1D transfer_function) :
  COLOR
{
   float index = tex3D(volume_texture, texUV);
   float4 result = tex1D(transfer_function, index);
   return result;
}
```

Windowing Transfer Function

Map input scalar range to output intensity range

- Select scalar range of interest
- Adjust contrast

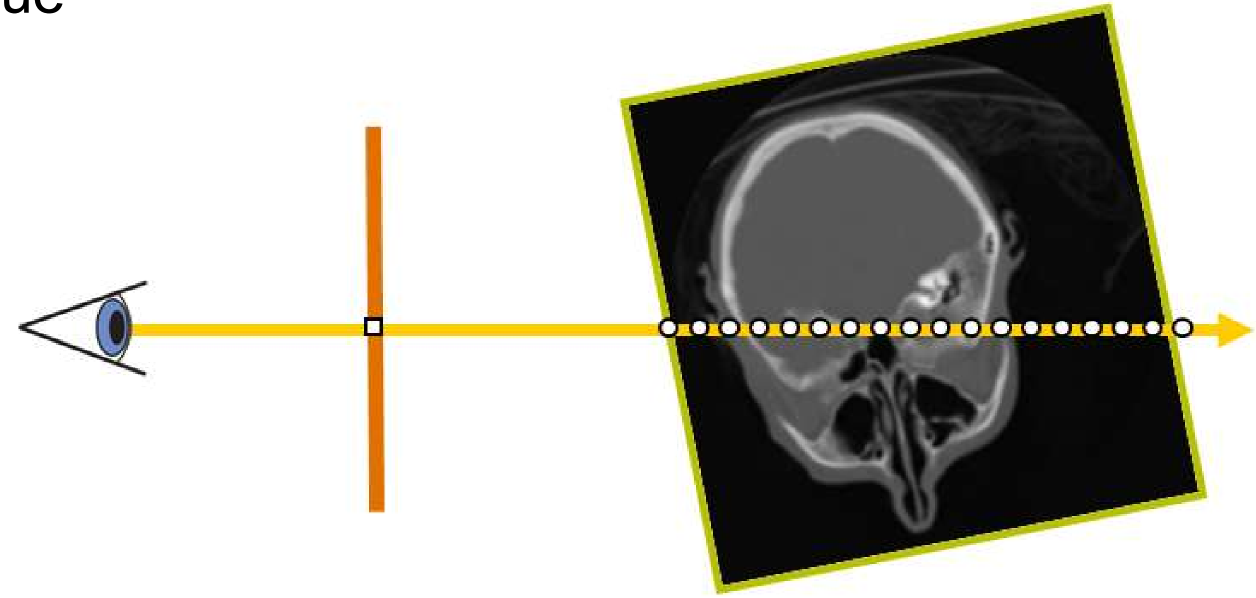


Implementation

Ray setup

Loop over ray

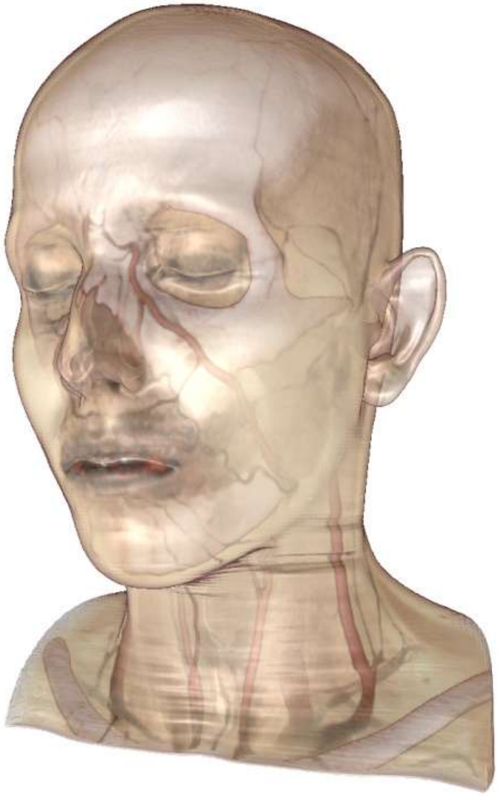
- Resample scalar value
- Classification
- **Shading**
- Compositing

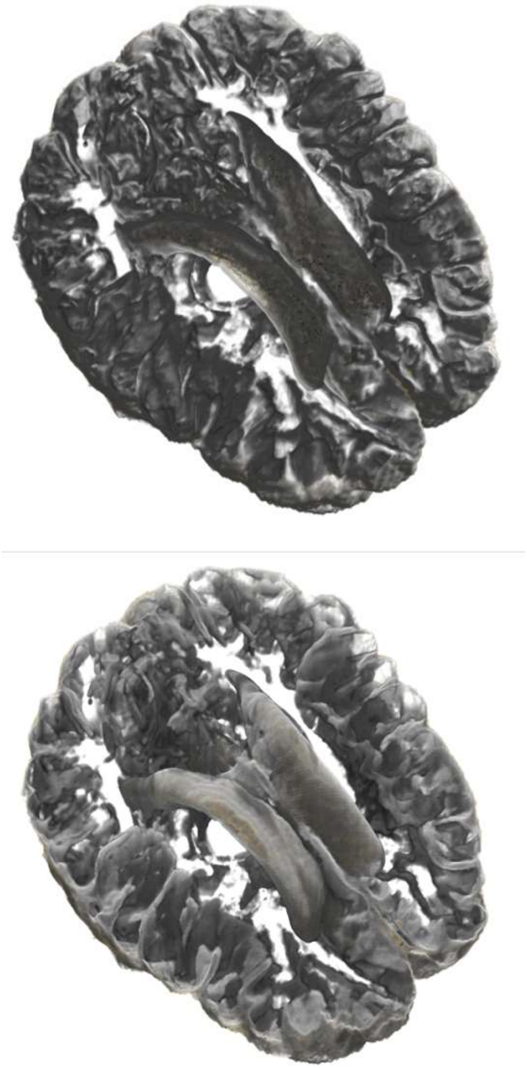


Volume Shading

Local illumination vs. global illumination

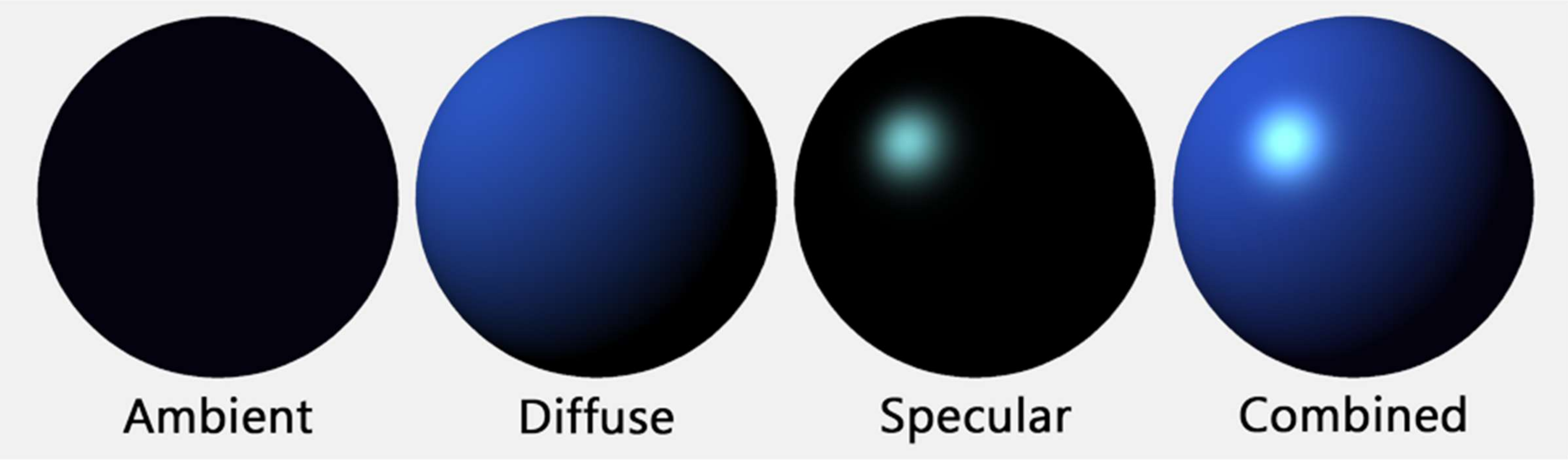
- Gradient-based or gradient-less
- Shadows, (multiple) scattering, …





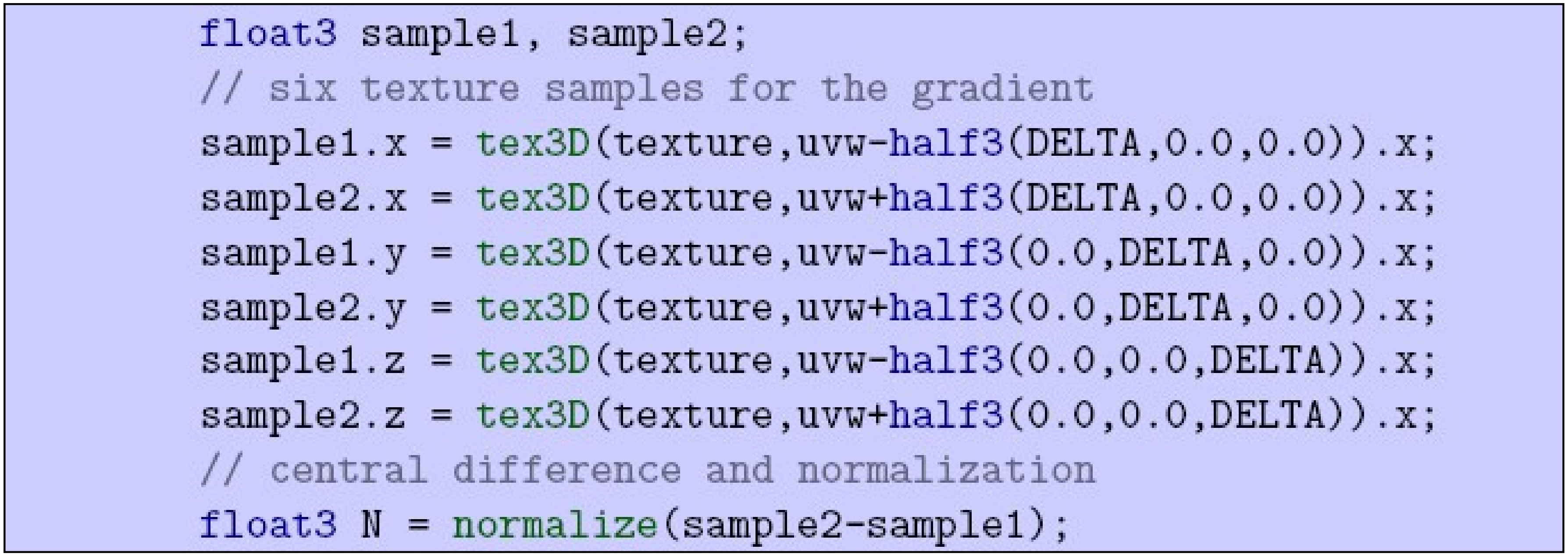
Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



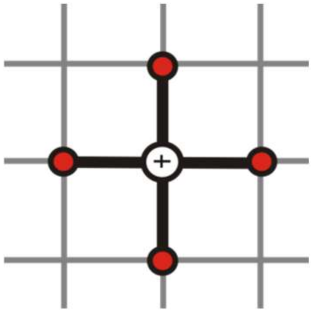
On-the-fly Gradient Estimation

$$\nabla f(x,y,z) \approx \frac{1}{2h} \begin{pmatrix} f(x+h,y,z) - f(x-h,y,z) \\ f(x,y+h,z) - f(x,y-h,z) \\ f(x,y,z+h) - f(x,y,z-h) \end{pmatrix}$$



```
float3 sample1, sample2;
// six texture samples for the gradient
sample1.x = tex3D(texture,uvw-half3(DELTA,0.0,0.0)).x;
sample2.x = tex3D(texture,uvw+half3(DELTA,0.0,0.0)).x;
sample1.y = tex3D(texture,uvw-half3(0.0,DELTA,0.0)).x;
sample2.y = tex3D(texture,uvw+half3(0.0,DELTA,0.0)).x;
sample1.z = tex3D(texture,uvw-half3(0.0,0.0,DELTA)).x;
sample2.z = tex3D(texture,uvw+half3(0.0,0.0,DELTA)).x;
// central difference and normalization
float3 N = normalize(sample2-sample1);
```

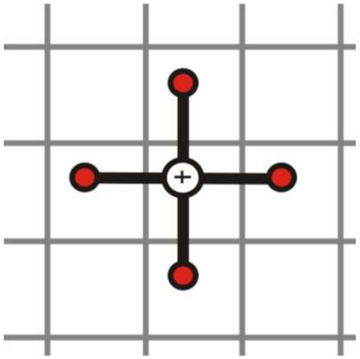
On-The-Fly Gradients



Reduce texture memory consumption!

Central differences before and after linear interpolation of values at grid points yield the same results

Caveat: texture filter precision



Filter kernel methods are expensive, but:

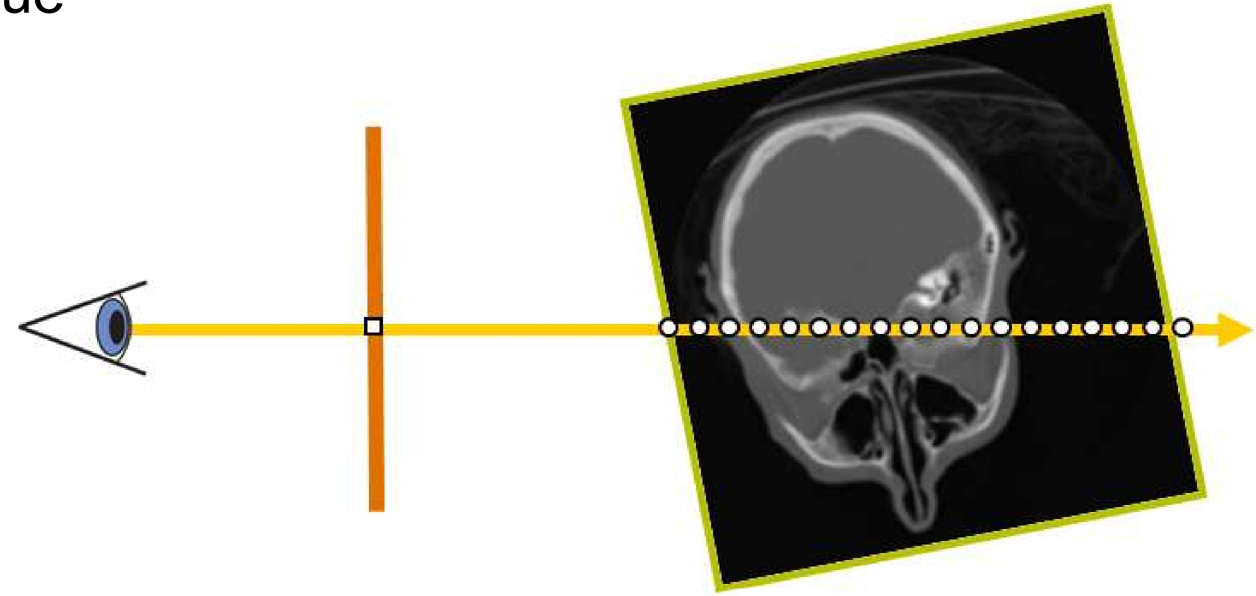
Tri-cubic B-spline kernels can be used in real-time (e.g., GPU Gems 2 Chapter “Fast Third-Order Filtering”)

Implementation

Ray setup

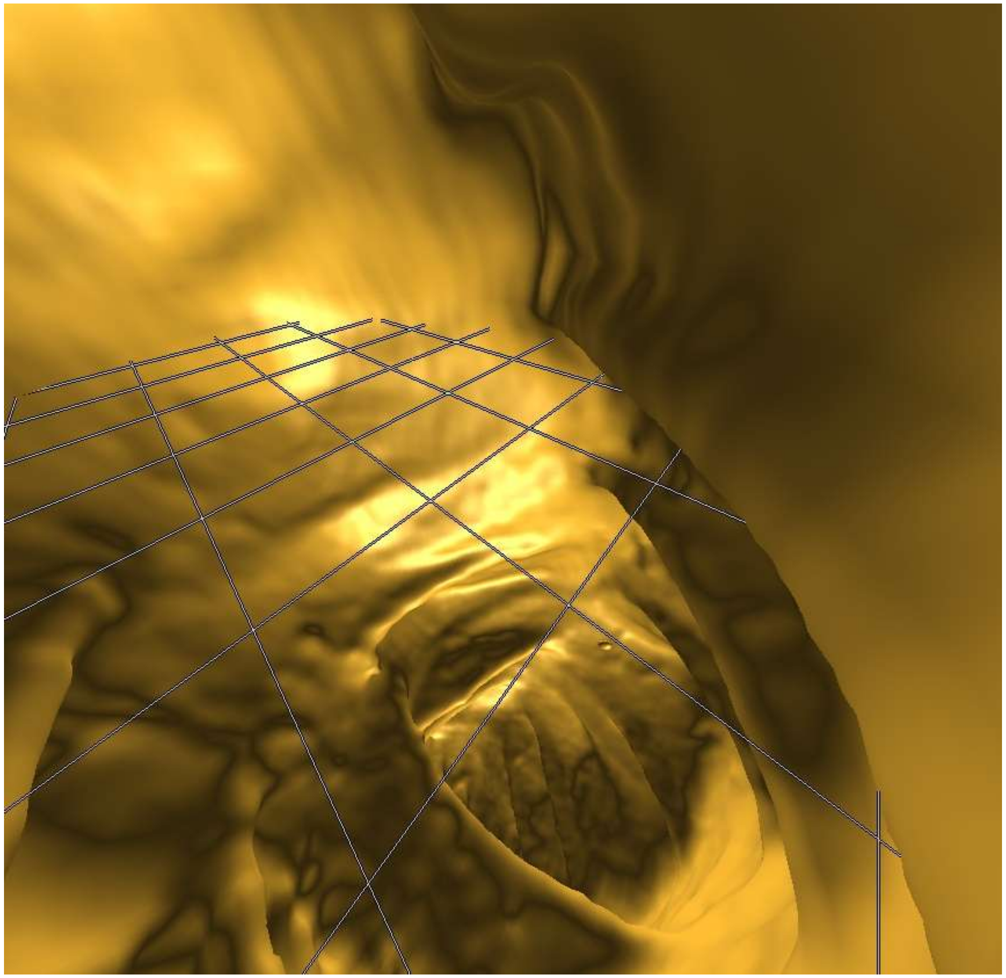
Loop over ray

- Resample scalar value
- Classification
- Shading
- **Compositing**

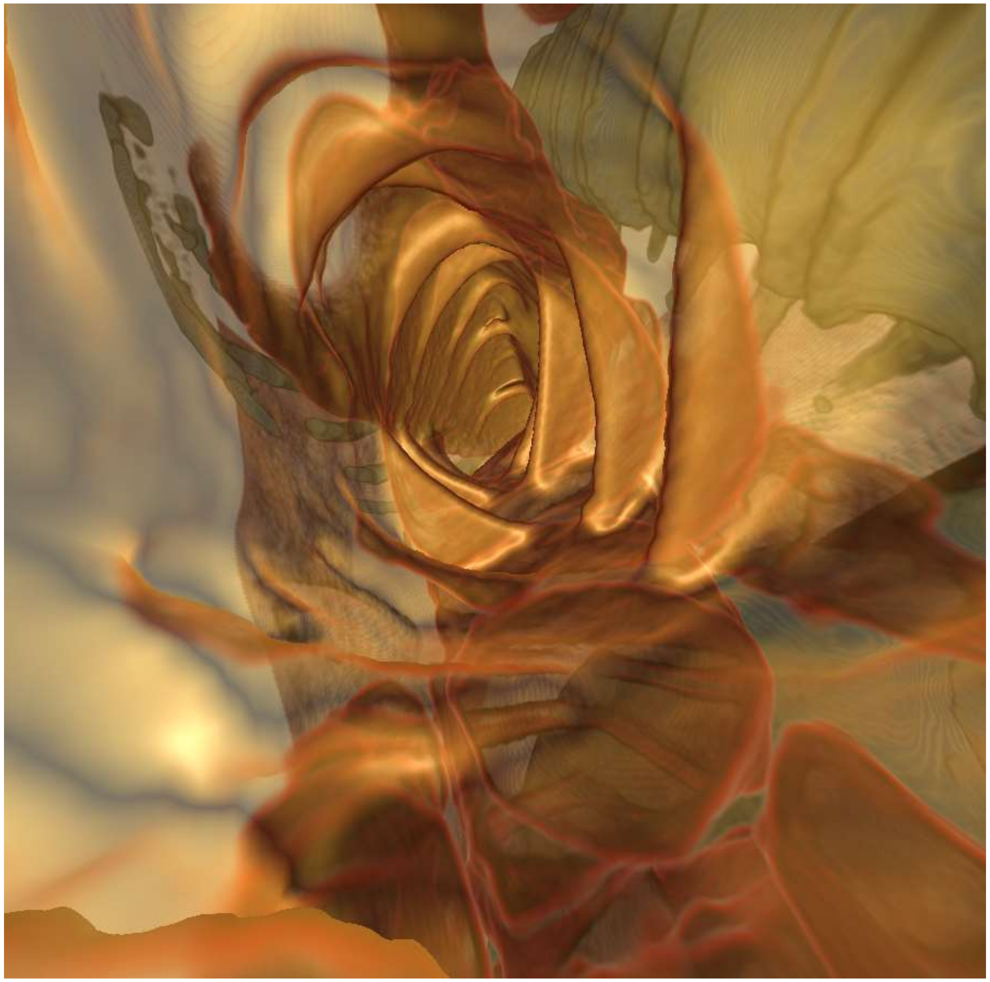


$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1})C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1})A_i \end{aligned}$$

Compositing

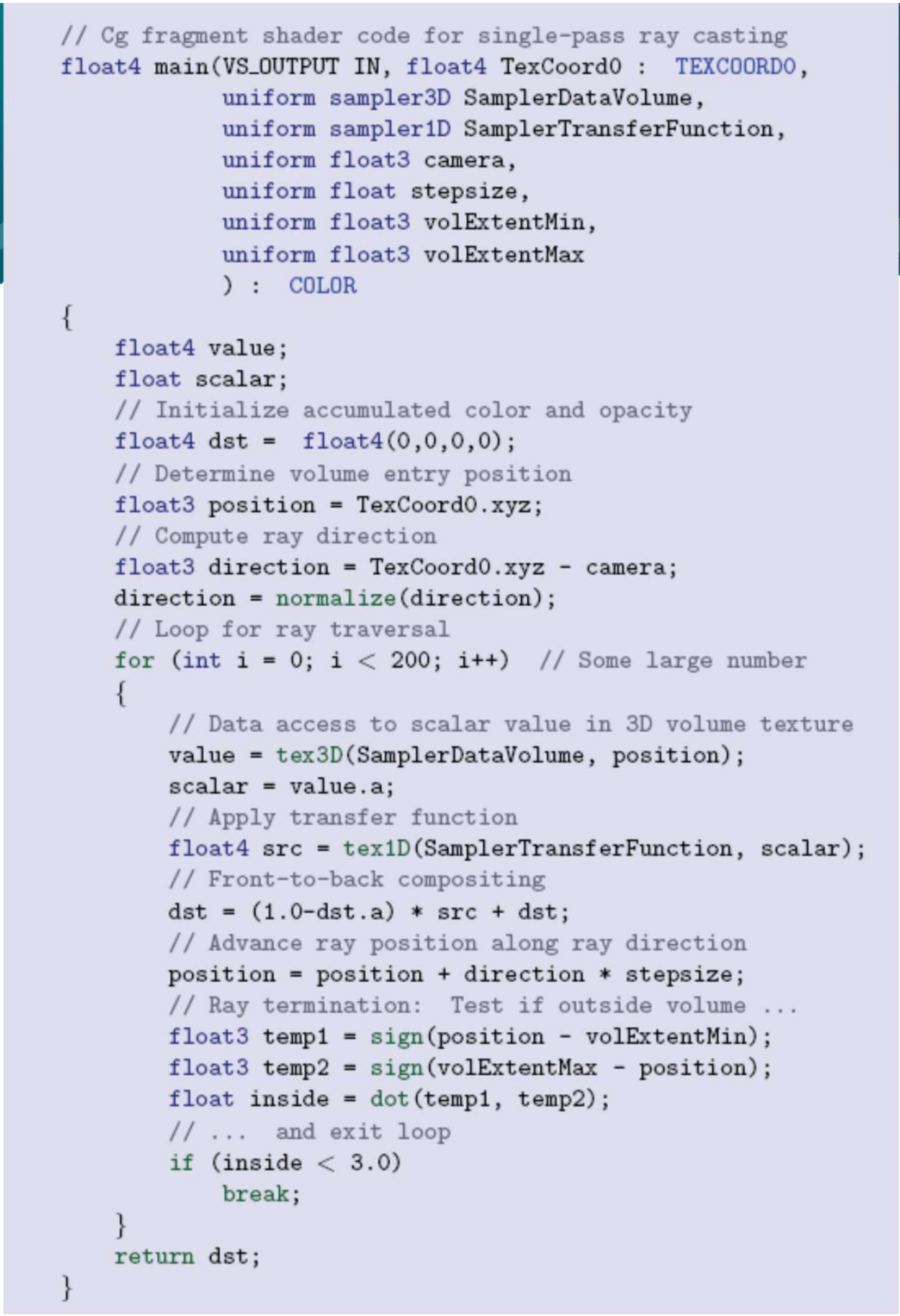


Compositing



Fragment Shader

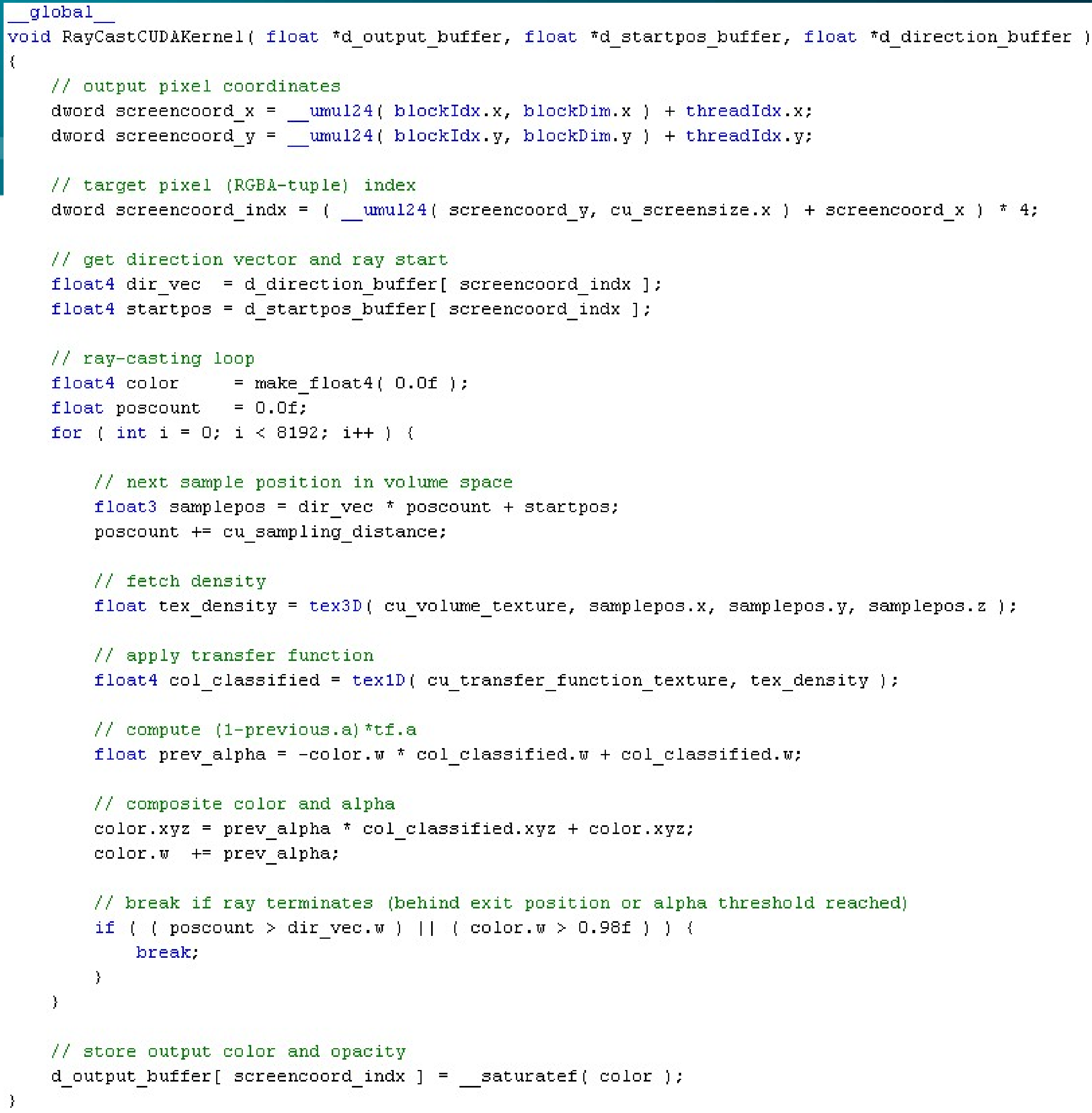
- Rasterize front faces of volume bounding box
- Texcoords are volume position in [0,1]
- Subtract camera position
- Repeatedly check for exit of bounding box



```
// Cg fragment shader code for single-pass ray casting
float4 main(VS_OUTPUT IN, float4 TexCoord0 :  TEXCOORD0,
            uniform sampler3D SamplerDataVolume,
            uniform sampler1D SamplerTransferFunction,
            uniform float3 camera,
            uniform float stepsize,
            uniform float3 volExtentMin,
            uniform float3 volExtentMax
            ) :  COLOR
{
    float4 value;
    float scalar;
    // Initialize accumulated color and opacity
    float4 dst =  float4(0,0,0,0);
    // Determine volume entry position
    float3 position = TexCoord0.xyz;
    // Compute ray direction
    float3 direction = TexCoord0.xyz - camera;
    direction = normalize(direction);
    // Loop for ray traversal
    for (int i = 0; i < 200; i++)  // Some large number
    {
        // Data access to scalar value in 3D volume texture
        value = tex3D(SamplerDataVolume, position);
        scalar = value.a;
        // Apply transfer function
        float4 src = tex1D(SamplerTransferFunction, scalar);
        // Front-to-back compositing
        dst = (1.0-dst.a) * src + dst;
        // Advance ray position along ray direction
        position = position + direction * stepsize;
        // Ray termination:  Test if outside volume ...
        float3 temp1 = sign(position - volExtentMin);
        float3 temp2 = sign(volExtentMax - position);
        float inside = dot(temp1, temp2);
        // ...  and exit loop
        if (inside < 3.0)
            break;
    }
    return dst;
}
```

CUDA Kernel

- Image-based ray setup
  - Ray start image
  - Direction image
- Ray-cast loop
  - Sample volume
  - Accumulate color and opacity
- Terminate
- Store output



```
__global__
void RayCastCUDAKernel( float *d_output_buffer, float *d_startpos_buffer, float *d_direction_buffer )
{
    // output pixel coordinates
    dword screencoord_x = __umul24( blockIdx.x, blockDim.x ) + threadIdx.x;
    dword screencoord_y = __umul24( blockIdx.y, blockDim.y ) + threadIdx.y;

    // target pixel (RGBA-tuple) index
    dword screencoord_indx = ( __umul24( screencoord_y, cu_screensize.x ) + screencoord_x ) * 4;

    // get direction vector and ray start
    float4 dir_vec  = d_direction_buffer[ screencoord_indx ];
    float4 startpos = d_startpos_buffer[ screencoord_indx ];

    // ray-casting loop
    float4 color     = make_float4( 0.0f );
    float poscount   = 0.0f;
    for ( int i = 0; i < 8192; i++ ) {

        // next sample position in volume space
        float3 samplepos = dir_vec * poscount + startpos;
        poscount += cu_sampling_distance;

        // fetch density
        float tex_density = tex3D( cu_volume_texture, samplepos.x, samplepos.y, samplepos.z );

        // apply transfer function
        float4 col_classified = tex1D( cu_transfer_function_texture, tex_density );

        // compute (1-previous.a)*tf.a
        float prev_alpha = -color.w * col_classified.w + col_classified.w;

        // composite color and alpha
        color.xyz = prev_alpha * col_classified.xyz + color.xyz;
        color.w   += prev_alpha;

        // break if ray terminates (behind exit position or alpha threshold reached)
        if ( ( poscount > dir_vec.w ) || ( color.w > 0.98f ) ) {
            break;
        }
    }

    // store output color and opacity
    d_output_buffer[ screencoord_indx ] = __saturatef( color );
}
```

Thank you.

Thanks for material

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- Philipp Muigg
- Christof Rezk-Salama