CS 247 – Scientific Visualization
Lecture 13: Scalar Field Visualization, Pt. 6

Markus Hadwiger, KAUST

Reading Assignment #8 (until Mar 23)

Read (required):

- Real-Time Volume Graphics, Chapter 1 (*Theoretical Background and Basic Approaches*), from beginning to 1.4.4 (inclusive)

Read (optional):

- Paper:
  Nelson Max, Optical Models for Direct Volume Rendering,
  IEEE Transactions on Visualization and Computer Graphics, 1995
  `http://dx.doi.org/10.1109/2945.468400`

The Gradient as Normal Vector

Gradient of the scalar field gives direction+magnitude of fastest change

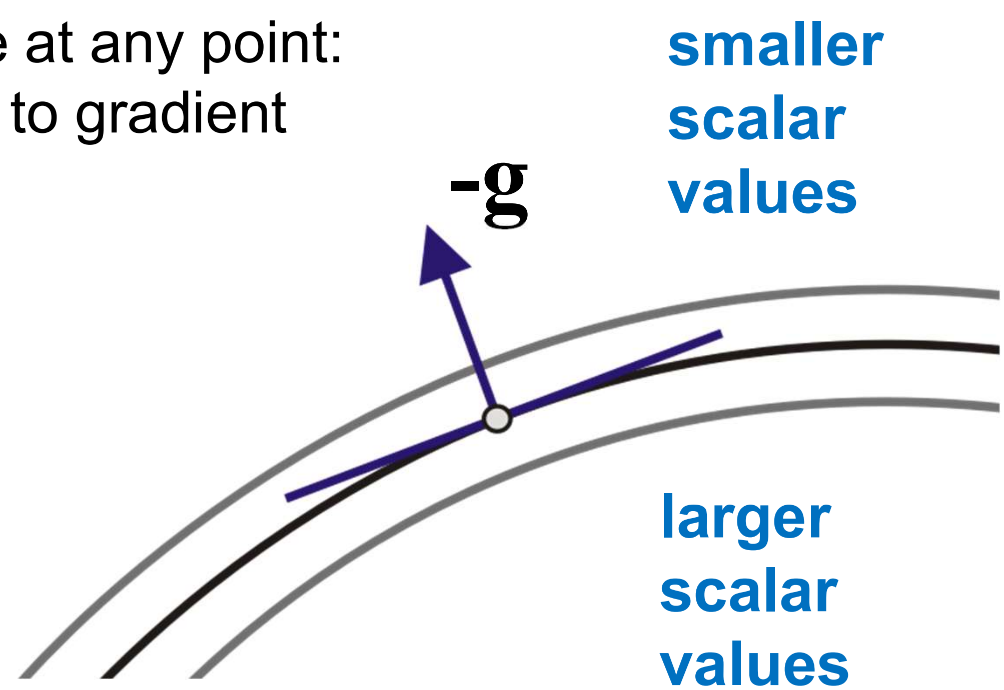
$$\mathbf{g} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)^{\mathbf{T}}$$

(only correct in Cartesian coordinates: see later)

Local approximation to isosurface at any point:
tangent plane = plane orthogonal to gradient

Normal of this isosurface:
normalized gradient vector
(negation is common convention)

$$\mathbf{n} = -\mathbf{g}/|\mathbf{g}|$$

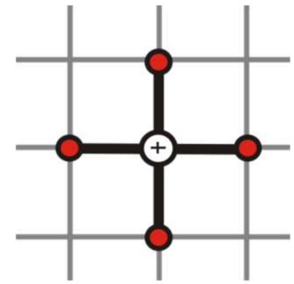


(Numerical) Gradient Reconstruction

We need to reconstruct the derivatives of a continuous function given as discrete samples

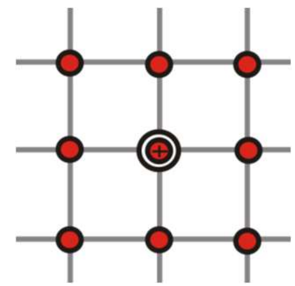
Central differences

- Cheap and quality often sufficient (2*3 neighbors in 3D)



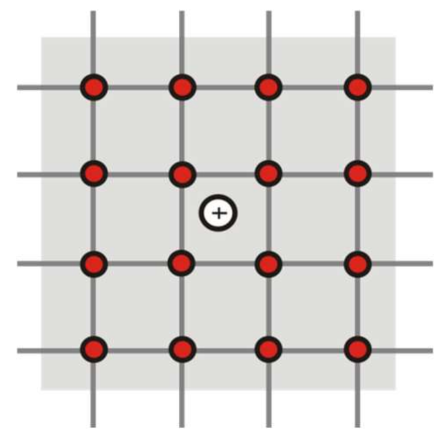
Discrete convolution filters on grid

- Image processing filters; e.g. Sobel (3^3 neighbors in 3D)



Continuous convolution filters

- Derived continuous reconstruction filters
- E.g., the cubic B-spline and its derivatives (4^3 neighbors)



Finite Differences

Obtain first derivative from Taylor expansion

$$\begin{aligned} f(x_0+h) &= f(x_0) + \frac{f'(x_0)}{1!}h + \frac{f''(x_0)}{2!}h^2 + \dots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}h^n \,. \end{aligned}$$

Forward differences / backward differences

$$f(x_0)' = \frac{f(x_0+h)-f(x_0)}{h} + o(h)$$

$$f(x_0)' = \frac{f(x_0)-f(x_0-h)}{h} + o(h)$$

Finite Differences

Central differences

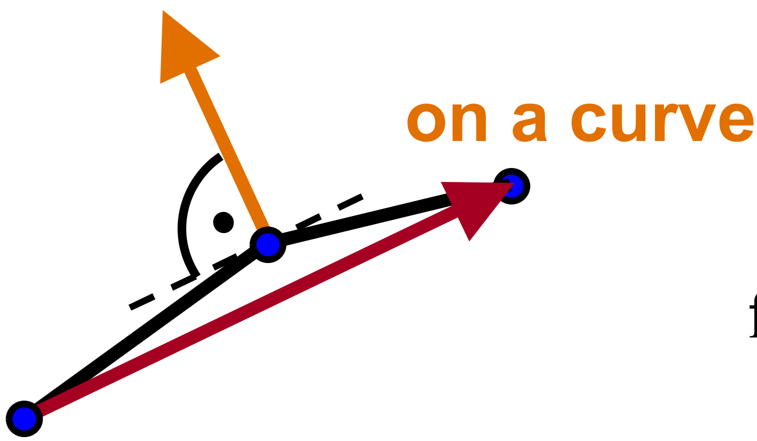
$$
\begin{aligned}
f(x_0 + h) &= f(x_0) + \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + o(h^3) \\
f(x_0 - h) &= f(x_0) - \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + o(h^3)
\end{aligned}
$$

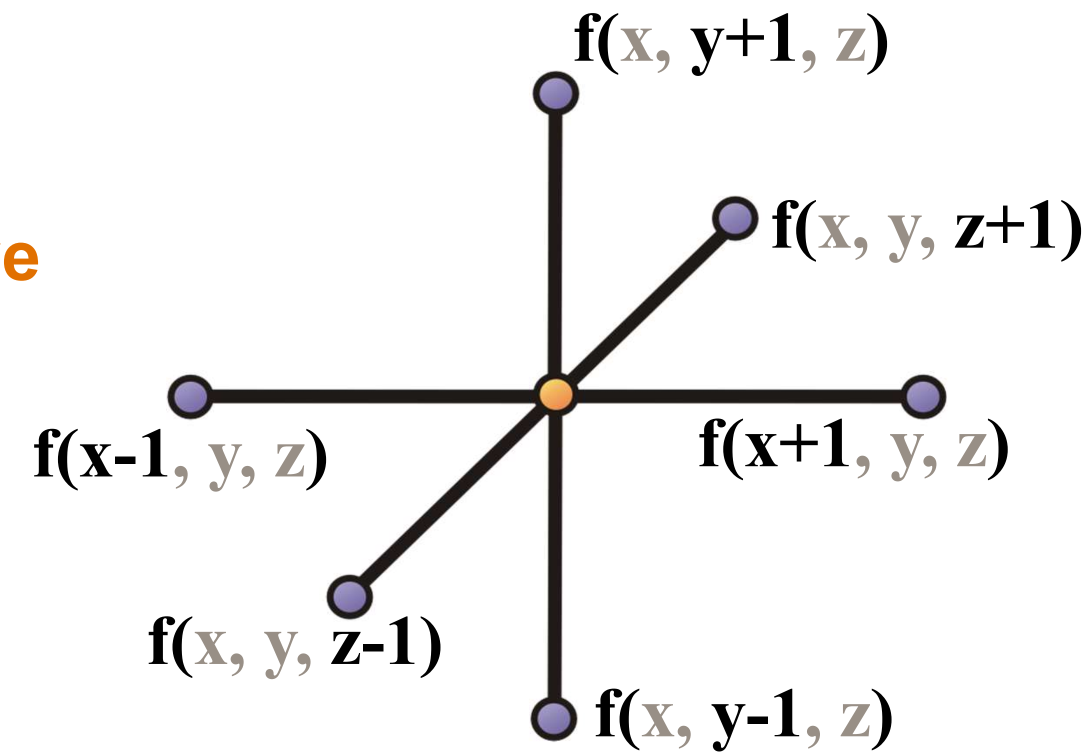
$$
f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h} + o(h^2)
$$

Central Differences

Need only two neighboring voxels per derivative

Most common method





$$g_x = 0.5(\, f(x+1, y, z) - f(x-1, y, z) \,)$$

$$g_y = 0.5(\, f(x, y+1, z) - f(x, y-1, z) \,)$$

$$g_z = 0.5(\, f(x, y, z+1) - f(x, y, z-1) \,)$$

in a volume

Gradients as Differential Forms (1-Forms)

The Gradient as a Differential Form

The gradient as a *differential* (differential 1-form) is the "primary" concept (also "total differential" or "total derivative")

$$df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy + \frac{\partial f}{\partial z}\, dz$$

A differential 1-form is a scalar-valued linear function that takes a (direction) vector as input, and gives a scalar as output

Each of the 1-forms df, dx, dy, dz takes direction vector as input, gives scalar output

In the expression of the gradient df above, all 1-forms on the right-hand side get the same vector as input

df is simply a linear combination of the coordinate differentials dx, dy, dz

The Gradient as a Differential Form

The gradient as a *differential* (differential 1-form) is the “primary” concept (also “total differential” or “total derivative”)

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy + \frac{\partial f}{\partial z}\,dz$$

The directional derivative and the gradient vector

$$D_{\mathbf{u}} f = df(\mathbf{u})$$
$$df(\mathbf{u}) = \nabla f \cdot \mathbf{u}$$

The gradient vector is then *defined*, such that:

$$\nabla f \cdot \mathbf{u} := df(\mathbf{u})$$

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama