CS 247 – Scientific Visualization
Lecture 9: Scalar Field Visualization, Pt. 2

Markus Hadwiger, KAUST

Reading Assignment #5 (until Mar 2)

Read (required):

- Gradients of scalar-valued functions

  `https://en.wikipedia.org/wiki/Gradient`

- Critical points

  `https://en.wikipedia.org/wiki/Critical_point_(mathematics)`

- Multivariable derivatives and differentials

  `https://en.wikipedia.org/wiki/Total_derivative`

  `https://en.wikipedia.org/wiki/Differential_of_a_function#Differentials_in_several_variables`

  `https://en.wikipedia.org/wiki/Hessian_matrix`

- Dot product, inner product (more general)

  `https://en.wikipedia.org/wiki/Dot_product`

  `https://en.wikipedia.org/wiki/Inner_product_space`

Scalar Fields

What are contours?

Set of points where the scalar field *s* has a given value *c*:

$$S(c) := \{x \in \mathbb{R}^n : f(x) = c\}$$

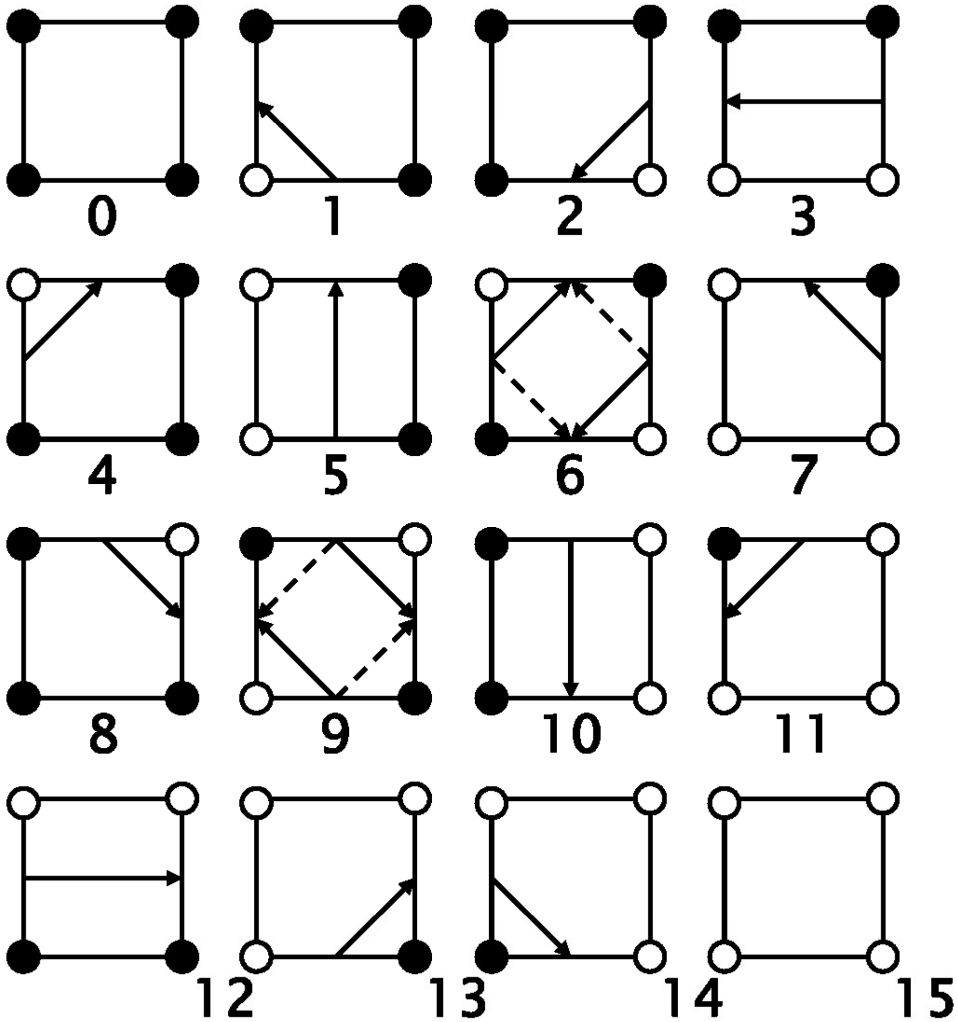
Examples in 2D:

- height contours on maps
- isobars on weather maps

Contouring algorithm:

- find intersection with grid edges
- connect points in each cell

Contours in a quadrangle cell



● $\tilde{f}(x_i) < 0$

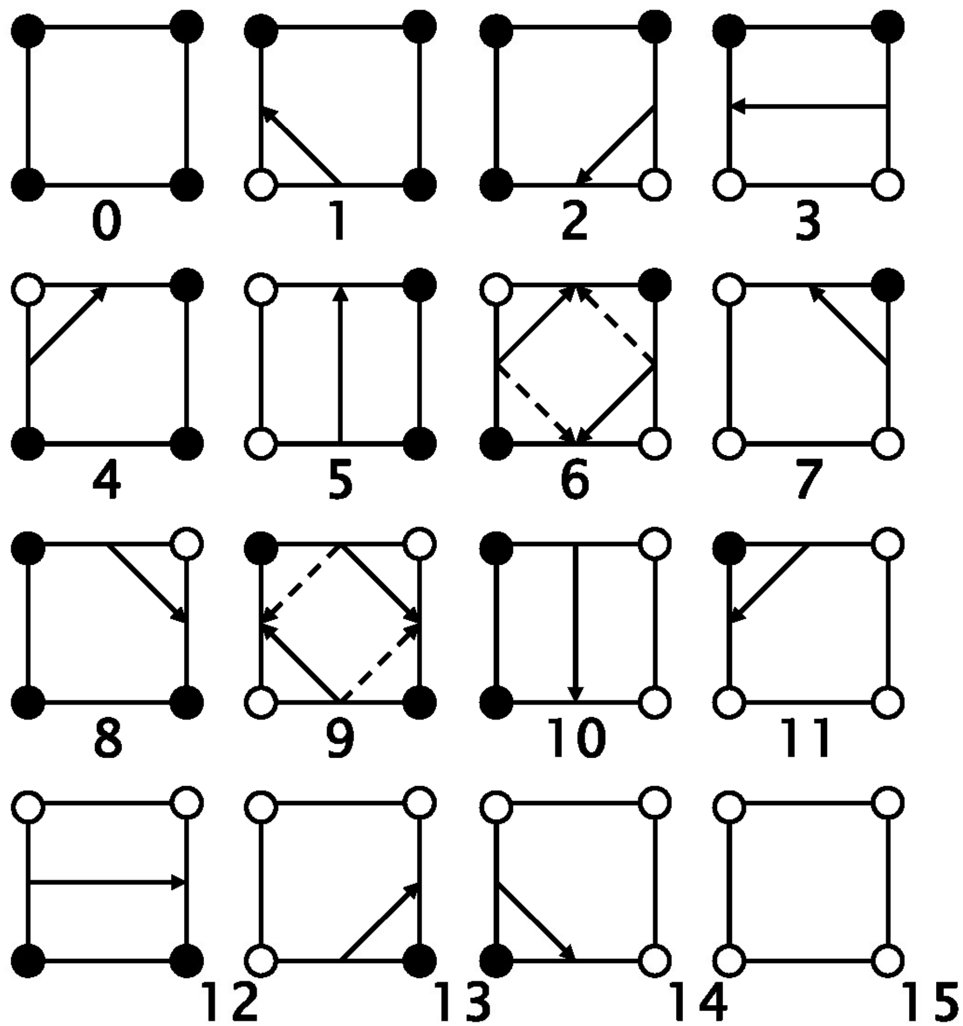
○ $\tilde{f}(x_i) > 0$

Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.



- ● $f(x_i) < c$
- ○ $f(x_i) \geq c$

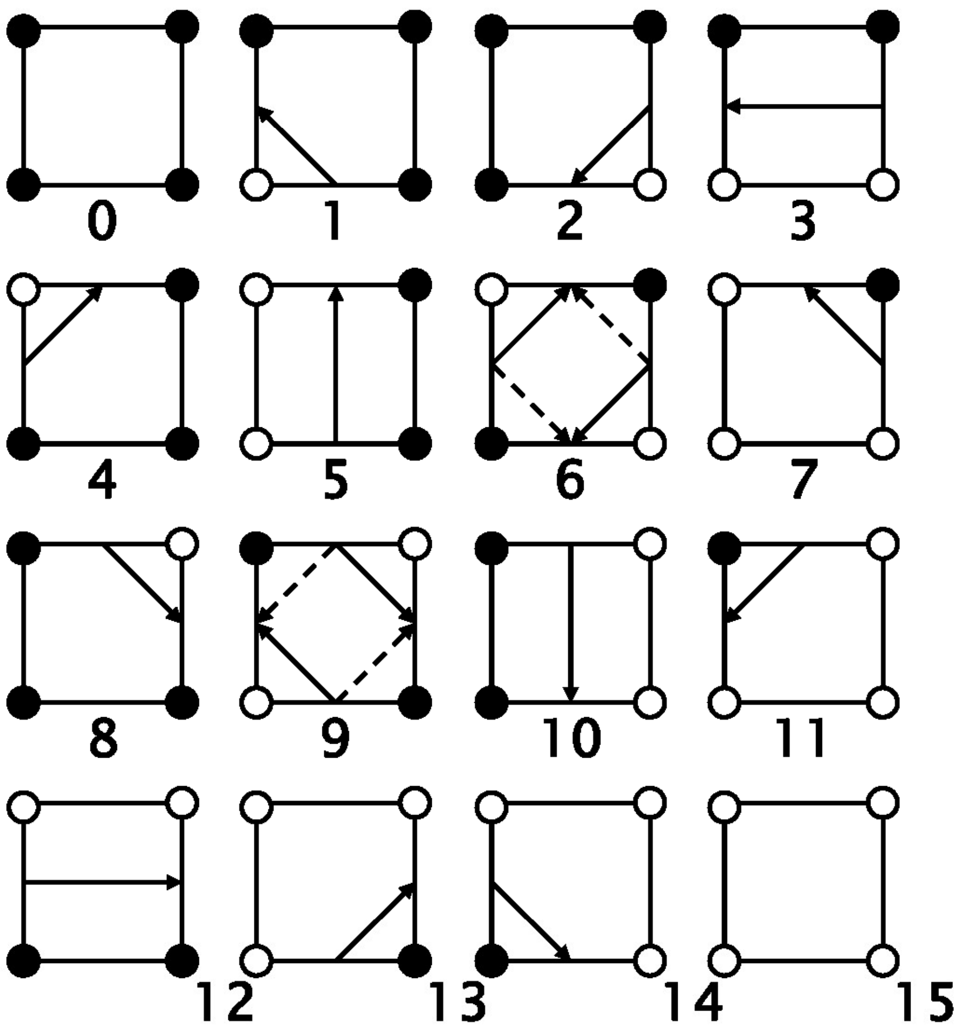
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



● $f(x_i) \leq c$

○ $f(x_i) > c$

Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Basic contouring algorithms:

- cell-by-cell algorithms: simple structure, but generate disconnected segments, require post-processing
- contour propagation methods: more complicated, but generate connected contours

"Marching squares" algorithm (systematic cell-by-cell):

- process nodes in ccw order, denoted here as x_0, x_1, x_2, x_3
- compute at each node $\mathbf{x}_i$ the reduced field $\tilde{f}(x_i) = f(x_i) - (c - \varepsilon)$ (which is forced to be nonzero)
- take its sign as the i^{th} bit of a 4-bit integer
- use this as an index for lookup table containing the connectivity information:

Orientability (1-manifold embedded in 2D)

not orientable

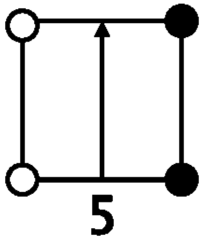
Moebius strip
(only one side!)

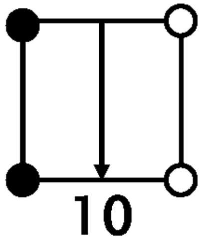
Orientability of 1-manifold:

Possible to assign consistent left/right orientation

Iso-contours

- Consistent side for scalar values…
  - greater than iso-value (e.g, *left* side)
  - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices
  (e.g., larger vertex index is “tip” of arrow;
  if (0,1) points “up”, “left” is left, …)





● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

not orientable

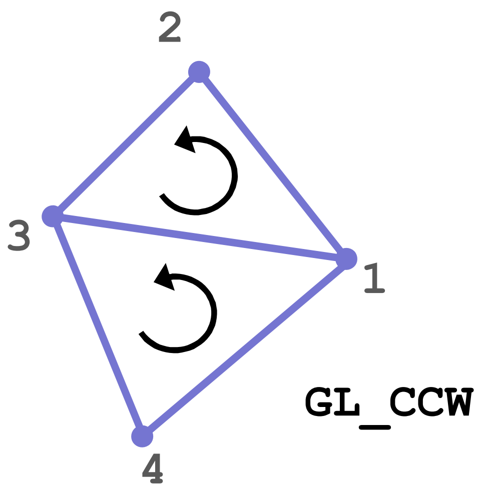
Moebius strip
(only one side!)

Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

Triangle meshes

- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”



Topological consistency

To avoid degeneracies, use symbolic perturbations:

If level *c* is found as a node value, set the level to $c-\varepsilon$ where ε is a symbolic infinitesimal.

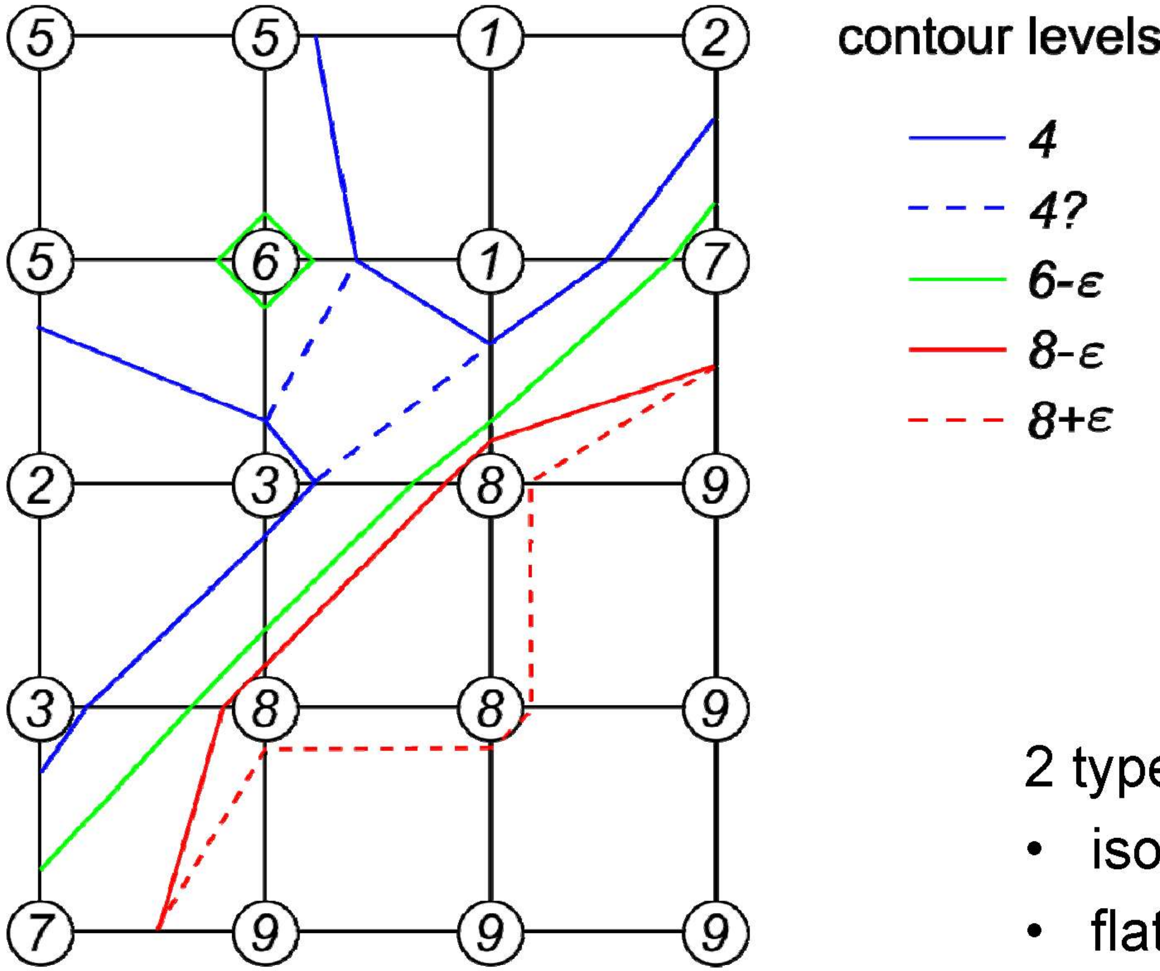
Then:

- contours intersect edges at some (possibly infinitesimal) distance from end points
- flat regions can be visualized by pair of contours at $c-\varepsilon$ and $c+\varepsilon$
- contours are topologically consistent, meaning:

Contours are closed, orientable, nonintersecting lines.

(except where the boundary is hit)

Example



2 types of degeneracies:

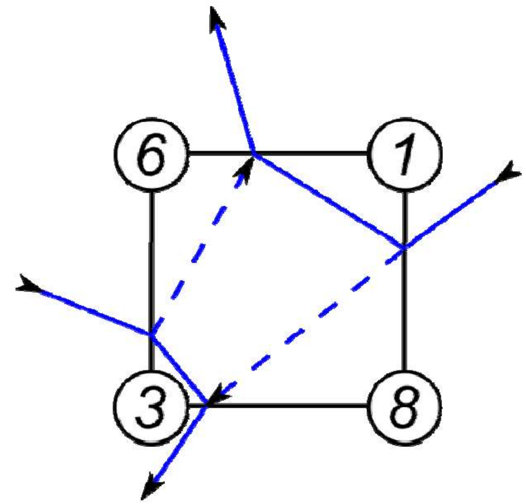
- isolated points (*c*=6)
- flat regions (*c*=8)

Ambiguities of contours

What is the correct contour of c=4?

Two possibilities, both are orientable:

- connect high values ————
- connect low values - - - - - - - - - -



Answer: correctness depends on interior values of $f(x)$.

But: different interpolation schemes are possible.

Better question: What is the correct contour with respect to bilinear interpolation?

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
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- Philipp Muigg
- Christof Rezk-Salama