King Abdullah University of Science and Technology KAUST

# CS 247 – Scientific Visualization
# Lecture 5: Data Representation, Pt. 2

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# Reading Assignment #3 (until Feb 16)

Read (required):

- Data Visualization book, finish Chapter 3 (read starting with 3.6)
- Data Visualization book, Chapter 5 until 5.3 (inclusive)

# Data Representation

# Data Space (Domain) vs. Data Type (Codomain)

| | 1D | 2D | 3D |
|---|---|---|---|
| 1D | y=f(x) | | spatial curve **x**(t) |
| 2D | | 2D flow **v**(**x**) | |
| 3D | CT data s(**x**) | | 3D flow **v**(**x**) |

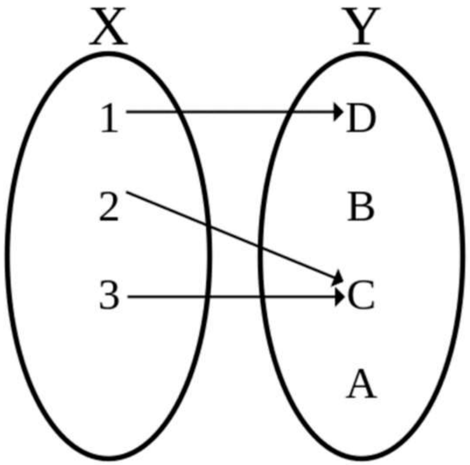
examples

# Data == Functions

# Mathematical Functions

Associates every element of a set (e.g., X) with *exactly one* element of another set (e.g., Y)

Maps from *domain* (X) to *codomain* (Y)




$$f : X \to Y$$
$$x \mapsto f(x)$$

Also important: *range/image*; *preimage*; continuity, differentiability, dimensionality, ...

Graph of a function (mathematical definition):

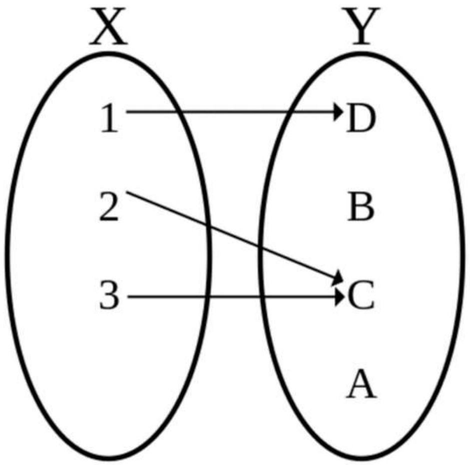
$$G(f) := \{(x, f(x)) | x \in X\} \subset X \times Y$$

# Mathematical Functions

Associates every element of a set (e.g., X) with *exactly one* element of another set (e.g., Y)

Maps from *domain* (X) to *codomain* (Y)

$$f: \mathbb{R}^n \to \mathbb{R}^m$$
$$x \mapsto f(x)$$

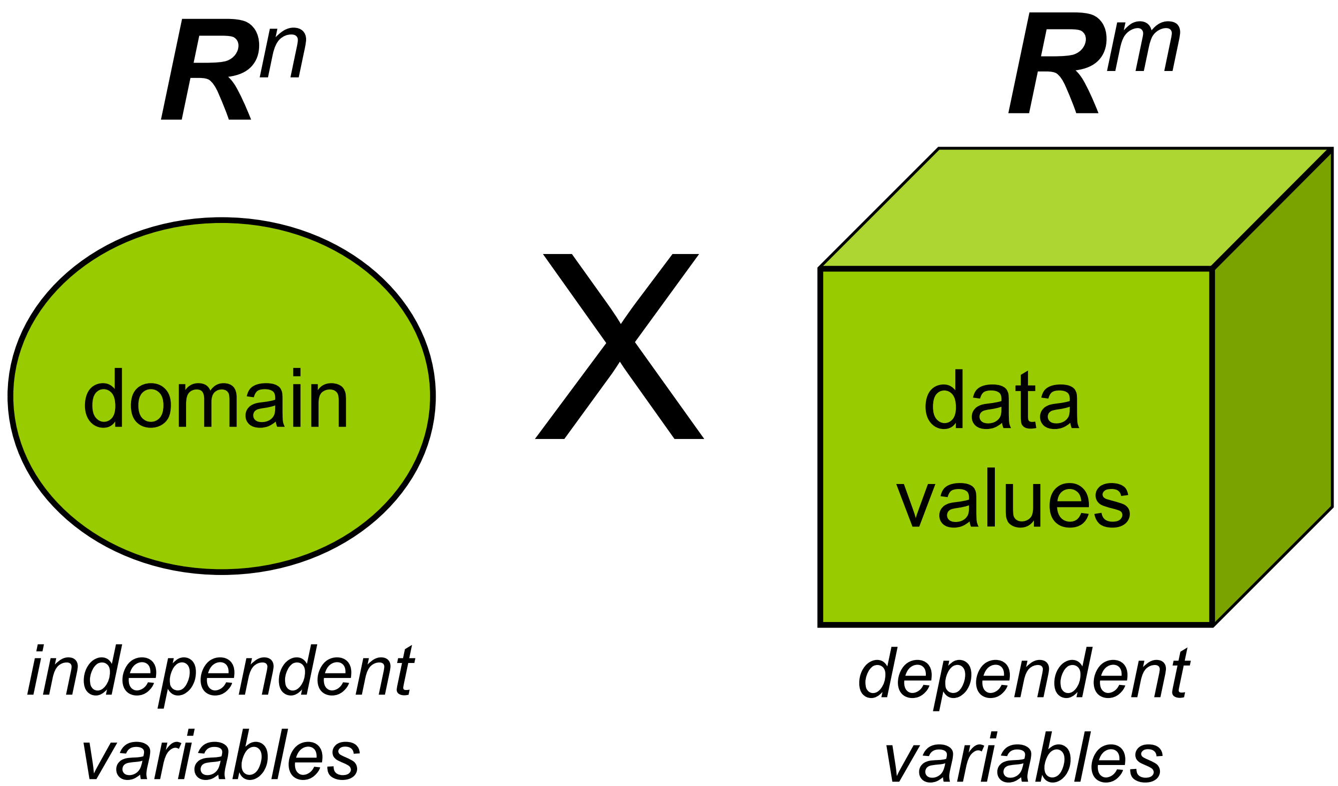



Also important: *range/image*; *preimage*; continuity, differentiability, dimensionality, ...

Graph of a function (mathematical definition):

$$G(f) := \{(x, f(x)) | x \in \mathbb{R}^n\} \subset \mathbb{R}^n \times \mathbb{R}^m \simeq \mathbb{R}^{n+m}$$

# Data Representation




scientific data $\subseteq \boldsymbol{R}^{n+m}$

# Example: Scalar Fields

2D scalar field

$$f : \mathbb{R}^2 \to \mathbb{R}$$
$$x \mapsto f(x)$$

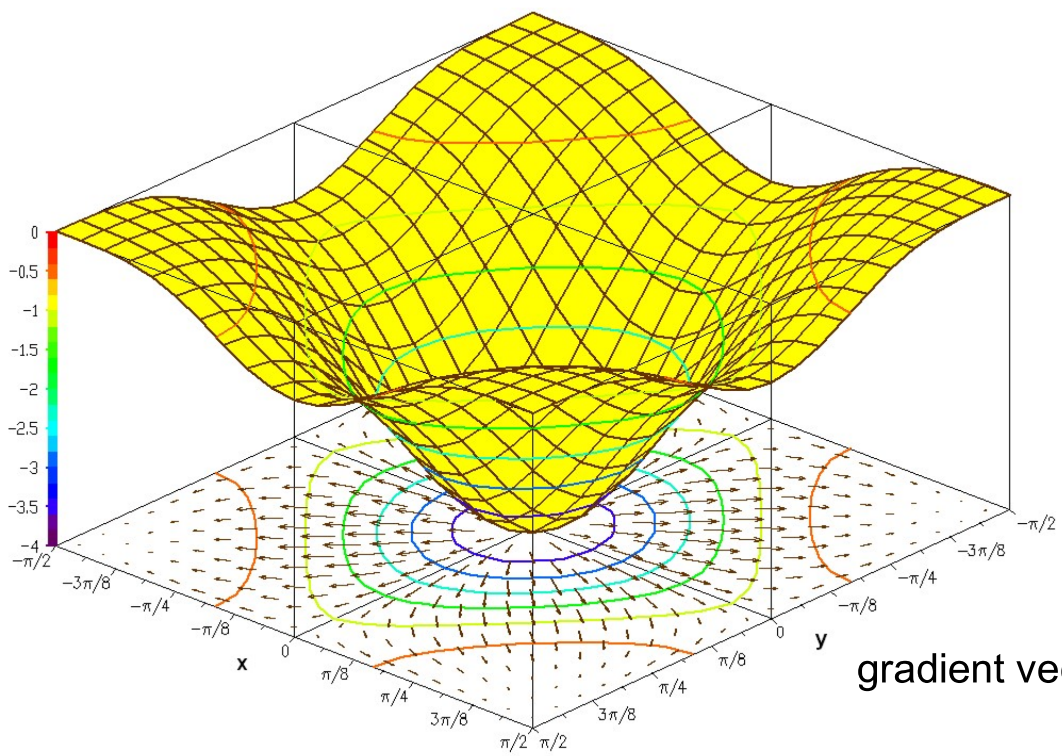
Graph: $G(f) := \{(x, f(x)) | x \in \mathbb{R}^2\} \subset \mathbb{R}^2 \times \mathbb{R} \simeq \mathbb{R}^3$

pre-image

$$S(c) := f^{-1}(c)$$

iso-contour

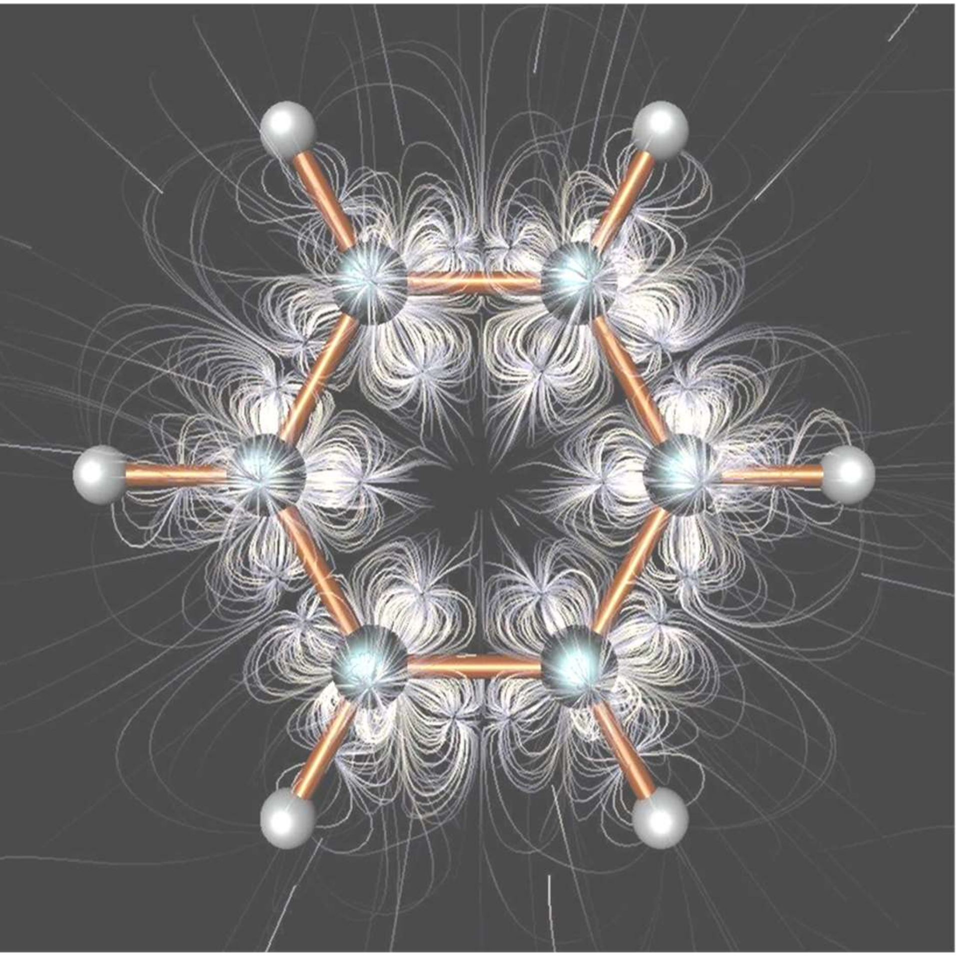
$$(\nabla f \neq 0)$$

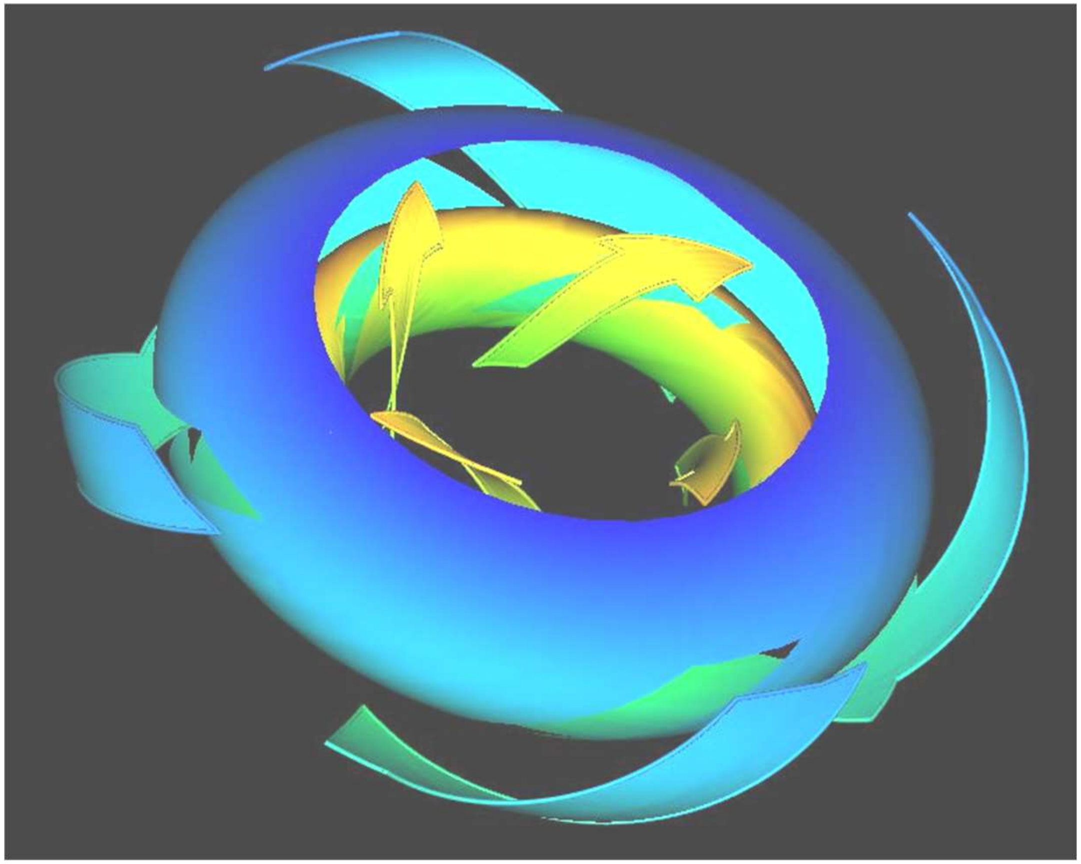



gradient vector field $\nabla f$

# Visualization Examples

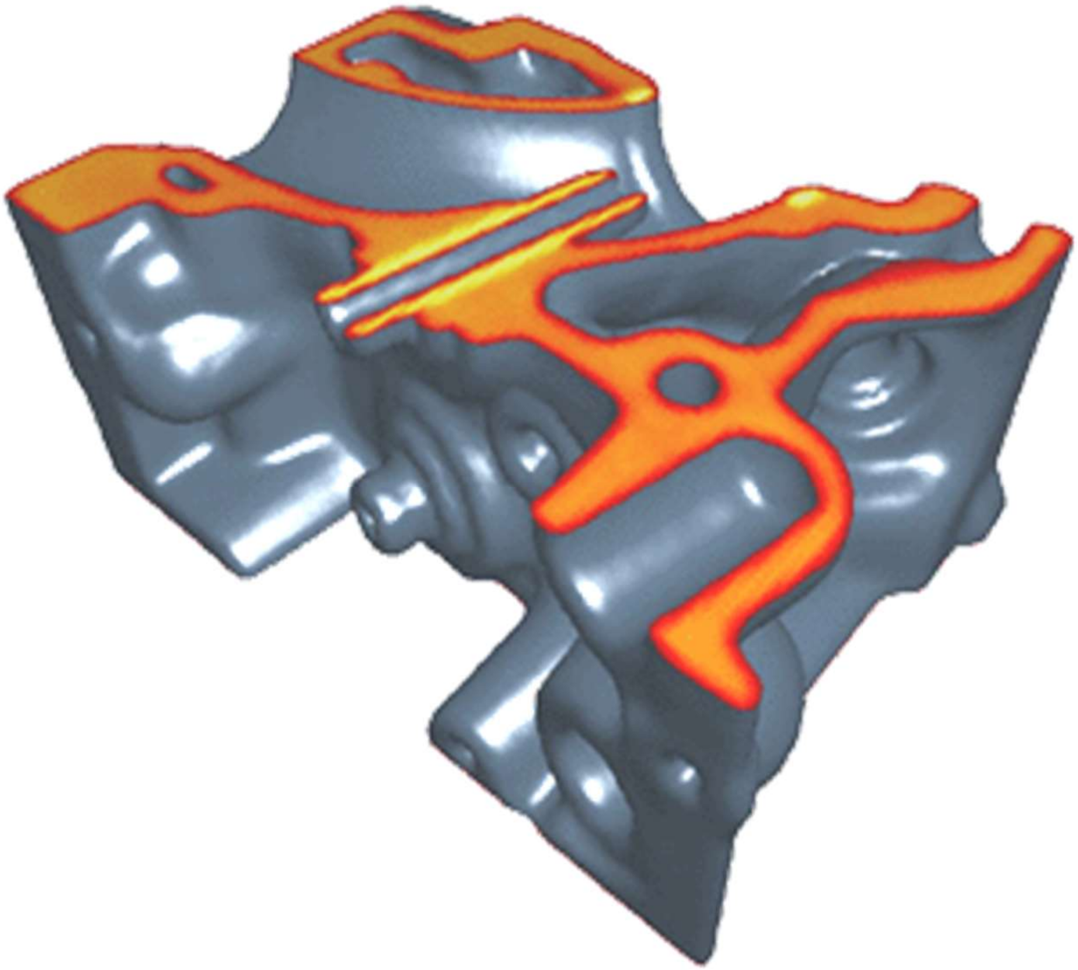
| data | description | visualization example |
|---|---|---|
| $R^3 \rightarrow R^3$ | 3D-flow | streamlines, streamsurfaces |

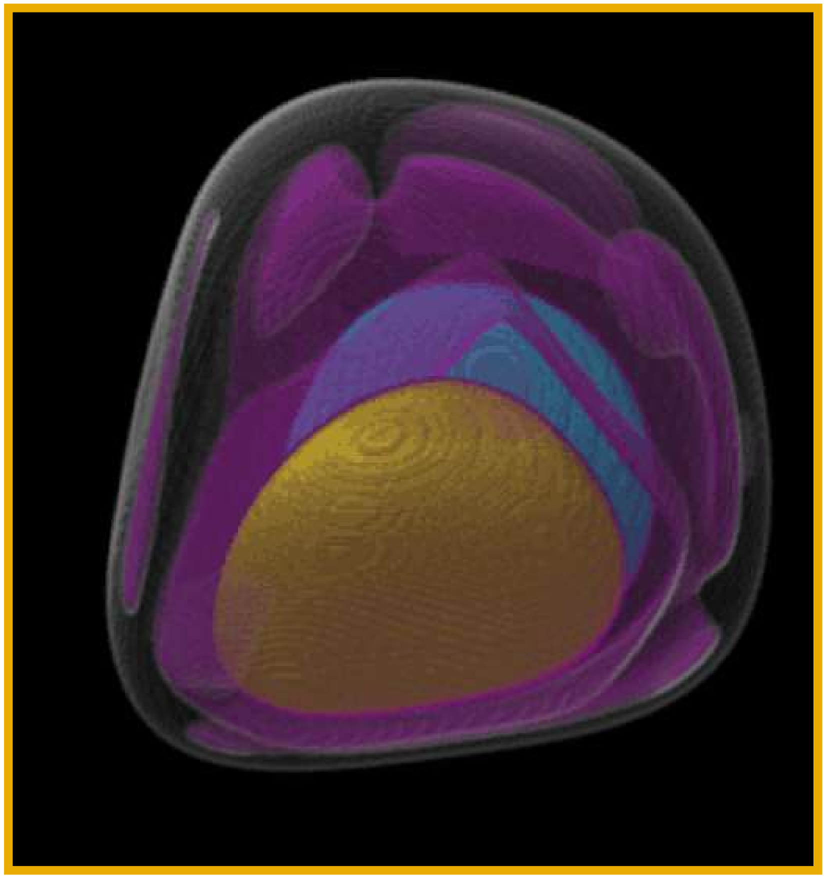




# Visualization Examples

| data | description | visualization example |
|---|---|---|
| $R^3 \rightarrow R^1$ | 3D-densities | iso-surfaces in 3D, volume rendering |



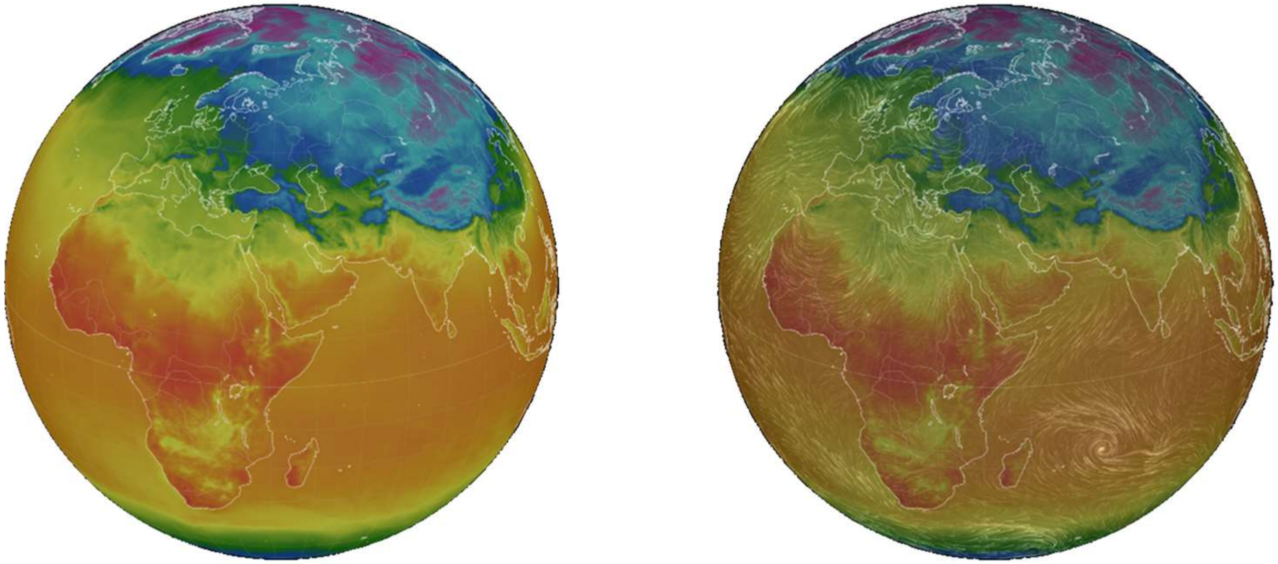


# Domain is Not Always Euclidean

Manifolds

…

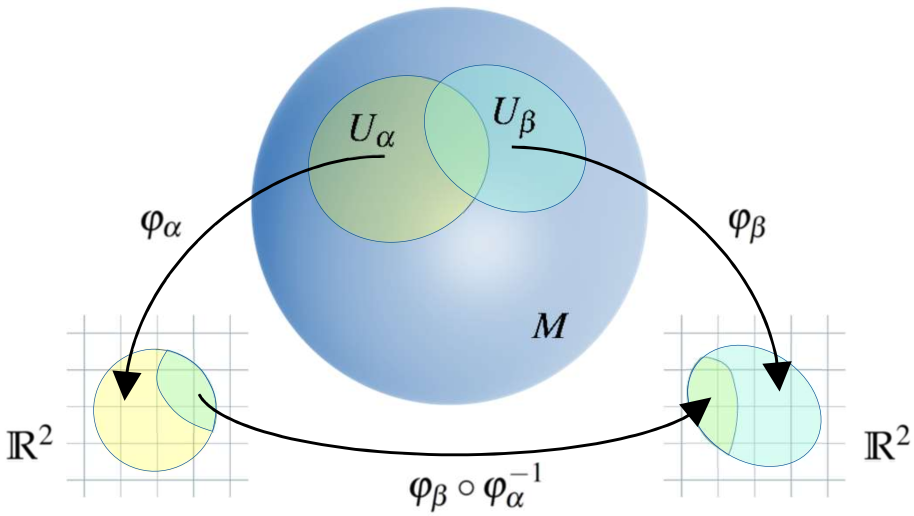
- Scalar, vector, tensor fields on manifolds

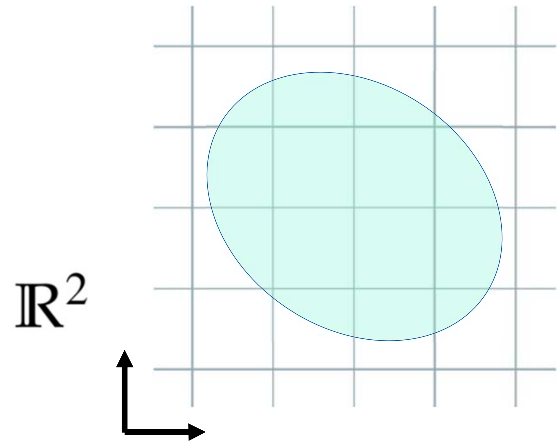


# Topological Manifolds

Every point of an *n*-manifold is homeomorphic (topologically equivalent) to a region of $\mathbb{R}^n$

Think about being able to assign coordinates to a region: coordinate chart; (collection of charts: atlas)



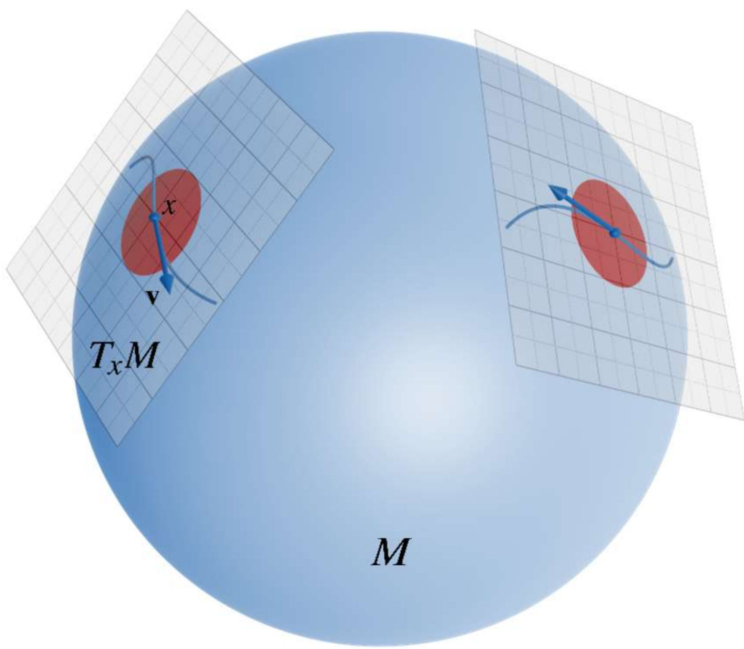



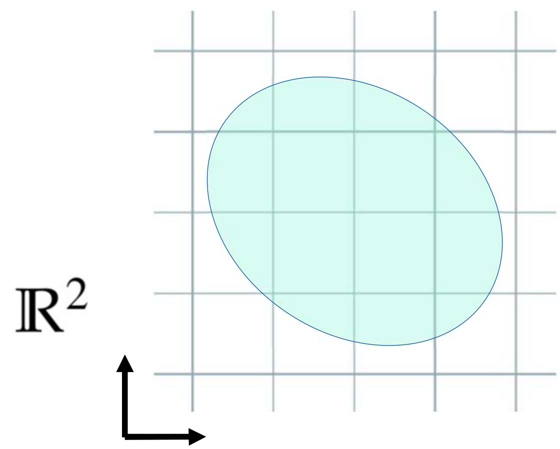
# Smooth Manifolds

Well-defined tangent space at every point

- Dimensionality of each tangent space is the same as that of manifold

Enables calculus on manifolds (and vector fields, tensor fields, …)



# Sampled Functions and Data Structures

# Data Representation

- Discrete (sampled) representations
  - The objects we want to visualize are often 'continuous'
  - But in most cases, the visualization data is given only at discrete locations in space and/or time
  - Discrete structures consist of samples, from which grids/meshes consisting of cells are generated
- Primitives in different dimensions

| dimension | cell | mesh |
|---|---|---|
| 0D | points | |
| 1D | lines (edges) | polyline(–gon) |
| 2D | triangles, quadrilaterals (rectangles) | 2D mesh |
| 3D | tetrahedra, prisms, hexahedra | 3D mesh |

# Domain

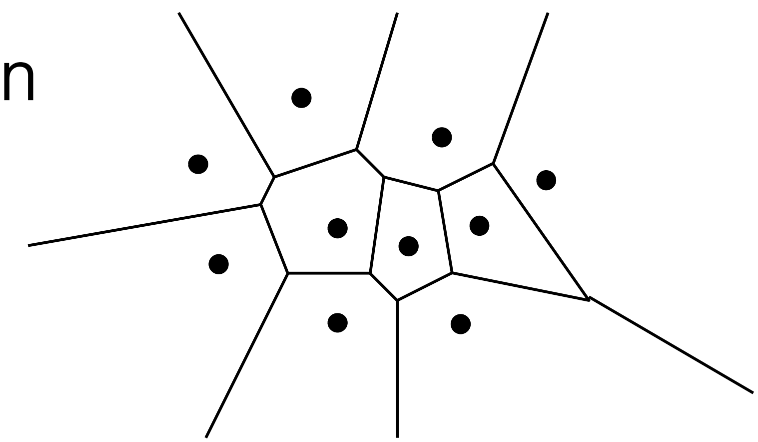
- The (geometric) shape of the domain is determined by the positions of sample points
- Domain is characterized by
  - Dimensionality: 0D, 1D, 2D, 3D, 4D, …
  - Influence: How does a data point influence its neighborhood?
  - Structure: Are data points connected? How? (Topology)

# Domain

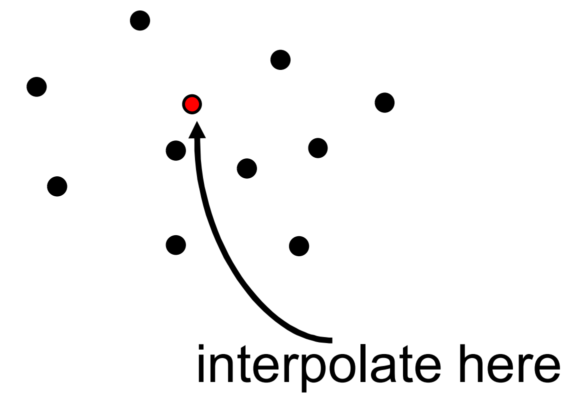
- Influence of data points
  - Values at sample points influence the data distribution in a certain region around these samples
  - To reconstruct the data at arbitrary points within the domain, the distribution of all samples has to be calculated
- Point influence
  - Only influence on point itself
- Local influence
  - Only within a certain region
    - Voronoi diagram
    - Cell-wise interpolation (see later in course)
- Global influence
  - Each sample might influence any other point within the domain
    - Material properties for whole object
    - Scattered data interpolation

# Domain

- Voronoi diagram
  - Construct a region around each sample point that covers all points that are closer to that sample than to every other sample
  - Each point within a certain region gets assigned the value of the sample point
  - Nearest-neighbor interpolation



# Domain




- Scattered data interpolation
  - At each point the weighted average of all sample points in the domain is computed
  - Weighting functions determine the support of each sample point
    - Radial basis functions simulate decreasing influence with increasing distance from samples
  - Schemes might be non-interpolating and expensive in terms of numerical operations

# Data Structures

- Requirements:
  - Efficiency of accessing data
  - Space efficiency
  - Lossless vs. lossy
  - Portability
    - Binary – less portable, more space/time efficient
    - Text – human readable, portable, less space/time efficient
- Definition
  - If points are arbitrarily distributed and no connectivity exists between them, the data is called scattered
  - Otherwise, the data is composed of cells bounded by grid lines
  - Topology specifies the structure (connectivity) of the data
  - Geometry specifies the position of the data

# Data Structures

- Some definitions concerning topology and geometry
  - In topology, qualitative questions about geometrical structures are the main concern
    - Does it have any holes in it?
    - Is it all connected together?
    - Can it be separated into parts?
- Underground map does not tell you how far one station is from the other, but rather how the lines are connected (topological map)

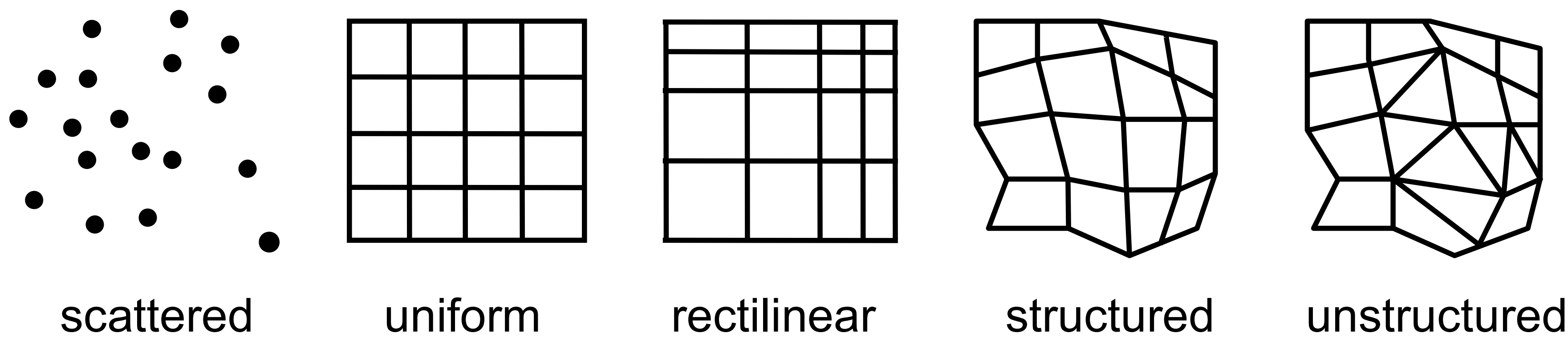
# Grids – General Questions

Important questions:

- Which data organization is optimal?
- Where do the data come from?
- Is there a neighborhood relationship?
- How is the neighborhood info stored?
- How is navigation within the data possible?
- What calculations with the data are possible ?
- Are the data structured (regular/irregular topology)?

# Data Structures

- Grid types
  - Grids differ substantially in the cells (basic building blocks) they are constructed from and in the way the topological information is given



# Thank you.

Thanks for material

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