King Abdullah University of Science and Technology

KAUST

CS 247 – Scientific Visualization
Lecture 25: Vector / Flow Visualization, Pt. 4

Markus Hadwiger, KAUST

Reading Assignment #13 (until May 7)

Read (required):

- Data Visualization book
  - Chapter 6.1 (Divergence and Vorticity)
  - Chapter 6.6 (Texture-Based Vector Visualization)
- Diffeomorphisms / smooth deformations
  `https://en.wikipedia.org/wiki/Diffeomorphism`
- Learn how convolution (the convolution of two functions) works:
  `https://en.wikipedia.org/wiki/Convolution`
- B. Cabral, C. Leedom:
  Imaging Vector Fields Using Line Integral Convolution, SIGGRAPH 1993
  `http://dx.doi.org/10.1145/166117.166151`

Quiz #3: May 7

Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

Content of questions

- Lectures (both actual lectures and slides)
- Reading assignments (except optional ones)
- Programming assignments (algorithms, methods)
- Solve short practical examples

The Flow / Flow Map of a Vector Field (2)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t* (t < s is allowed: map forward or backward in time)

$$\boxed{\psi_{t,s}(x)}$$

with

$$\psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\,\mathrm{d}\tau$$

$$\psi_{s,s}(x) = x$$

$$\psi_{t,r}\big(\psi_{r,s}(x)\big) = \psi_{t,s}(x)$$

The Flow / Flow Map of a Vector Field (3)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t*
  (t < s is allowed: map forward or backward in time)

$\boxed{\psi_{t,s}(x)}$ $\quad \psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\,\mathrm{d}\tau$

Can write explicitly as function of t, *with s and x fixed*

$t \mapsto \psi_{t,s}(x)$ $\quad \rightarrow$ path line

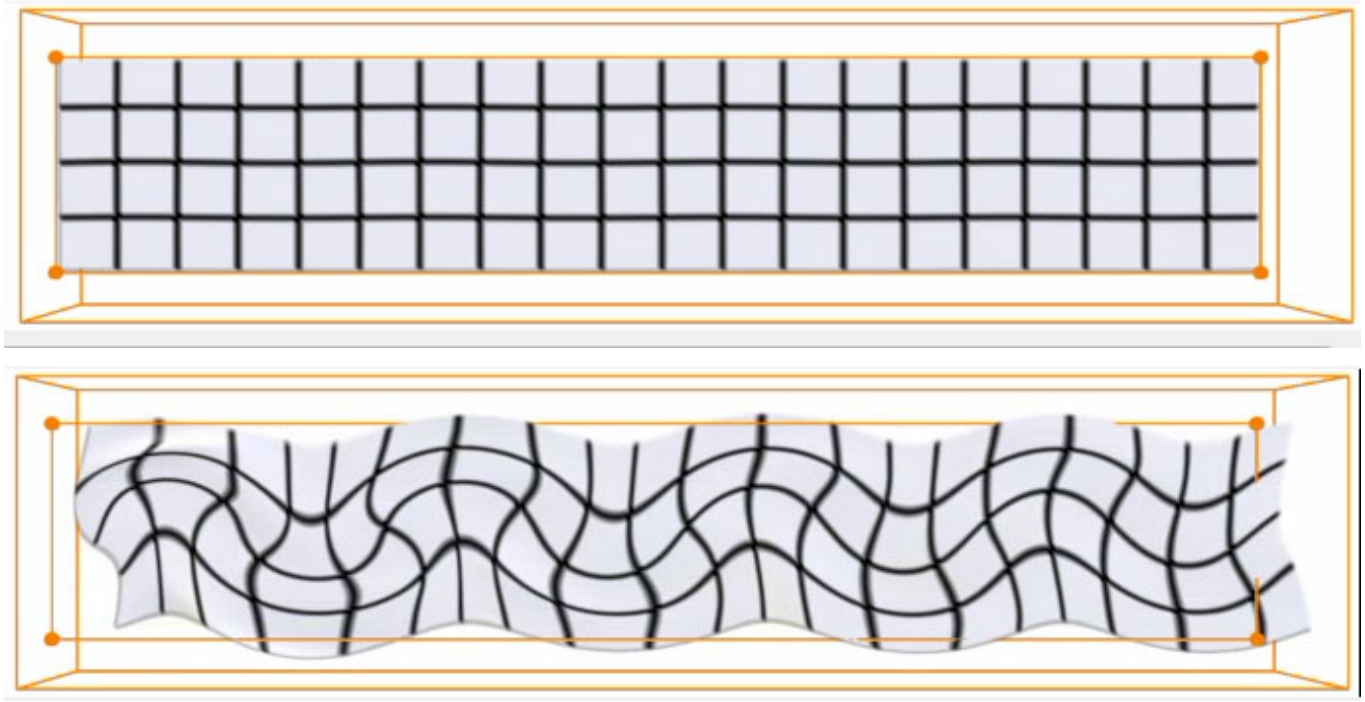
Can write explicitly as function of s, *with t and x fixed*

$s \mapsto \psi_{t,s}(x)$ $\quad \rightarrow$ streak line

$\psi_{t,s}(x)$ is also often written as ***flow map*** $\phi_t^{\tau}(x)$ (with t:=s and either τ:=t or τ:=t-s)

The Flow / Flow Map of a Vector Field (4)

Can map a whole set of points (or the entire domain) through the flow map (this map is a *diffeomorphism*): $t \mapsto \psi_{t,s}(U)$



U

$= \psi_{s,s}(U)$

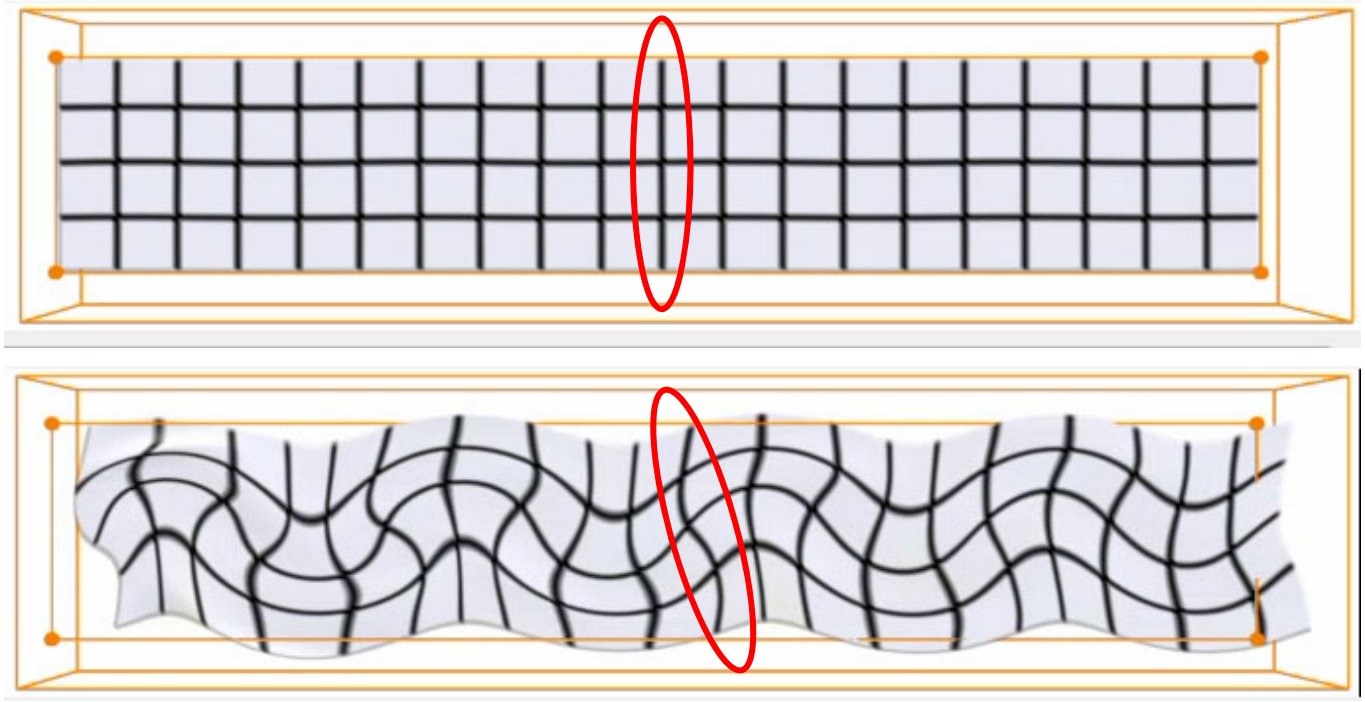
$\psi_{t,s}(U)$

(this is a *time surface!*)

The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (s) to another time (t)

$$t \mapsto \psi_{t,s}(c(\lambda))$$



$$c(\lambda)$$

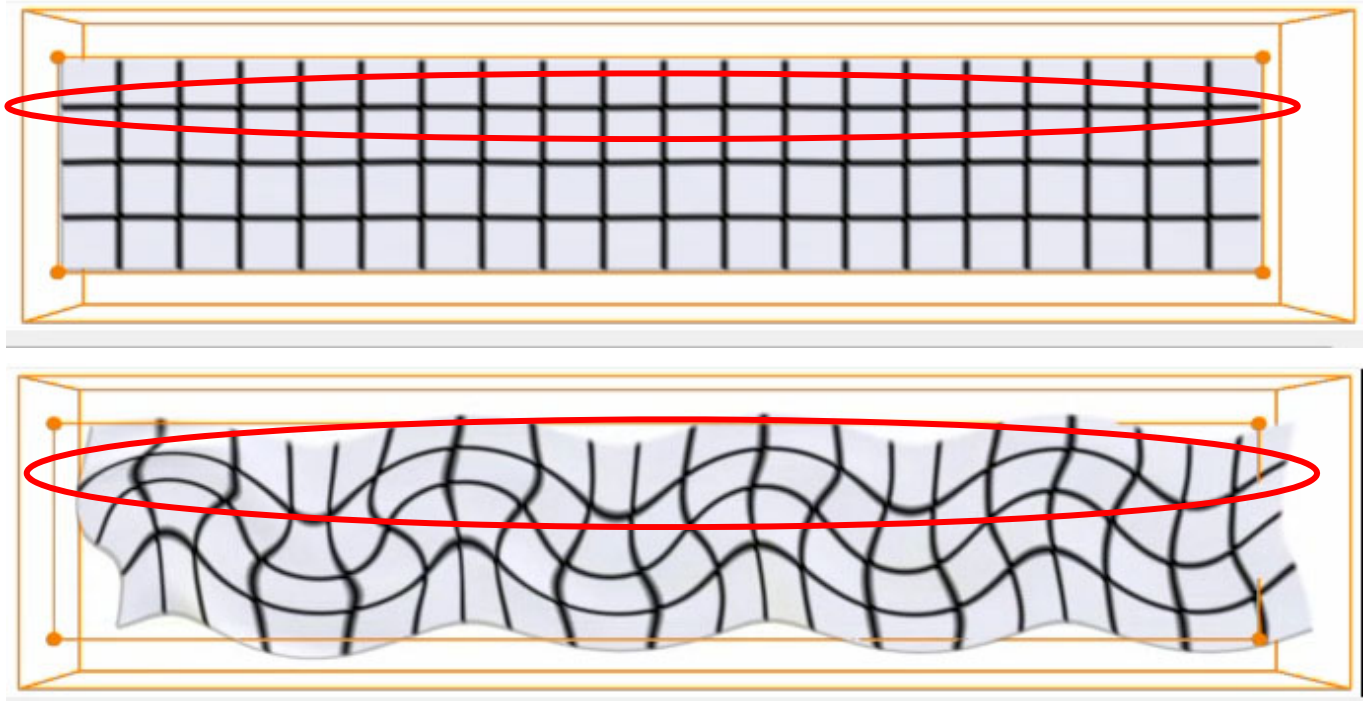
$$= \psi_{s,s}(c(\lambda))$$

$$\psi_{t,s}(c(\lambda))$$

The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (s) to another time (t)

$$t \mapsto \psi_{t,s}(c(\lambda))$$



$$c(\lambda)$$

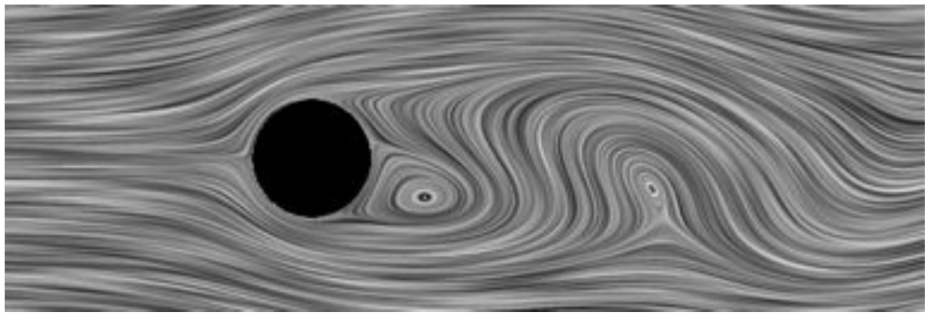
$$= \psi_{s,s}(c(\lambda))$$

$$\psi_{t,s}(c(\lambda))$$

Line Integral Convolution (LIC)

Line Integral Convolution

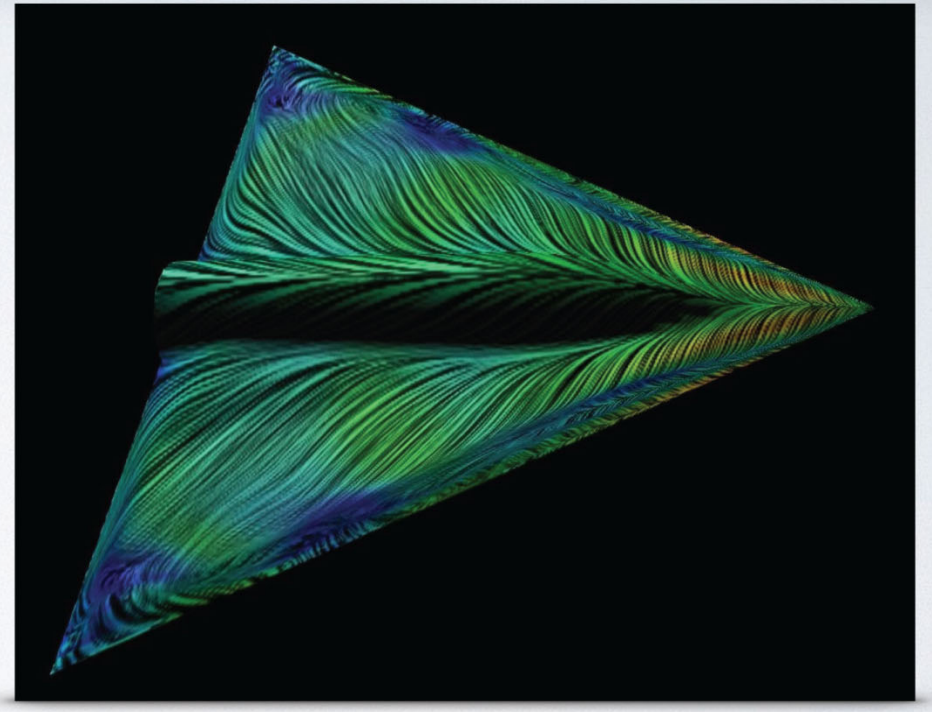
- Line Integral Convolution (LIC)
  - Visualize dense flow fields by imaging its integral curves
  - Cover domain with a random texture (so called ‚input texture‘, usually stationary white noise)
  - Blur (convolve) the input texture along stream lines using a specified filter kernel
- Look of 2D LIC images
  - Intensity distribution along stream lines shows high correlation
  - No correlation between neighboring stream lines



Line Integral Convolution I

- Line Integral Convolution (LIC):
  - goal: general overview of flow
  - approach: use dense textures
  - idea: flow ↔ visual correlation

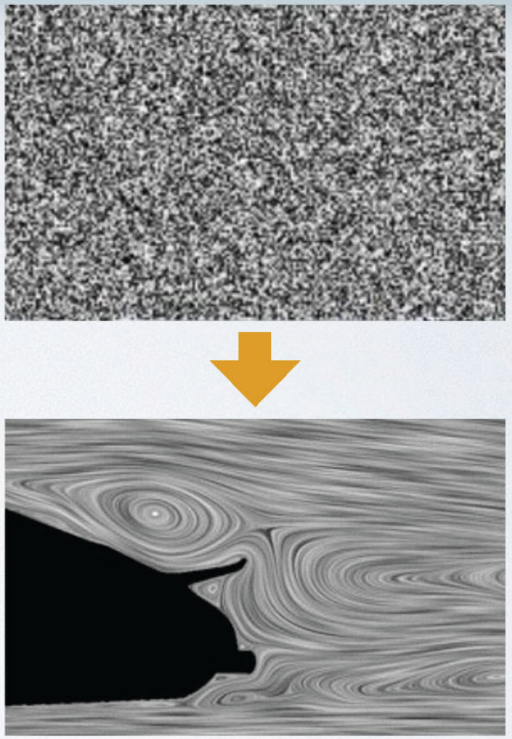
Line Integral Convolution I



- Line Integral Convolution (LIC):
  - goal: general overview of flow
  - approach: use dense textures
  - idea: flow ↔ visual correlation

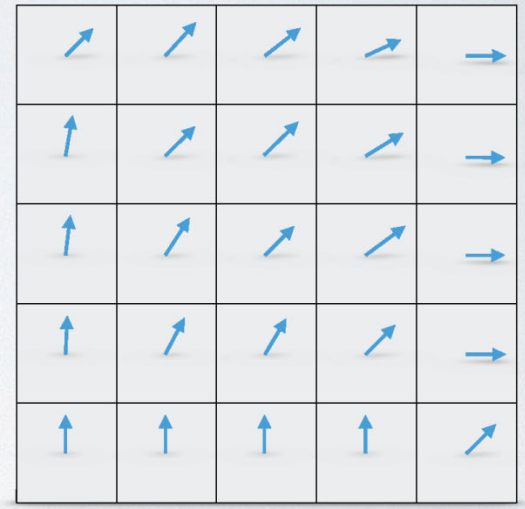
Line Integral Convolution II

- Idea
  - global visualization technique
  - dense representation
  - start with random texture
  - smear along stream lines
- Only for stream lines!
  (steady flow, i.e. time-independent fields)



Line Integral Convolution III

- How LIC works
  - visualize dense flow fields by imaging integral curves
  - cover domain with a random texture
    ('input texture', usually stationary white noise)
  - blur (convolve) the input texture along stream lines



Line Integral Convolution III

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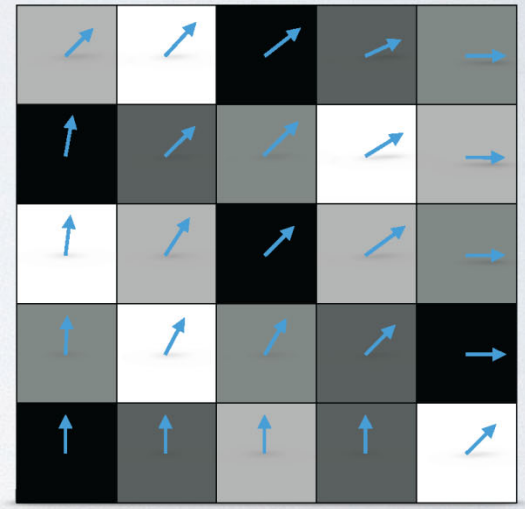

Line Integral Convolution III

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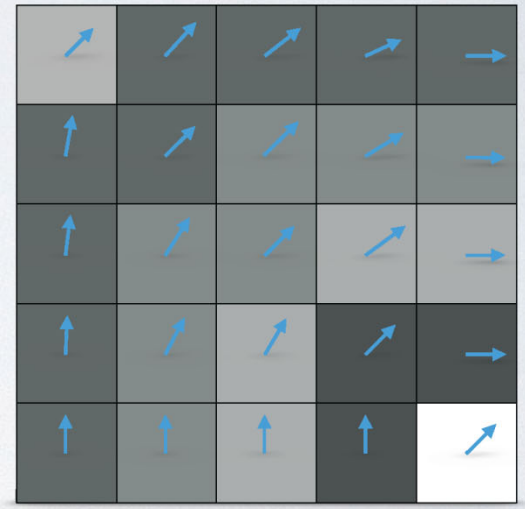
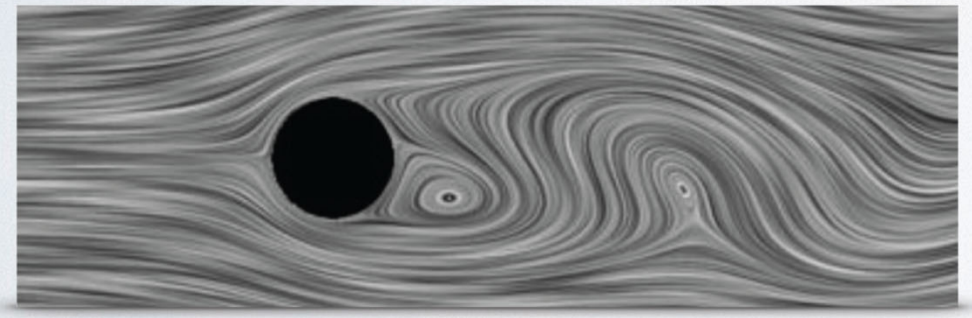
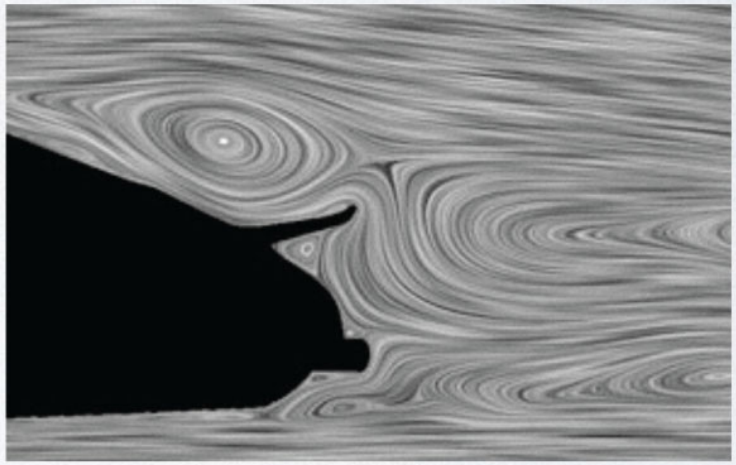


Line Integral Convolution IV

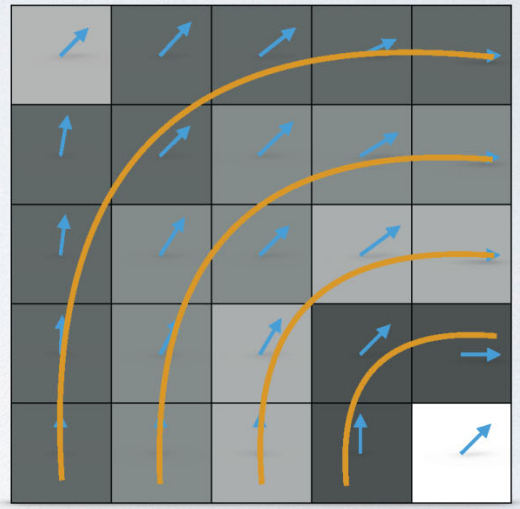
- Look of 2D LIC images
  - intensity along stream lines shows high correlation
  - no correlation between neighboring stream lines





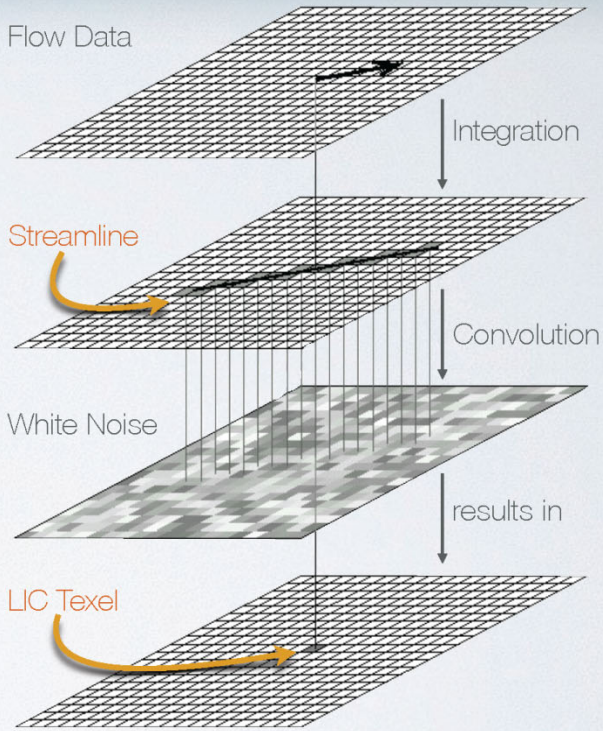
LIC Approach - Goal

- For every texel: let the texture value
  - correlate with neighboring texture values along the flow (in flow direction)
  - not correlate with neighboring texture values across the flow (normal to flow direction)
- Result: along streamlines the texture values are correlated ⇨ visually coherent!



LIC Approach - Steps

- Idea: "smear" white noise (no a priori correlations) along flow
- Calculation of a texture value:
  - follow streamline through point
  - filter white noise along streamline



Convolution Example

Gaussian Blur

en.wikipedia.org/wiki/Gaussian_blur

Cut off filter kernel after an extent of, e.g., 3*standard deviation in each direction

Example:

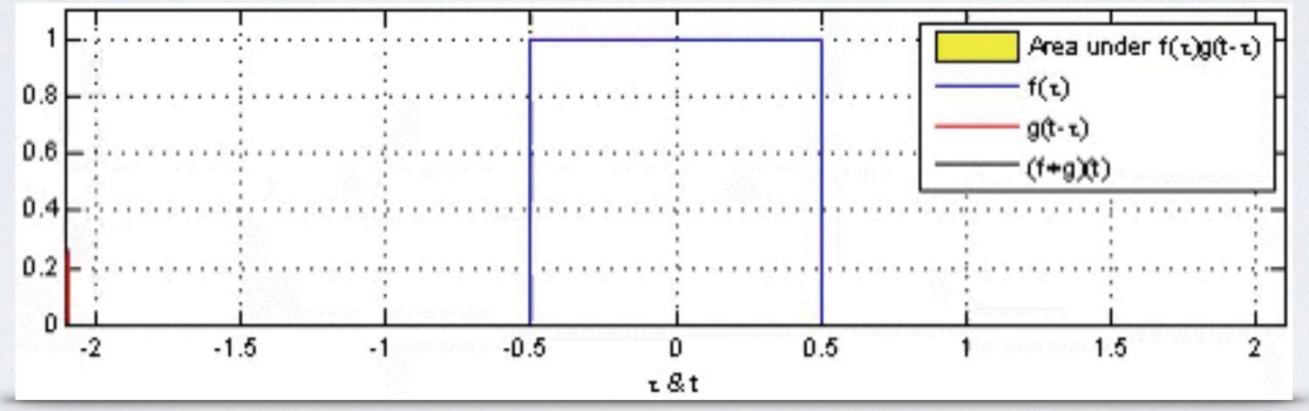
0.00000067	0.00002292	**0.00019117**	0.00038771	**0.00019117**	0.00002292	0.00000067
0.00002292	0.00078634	0.00655965	0.01330373	0.00655965	0.00078633	0.00002292
0.00019117	0.00655965	0.05472157	0.11098164	0.05472157	0.00655965	**0.00019117**
0.00038771	0.01330373	0.11098164	**0.22508352**	0.11098164	0.01330373	0.00038771
0.00019117	0.00655965	0.05472157	0.11098164	0.05472157	0.00655965	**0.00019117**
0.00002292	0.00078633	0.00655965	0.01330373	0.00655965	0.00078633	0.00002292
0.00000067	0.00002292	**0.00019117**	0.00038771	**0.00019117**	0.00002292	0.00000067

Note that 0.22508352 (the central one) is 1177 times larger than 0.00019117 which is just outside 3σ.

Can do multiple iterations to achieve larger effective filter size

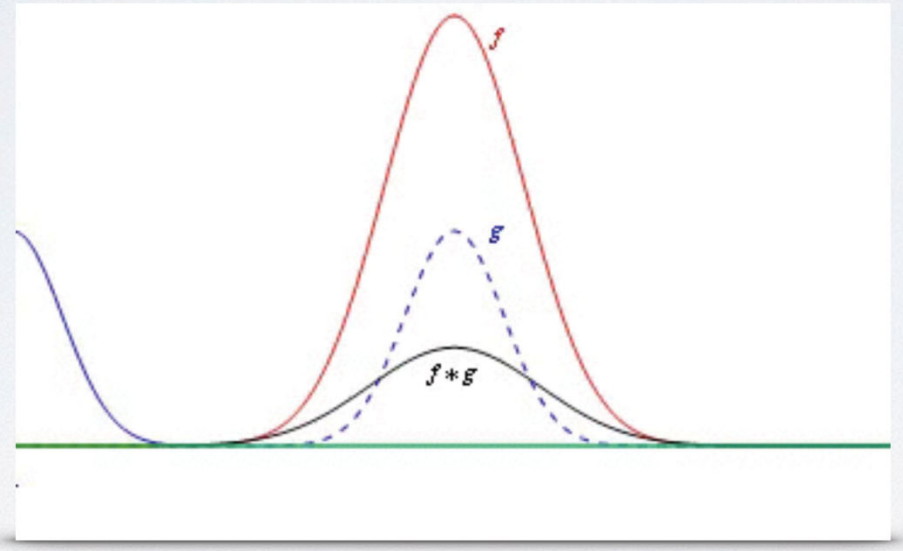
LIC Approach - 1D Convolution I

- Convolution defined as $(f * g)(x) := \int_{\mathbb{R}^n} f(\tau)g(x-\tau)d\tau$



LIC Approach - 1D Convolution II

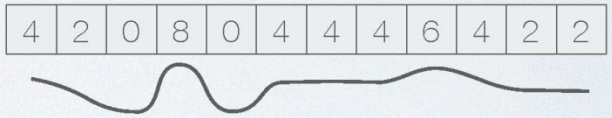
- Convolution defined as $(f * g)(x) := \int_{\mathbb{R}^n} f(\tau)g(x-\tau)d\tau$



LIC Approach - 1D Convolution III

$k(x)$ convolution kernel

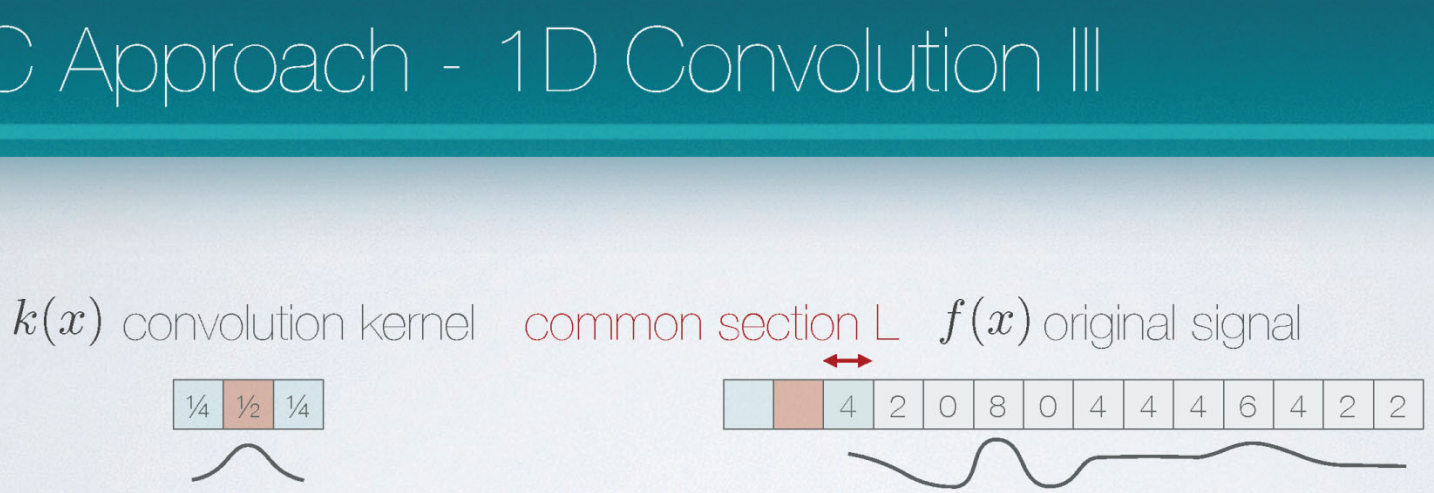
$f(x)$ original signal



$$(f * k)(x) = \int_{-L/2}^{L/2} f(\tau) k(x - \tau) d\tau$$

$(f * k)(x)$ smoothed signal

LIC Approach - 1D Convolution III



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LIC Approach - 1D Convolution III

$k(x)$ convolution kernel

common section L

$f(x)$ original signal

¼ · 0 + ½ · 4 + ¼ · 2

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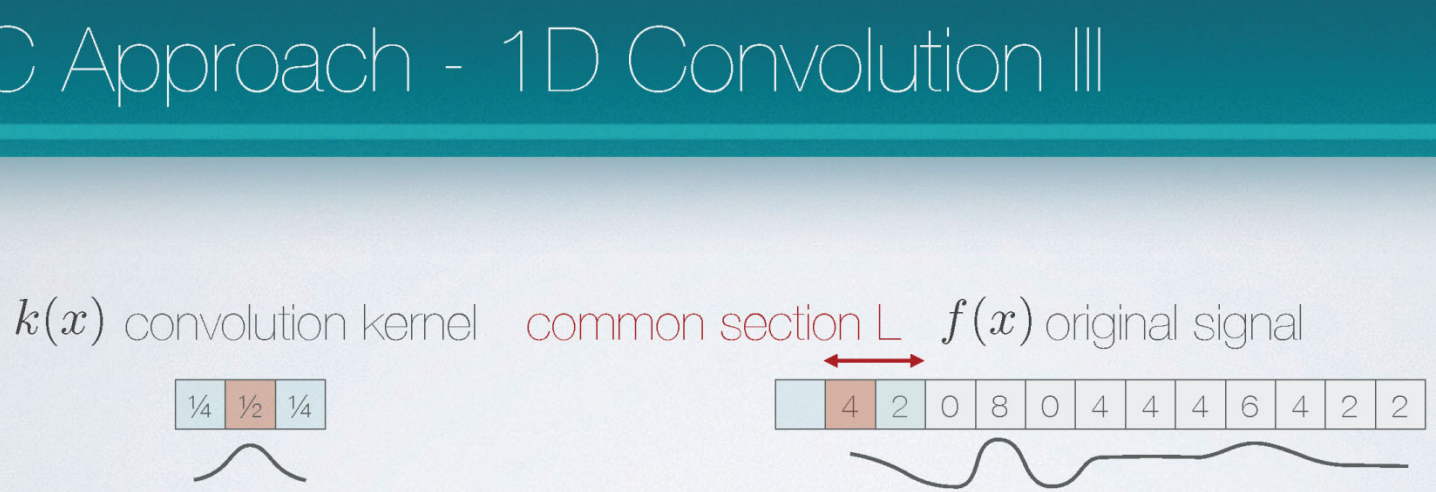
LIC Approach - 1D Convolution III



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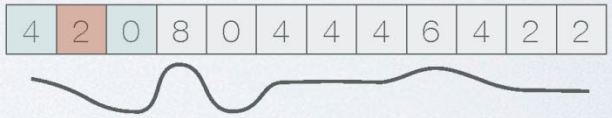
LIC Approach - 1D Convolution III

$k(x)$ convolution kernel

$f(x)$ original signal



¼ · 4 + ½ · 2 + ¼ · 0

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LIC Approach - 1D Convolution III

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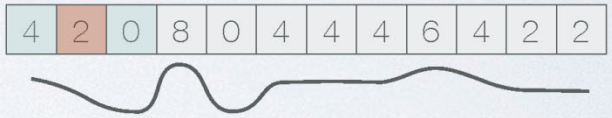

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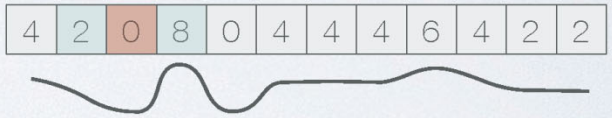

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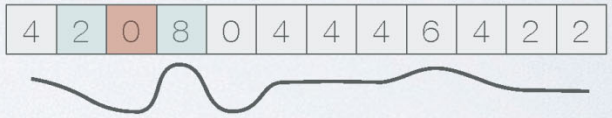

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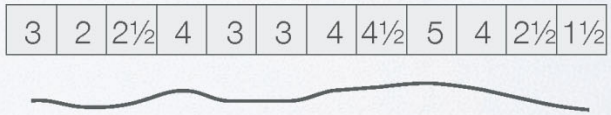
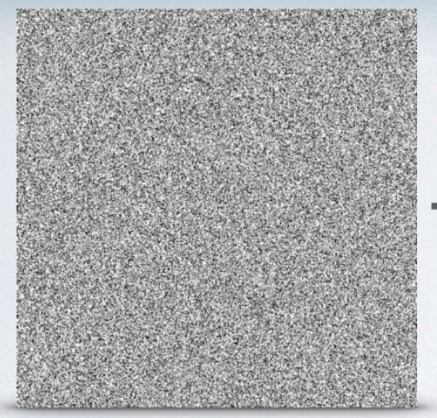


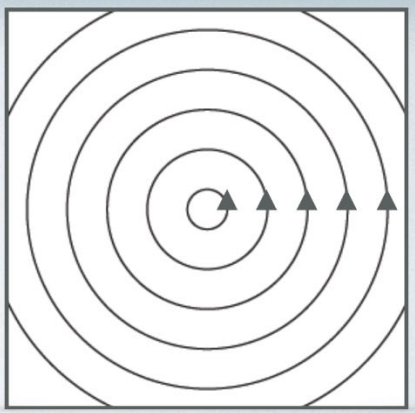
LIC Approach - 1D Convolution IV



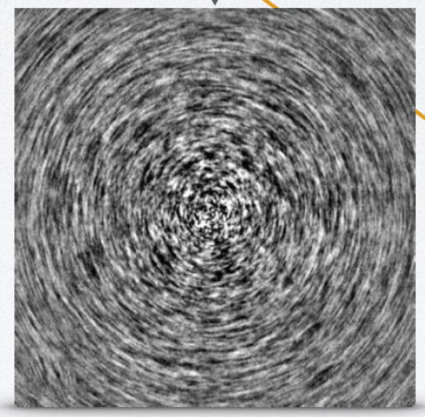
Input Noise

Convolution

$$\int T(\boldsymbol{x}(t+s))k(s)ds$$



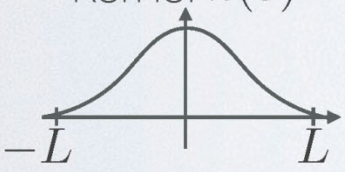
Vector Field



Streamline Tracing

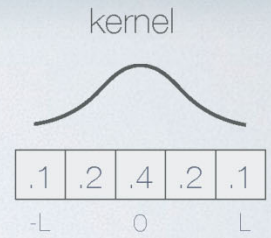
Final Image

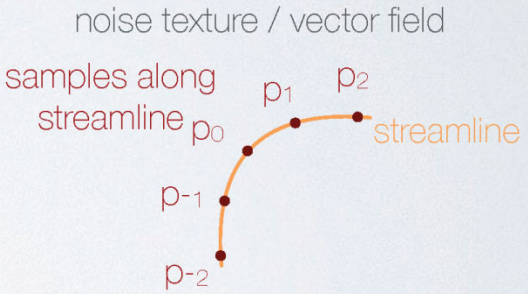
kernel $k(s)$



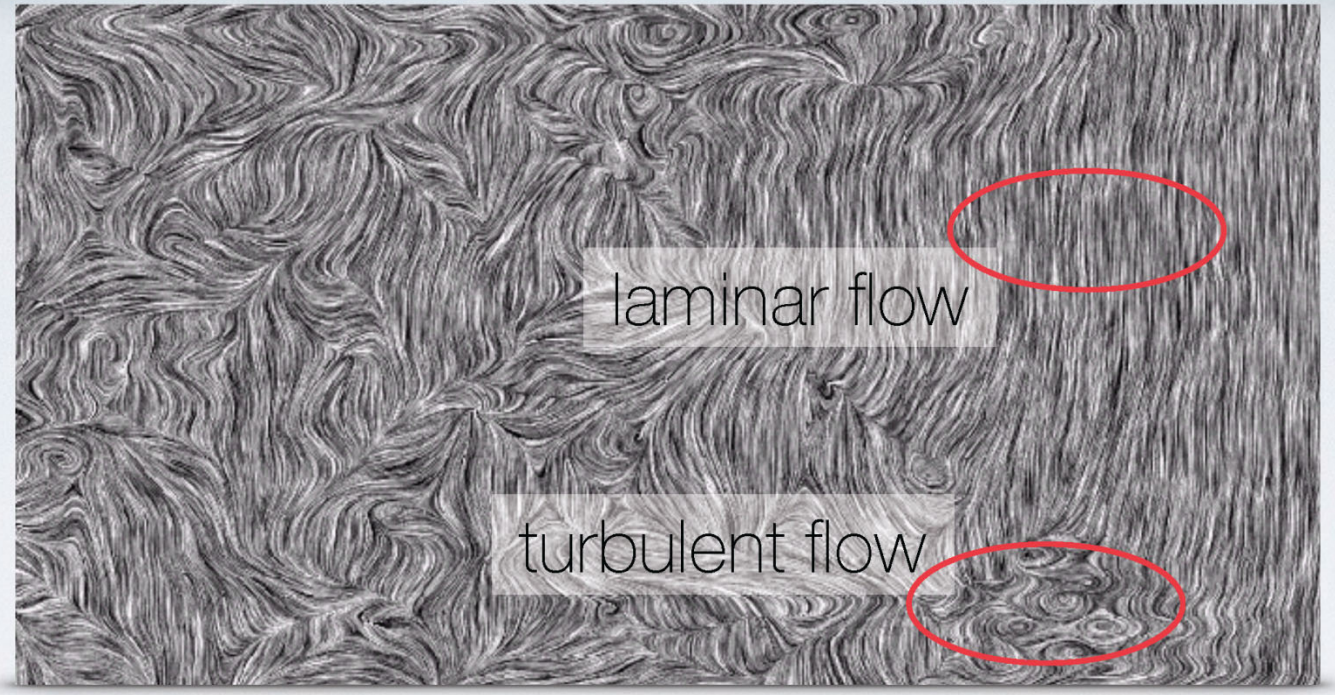
LIC - Algorithm

```
for each pixel //perfect fit for fragment shader

  t = texture( position, noise_texture );

  smoothed_value = kernel_value(center) * t;
  P+ = p- = position;

  for 1 to L // loop over kernel

    v+ = texture( p+, vector_texture );
    p+ = streamlineIntegration(p+, v+);
    smoothed_value +=
        kernel_value * texture( p+, noise_texture );

    v- = -texture( p-, vector_texture );
    p- = streamlineIntegration(p-, v-);
    smoothed_value +=
        kernel_value * texture( p-, noise_texture );
```





LIC - 2D Example



Linear Algebra Approach (1)

- Toeplitz matrix: constant diagonals

$$\mathbf{T} := (t_{ij}) \text{ with } t_{ij} := t_{i-j}$$

$$\mathbf{T}^{N\times N} := \begin{bmatrix} t_0 & t_{(-1)} & t_{(-2)} & \cdots & t_{(-(N-2))} & t_{(-(N-1))} \\ t_1 & t_0 & t_{(-1)} & \cdots & t_{(-(N-3))} & t_{(-(N-2))} \\ t_2 & t_1 & t_0 & \cdots & t_{(-(N-4))} & t_{(-(N-3))} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ t_{N-2} & t_{N-3} & t_{N-4} & \cdots & t_0 & t_{(-1)} \\ t_{N-1} & t_{N-2} & t_{N-3} & \cdots & t_1 & t_0 \end{bmatrix}$$

Linear Algebra Approach (2)

- Circulant matrix: special case of Toeplitz matrix

$$\mathbf{C} := (c_{ij}) \text{ where } c_{ij} := c_{(i-j) \bmod N}$$

$$\mathbf{C}^{N \times N} := \begin{bmatrix} c_0 & c_{N-1} & c_{N-2} & \cdots & c_2 & c_1 \\ c_1 & c_0 & c_{N-1} & \cdots & c_3 & c_2 \\ c_2 & c_1 & c_0 & \cdots & c_4 & c_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{N-2} & c_{N-3} & c_{N-4} & \cdots & c_0 & c_{N-1} \\ c_{N-1} & c_{N-2} & c_{N-3} & \cdots & c_1 & c_0 \end{bmatrix}$$

- Periodic convolution: multiply **C** with (periodic) signal in column vector
- The Fourier transform *diagonalizes* circulant matrices

Thank you.

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