King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 21: Volume Visualization, Pt. 7

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Reading Assignment #11 (until Apr 16)

Read (required):

- Real-Time Volume Graphics, Chapter 10
  (Transfer Functions Reloaded)
- Paper:

  Joe Kniss, Gordon Kindlmann, Charles Hansen,

  Multidimensional Transfer Functions for Interactive Volume Rendering,
  IEEE Transactions on Visualization and Comp. Graph. (TVCG) 2002

  `https://ieeexplore.ieee.org/document/1021579`

Read (optional):

- Real-Time Volume Graphics, Chapter 14
  (Non-Photorealistic and Illustrative Techniques)

More on Transfer Functions

2D (or higher) Transfer Functions

Transfer function look-up with more than one attribute

- T(scalar value, ... additional attributes ...)

Additional attributes:

- Derivatives (most common: gradient magnitude)
- Segmentation information (integer label IDs)
- Curvature (of isosurface going through each point)
- Spatial position
- ...

2D (or higher) Transfer Functions

Derivatives indicate where material boundaries are located

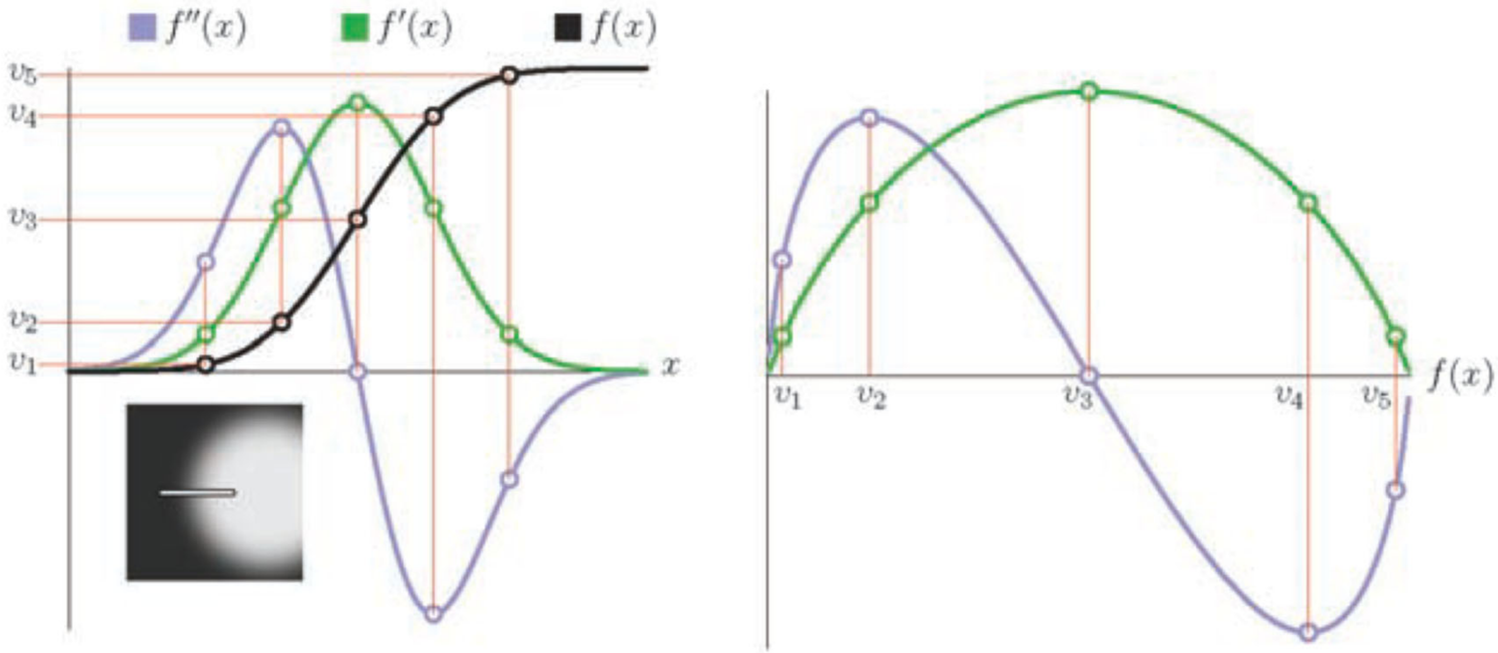


Figure 10.2. Relationships between f, f', f'' in an ideal boundary.

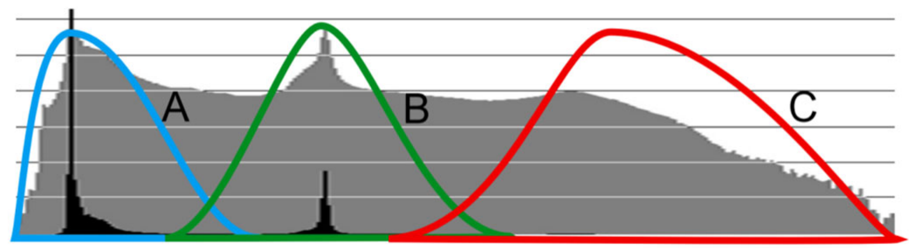
2D Transfer Functions

1D transfer function

Horizontal axis: scalar value

Vertical axis: number of voxels

1D histogram



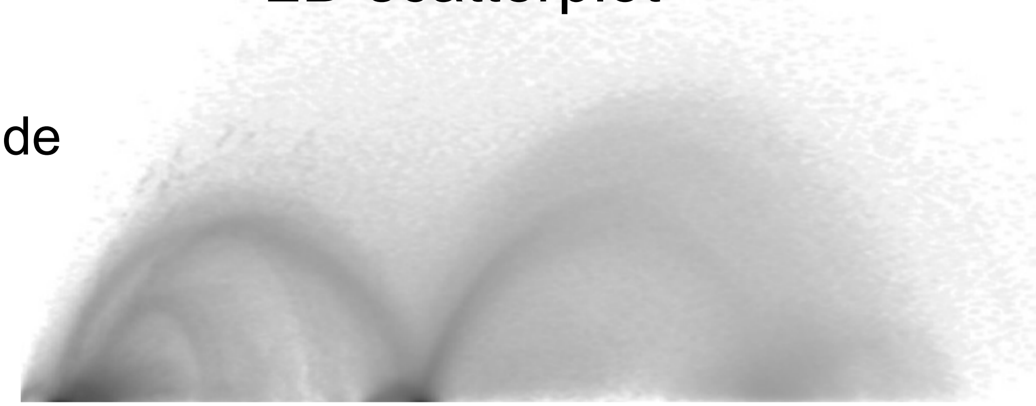
2D transfer function

Horizontal axis: scalar value

Vertical axis: gradient magnitude

Brightness: number of voxels
(here: darker means more)

2D scatterplot



[Kniss et al. 2002]

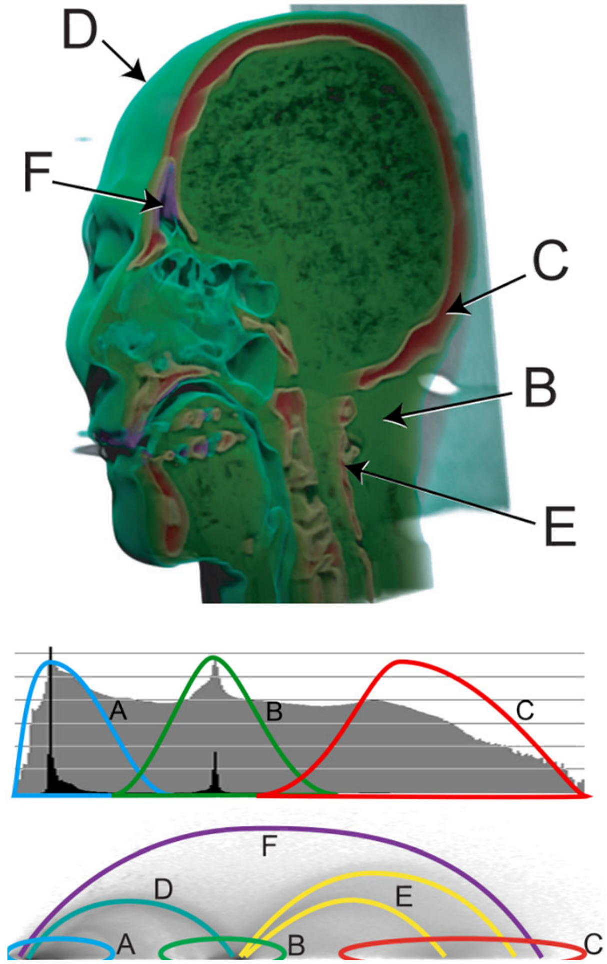
2D Transfer Functions

1D transfer function

- Horizontal axis: scalar value
- Vertical axis: number of voxels

2D transfer function

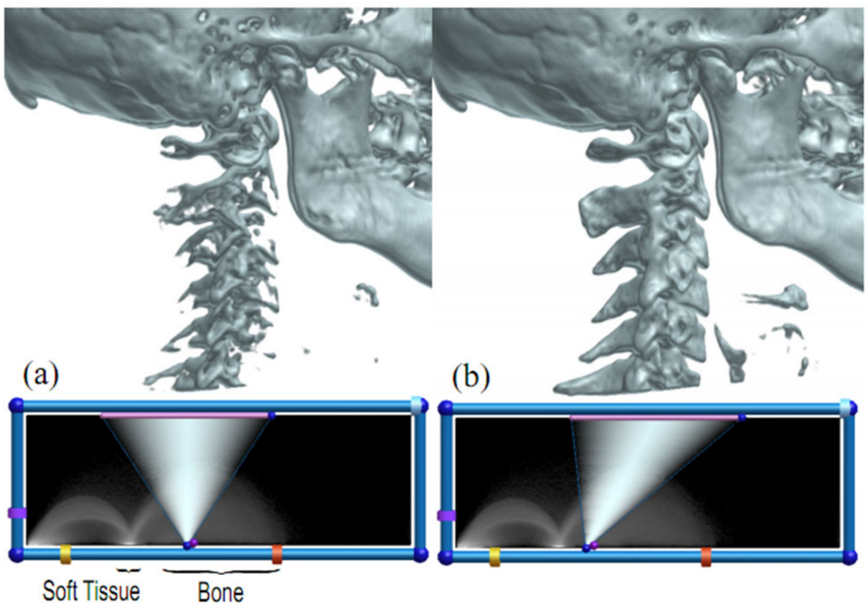
- Horizontal axis: scalar value
- Vertical axis: gradient magnitude

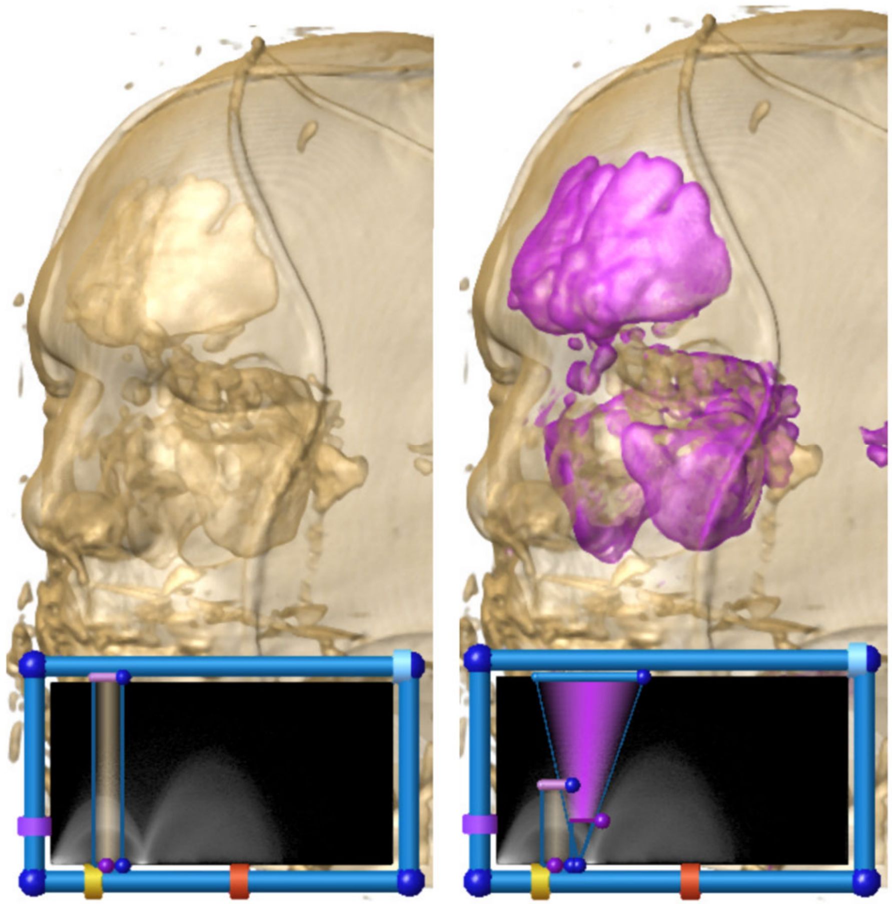


[Kniss et al. 2002]

2D Transfer Functions

Comparisons





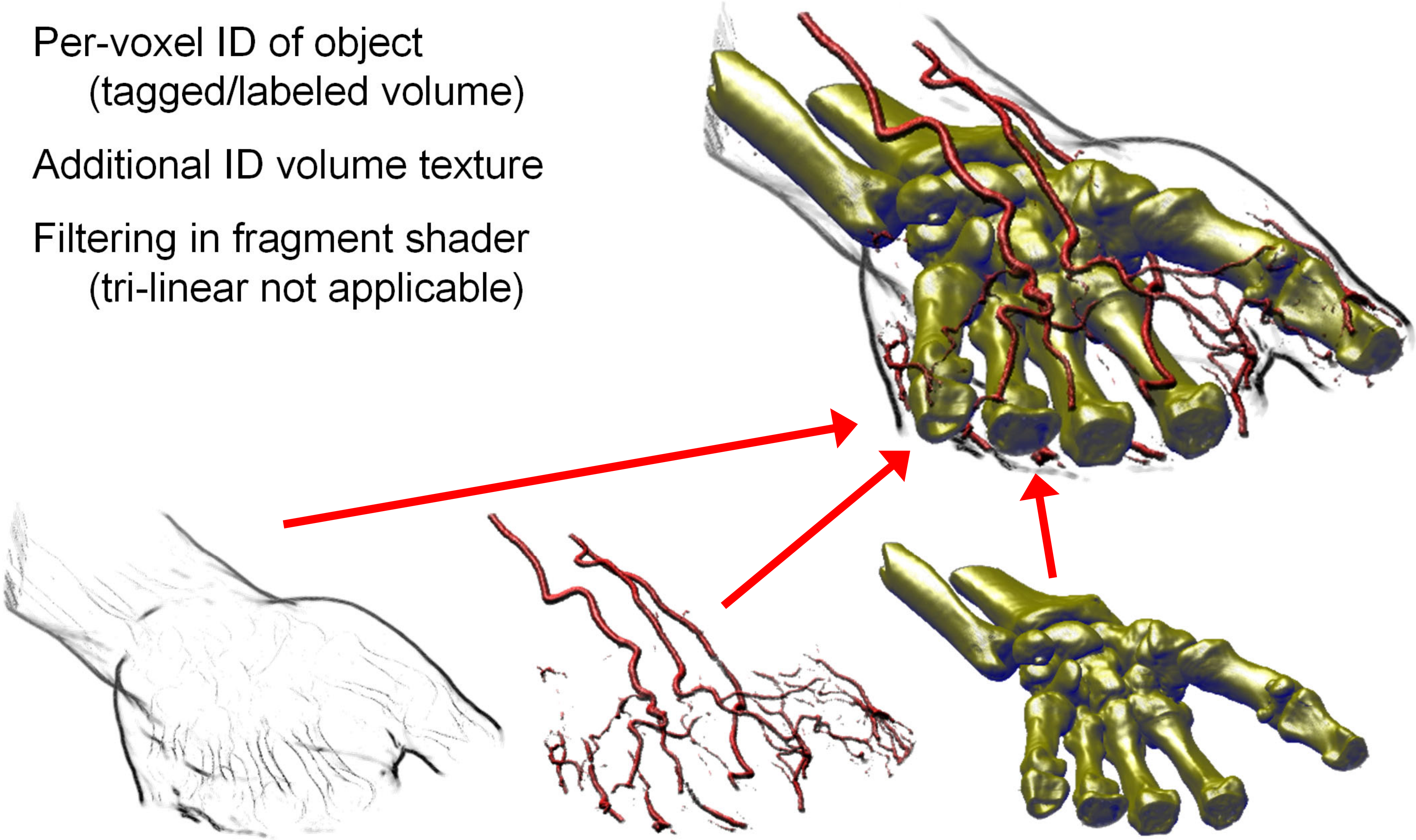
[Kniss et al. 2002]

Rendering Segmented Volumes (1)

Per-voxel ID of object
(tagged/labeled volume)

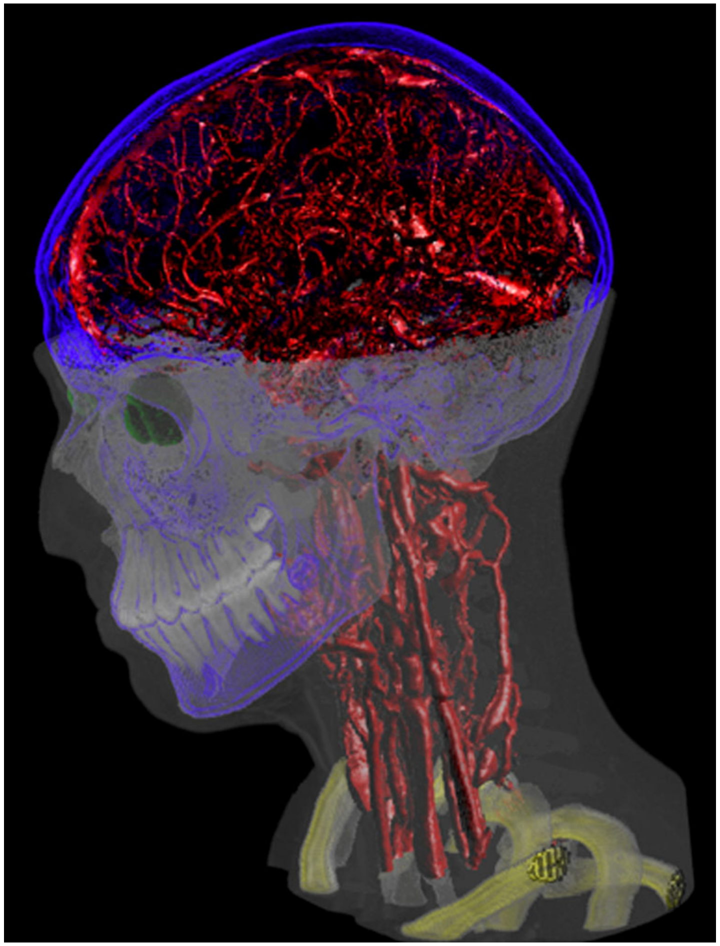
Additional ID volume texture

Filtering in fragment shader
(tri-linear not applicable)



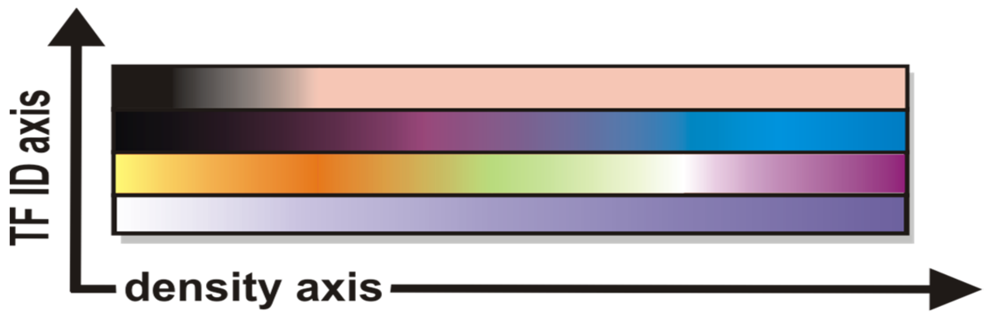
Rendering Segmented Volumes (2)

- Focus and context
- Per-object transfer function
- Per-object rendering mode
- Per-object compositing



Per-Object Transfer Functions

Put all transfer functions in one global TF texture



index with object ID
as additional axis

```
tf_coords.x = tex3D( density_tex, sample_pos );
tf_coords.y = tex3D( objectid_tex, sample_pos );
classified_sample.rgba = tex2D( tf_tex, tf_coords );
```

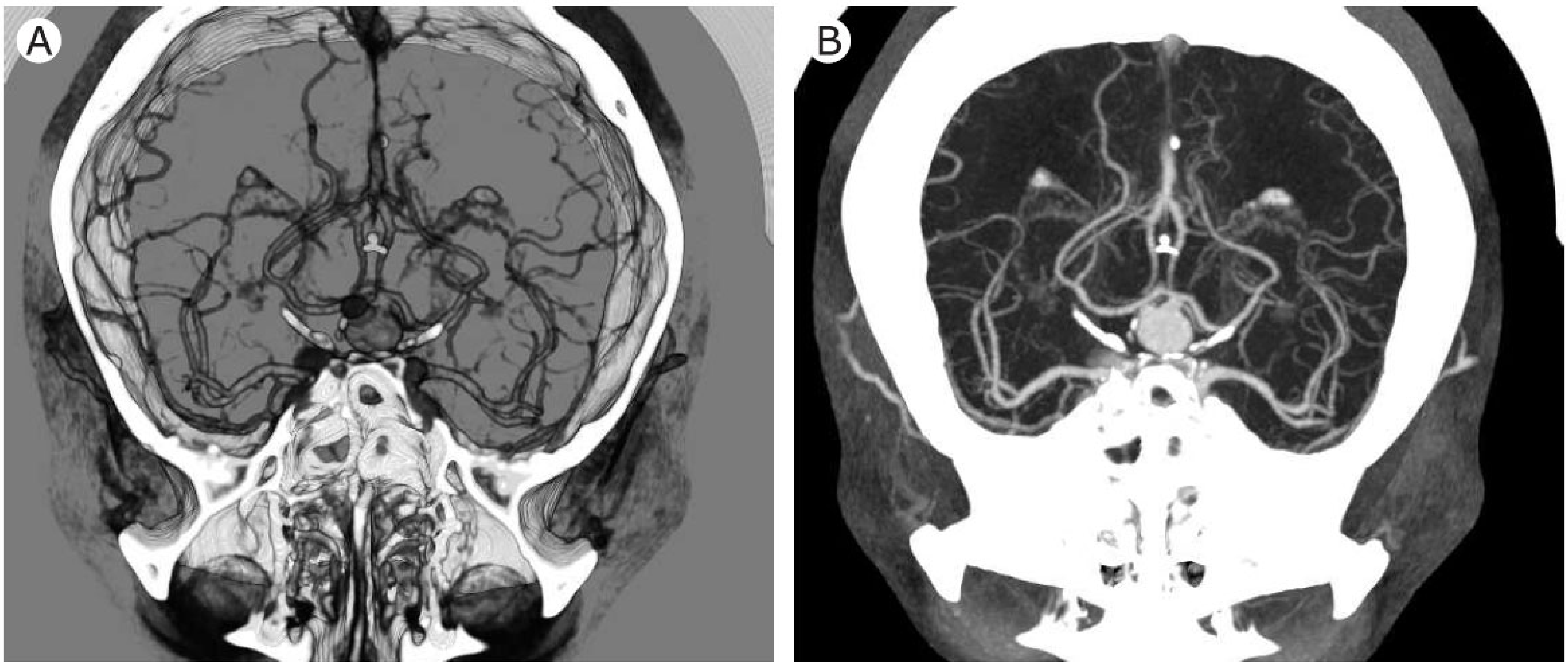
1D transfer functions → 2D texture

2D transfer functions → 3D texture

Maximum Intensity Projection

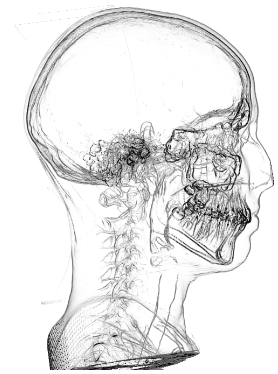
Alternative compositing mode (no alpha blending)

Keeps structure of maximum intensity visible



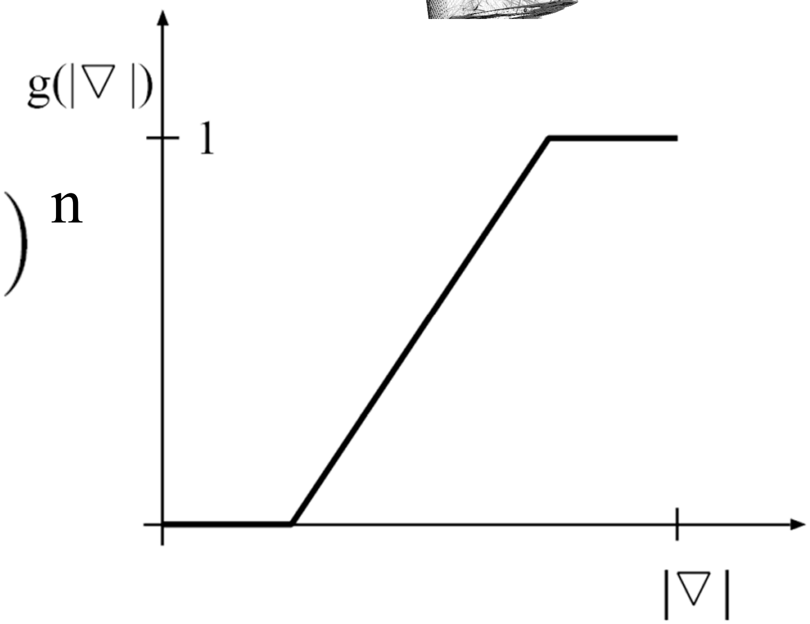
Volumetric Boundary Contours (1)

Based on view direction and gradient magnitude



Global boundary detection
instead of isosurface

Gradient magnitude window g(.)



$$\mathbf{I} = g\big(|\nabla f|\big) \cdot \big(1 - |\mathbf{v} \cdot \mathbf{n}|\big)^{n}$$

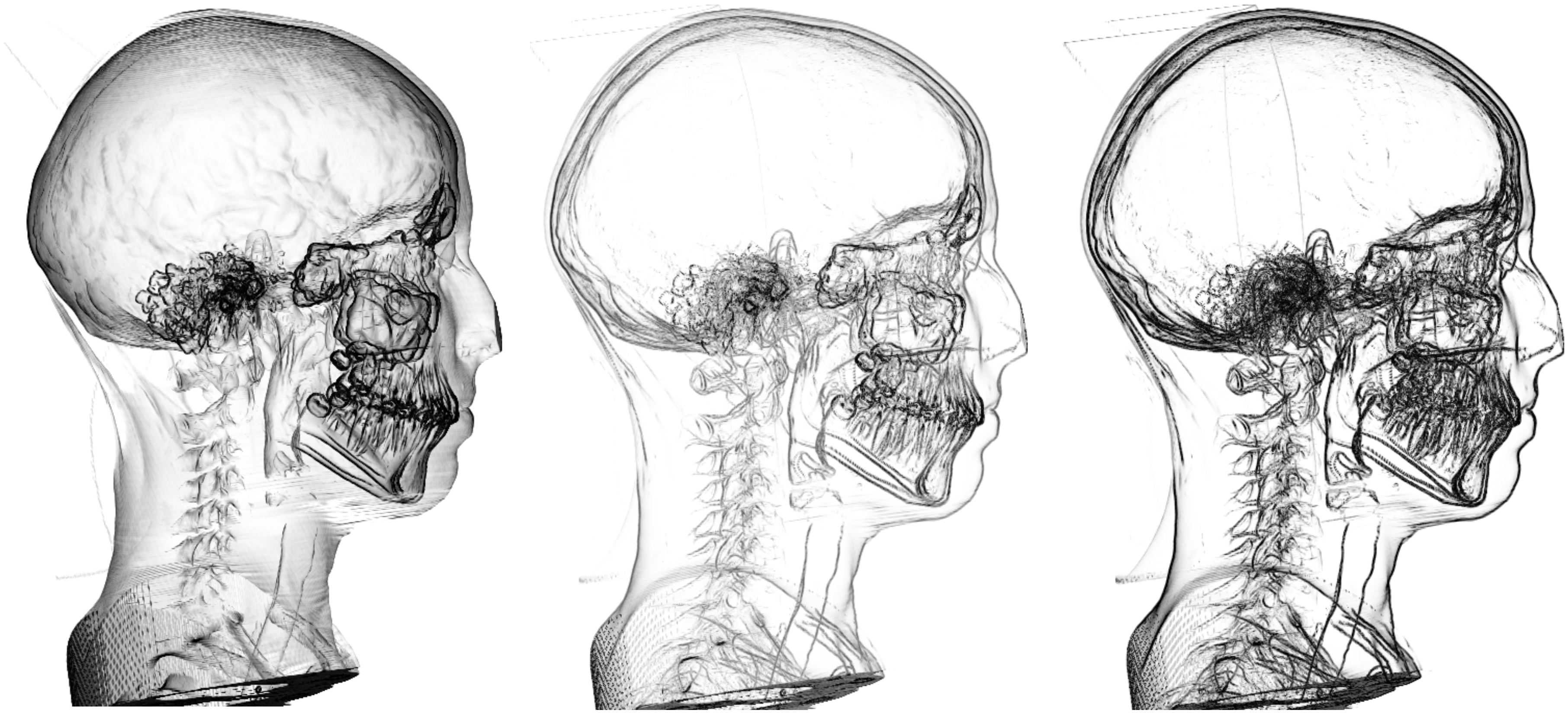
Exponent determines silhouette range

Does not work for distance fields!

Volumetric Boundary Contours (2)

Gradient magnitude window is main parameter

Exponent between 4 and 16 is good choice

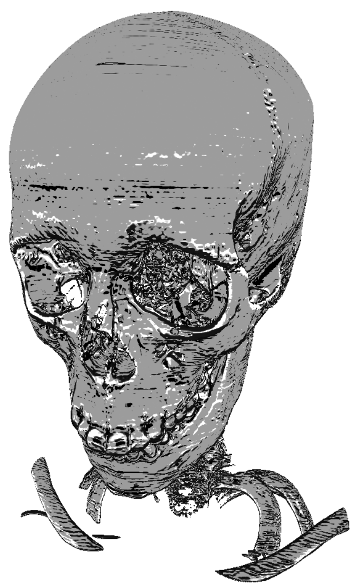


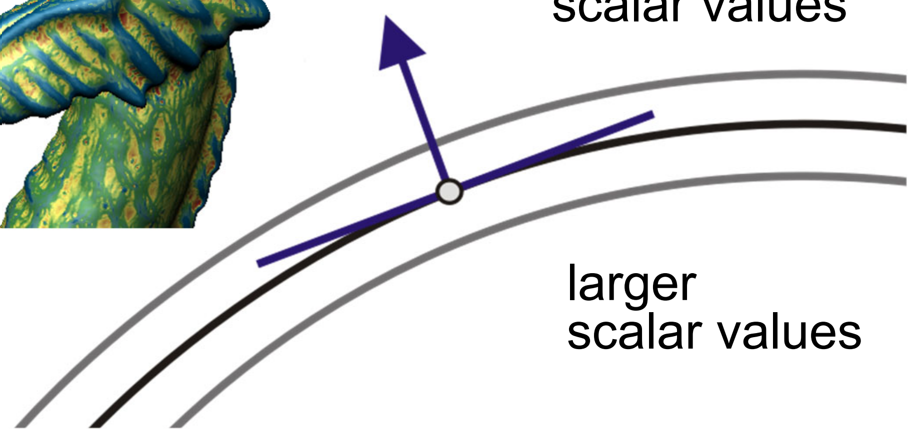
Curvature-Based Transfer Functions

Curvature-Based Isosurface Illustration

Curvature measure color mapping

Curvature directions; ridges and valleys





- Implicit surface curvature
- Isosurface through a point

(Extrinsic) Curvature

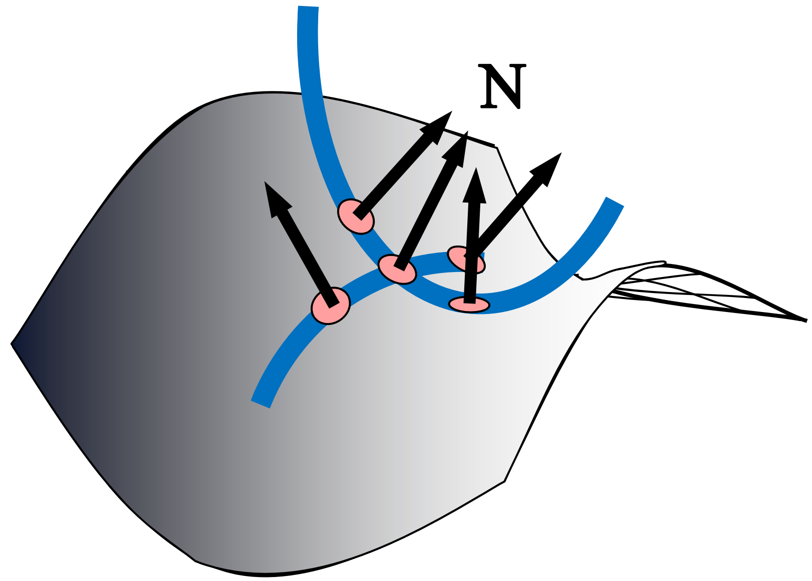
How fast do positional changes (in different directions) on the surface change the normal vector?

- Gauss map: assigns normal to each point

$$\begin{aligned} \mathrm{N}: M &\to \mathbb{S}^2, \\ x &\mapsto \mathrm{N}(x). \end{aligned}$$

- Differential of Gauss map:
  Shape operator / Weingarten map

$$\begin{aligned} \mathrm{dN}: T_x M &\to T_{\mathrm{N}(x)}\mathbb{S}^2, \\ \mathbf{v} &\mapsto \mathrm{dN}(\mathbf{v}). \end{aligned}$$

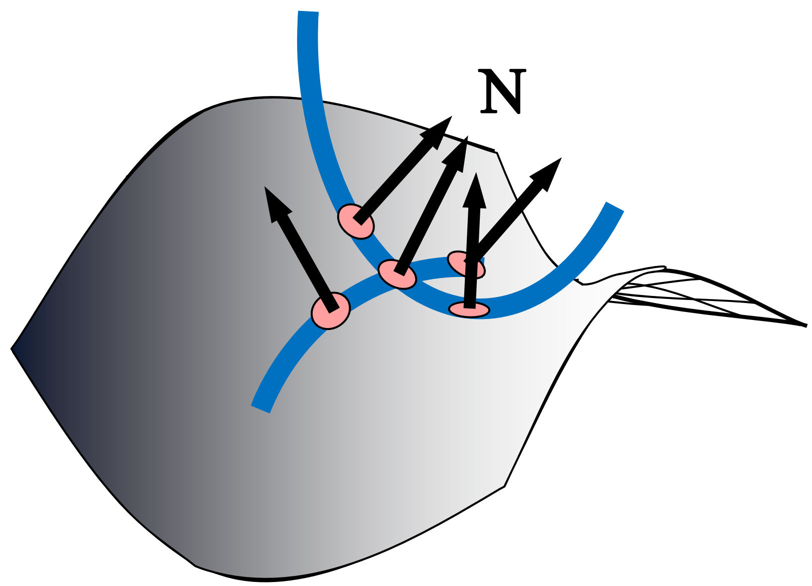


courtesy of Gordon Kindlmann

(Extrinsic) Curvature

Analyze shape operator S

- Eigenvalues: principal curvatures (magnitudes)
  - First and second principal curvature
  - Maximum: κ_1
  - Minimum: κ_2
- Eigenvectors: principal curv. directions
- Gaussian curvature (intrinsic!): $\kappa_1 \kappa_2$



courtesy of Gordon Kindlmann

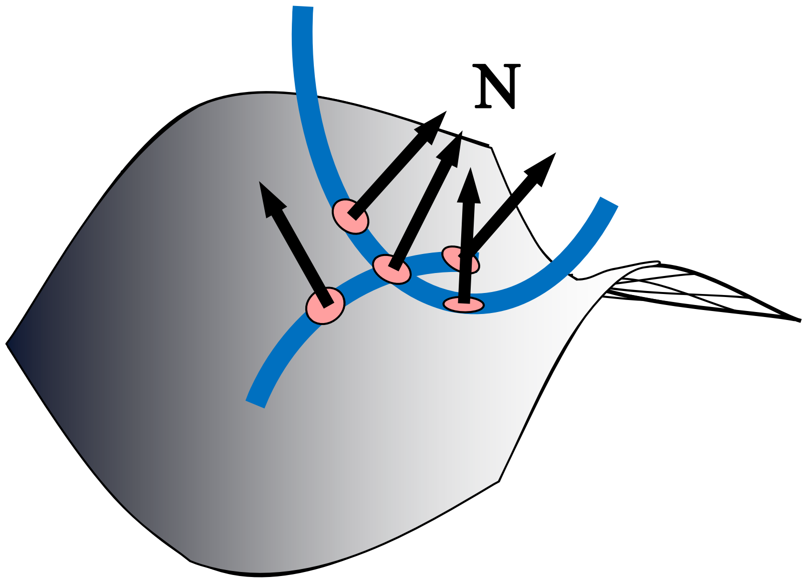
(Extrinsic) Curvature Computation

Simple recipe for implicit isosurfaces in volume

- Build on gradient and Hessian matrix
- Hessian contains curvature information

Transform Hessian into tangent space

- Curvature magnitudes:
  Eigenvalues of 2x2 matrix
- Curvature directions:
  Eigenvectors of 2x2 matrix



courtesy of Gordon Kindlmann

Alternative:

- Compute in 3D (see Real-Time Volume Graphics, 14.4.4)

Curvature and Shape Operator

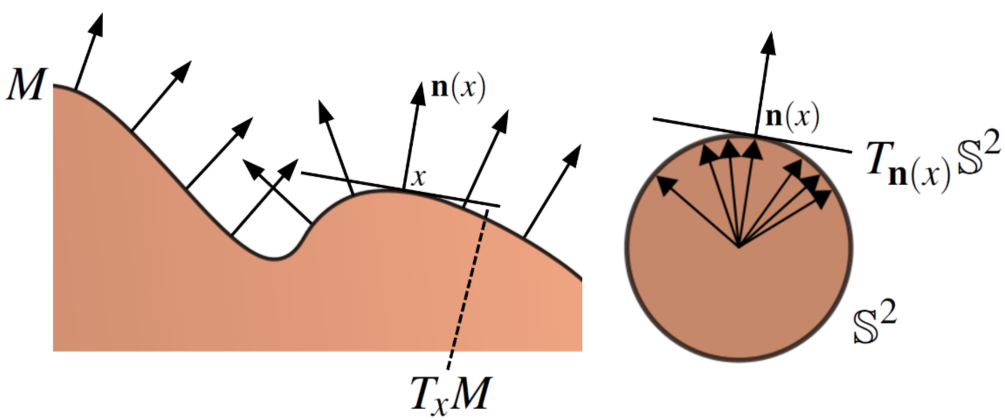
Gauss map

$$\mathbf{n}\colon M \to \mathbb{S}^2$$
$$x \mapsto \mathbf{n}(x)$$

Differential of Gauss map

$$d\mathbf{n}\colon TM \to T\mathbb{S}^2$$
$$\mathbf{v} \mapsto d\mathbf{n}(\mathbf{v})$$

$$(d\mathbf{n})_x\colon T_xM \to T_{\mathbf{n}(x)}\mathbb{S}^2$$
$$\mathbf{v} \mapsto d\mathbf{n}(\mathbf{v})$$



Shape operator (Weingarten map)

$$\mathbf{S}\colon TM \to TM$$

$$\mathbf{S}_x\colon T_xM \to T_xM$$
$$\mathbf{v} \mapsto \mathbf{S}_x(\mathbf{v}) = d\mathbf{n}(\mathbf{v})$$

$$T_{\mathbf{n}(x)}\mathbb{S}^2 \cong T_xM$$

Principal curvature magnitudes and directions are eigenvalues and eigenvectors of shape operator **S**

Curvature and Shape Operator

Gauss map

$$\mathbf{n}: M \to \mathbb{S}^2$$
$$x \mapsto \mathbf{n}(x)$$

Differential of Gauss map

$$d\mathbf{n}: TM \to T\mathbb{S}^2$$
$$\mathbf{v} \mapsto d\mathbf{n}(\mathbf{v})$$

$$(d\mathbf{n})_x: T_xM \to T_{\mathbf{n}(x)}\mathbb{S}^2$$
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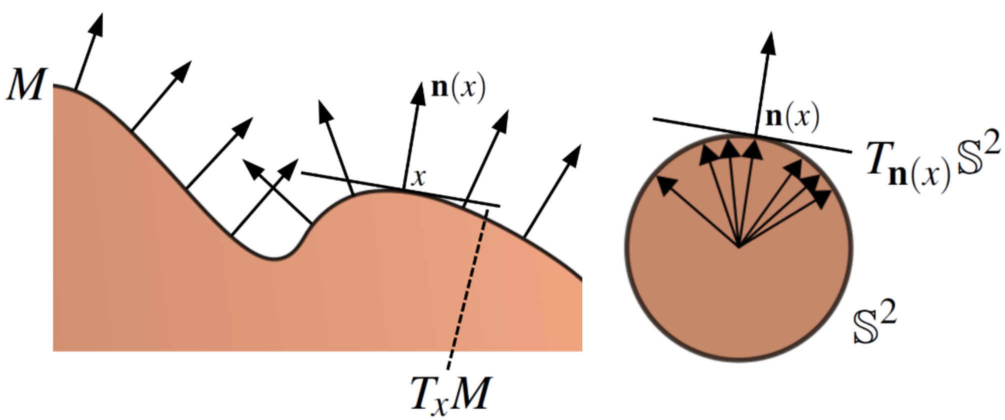


Shape operator (Weingarten map)

$$\mathbf{S}: TM \to TM$$

$$\mathbf{S}_x: T_xM \to T_xM$$
$$\mathbf{v} \mapsto \mathbf{S}_x(\mathbf{v}) = \nabla_{\mathbf{v}}\mathbf{n}$$

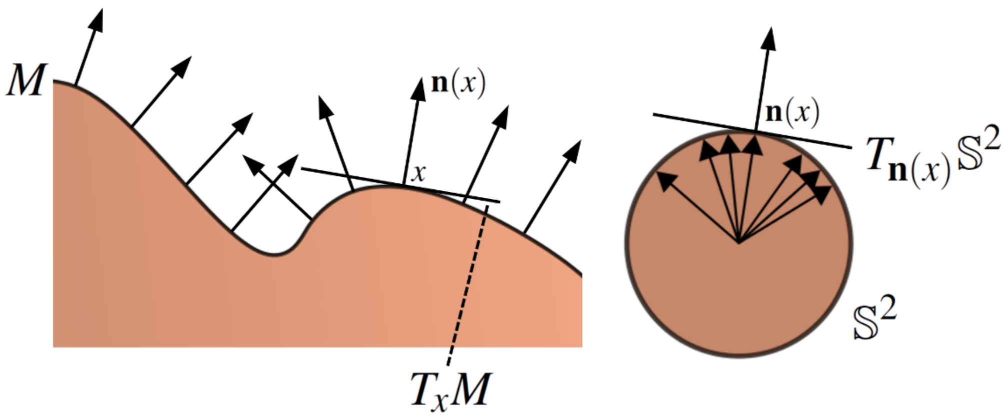
Principal curvature magnitudes and directions are eigenvalues and eigenvectors of shape operator **S**

$$T_{\mathbf{n}(x)}\mathbb{S}^2 \cong T_xM$$

Curvature and Shape Operator

Gauss map

$$\mathbf{n}: M \to \mathbb{S}^2$$
$$x \mapsto \mathbf{n}(x)$$



Differential of Gauss map

$$d\mathbf{n}: TM \to T\mathbb{S}^2$$
$$\mathbf{v} \mapsto d\mathbf{n}(\mathbf{v})$$

$$(d\mathbf{n})_x: T_x M \to T_{\mathbf{n}(x)}\mathbb{S}^2$$
$$\mathbf{v} \mapsto d\mathbf{n}(\mathbf{v})$$

Shape operator (Weingarten map)

$$\mathbf{S}: TM \to TM$$

$$\mathbf{S}_x: T_x M \to T_x M$$
$$\mathbf{v} \mapsto \mathbf{S}_x(\mathbf{v}) = -\nabla_{\mathbf{v}}\mathbf{n}$$

(sign is convention)

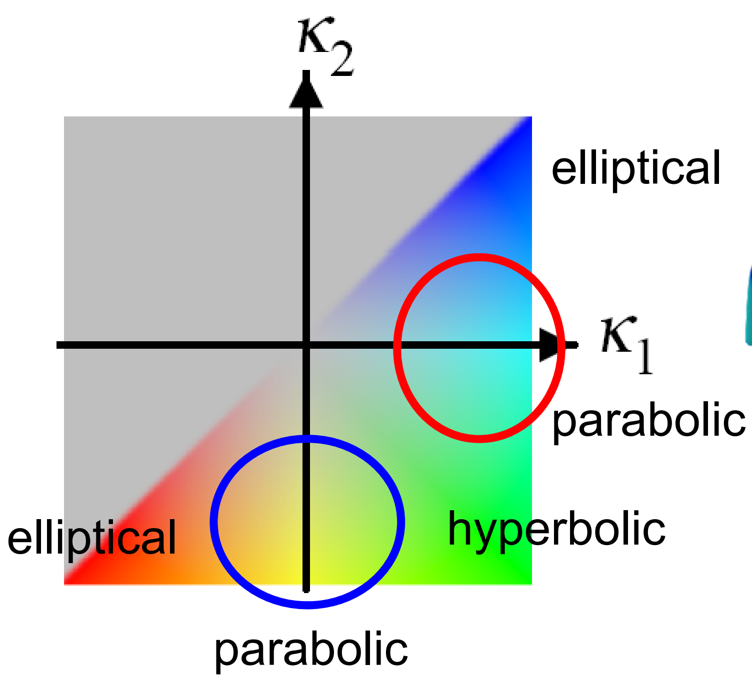
Principal curvature magnitudes and directions are eigenvalues and eigenvectors of shape operator **S**

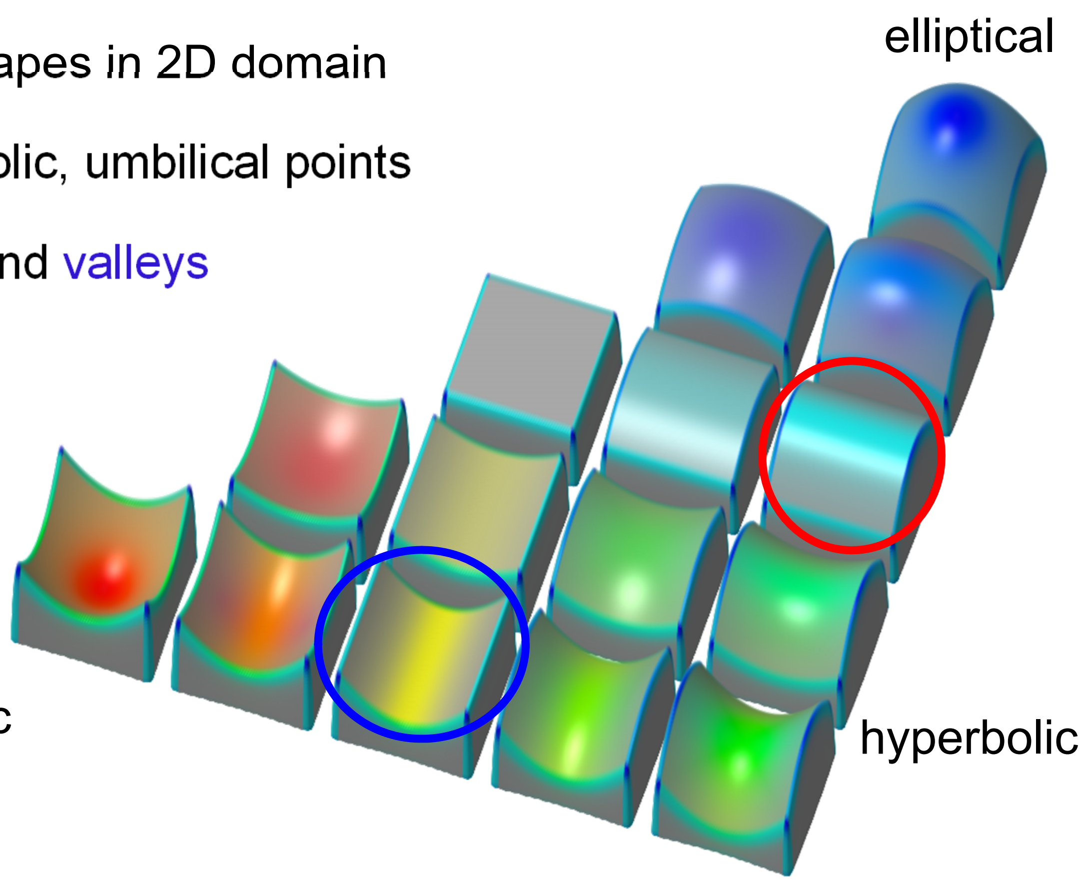
$$T_{\mathbf{n}(x)}\mathbb{S}^2 \cong T_x M$$

The Principal Curvature Domain

Maximum/minimum principal curvature magnitude

- Identification of different shapes in 2D domain
- Elliptical, parabolic, hyberbolic, umbilical points
- Feature lines: e.g., ridges and valleys

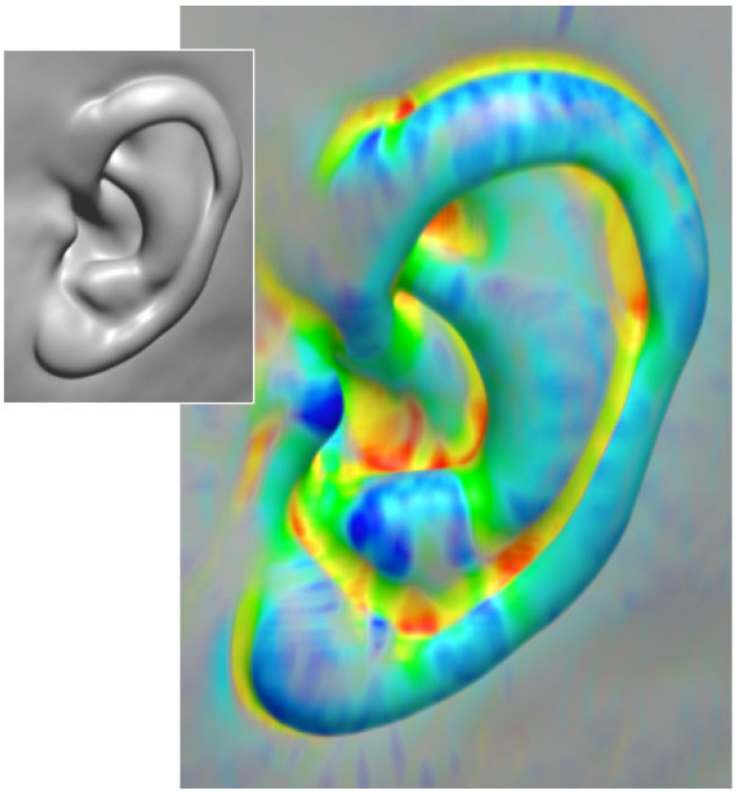


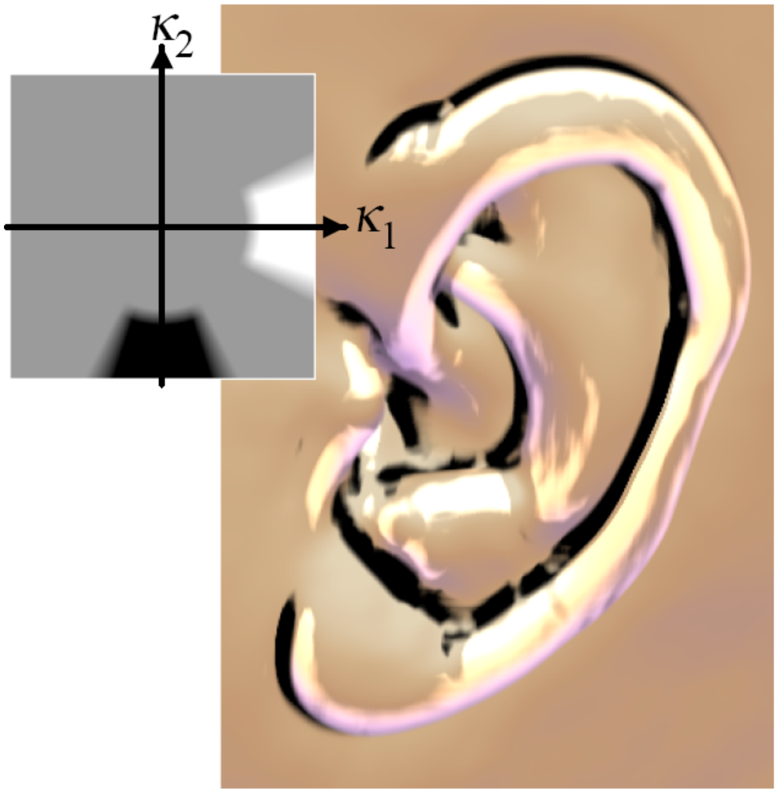


courtesy of Gordon Kindlmann

Curvature Transfer Functions

- Color coding of curvature domain
- Paint features: ridge and valley lines

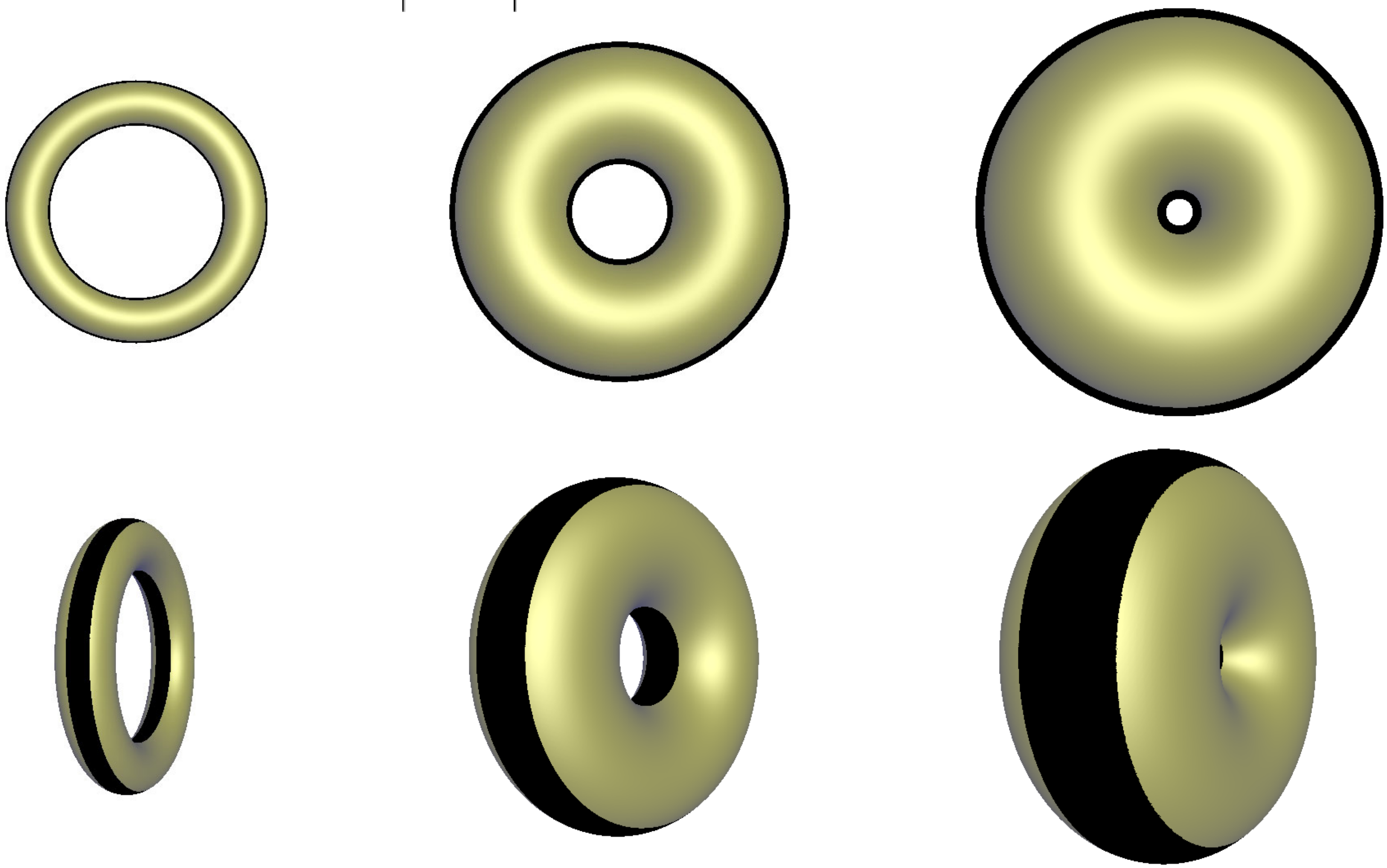




courtesy of Gordon Kindlmann

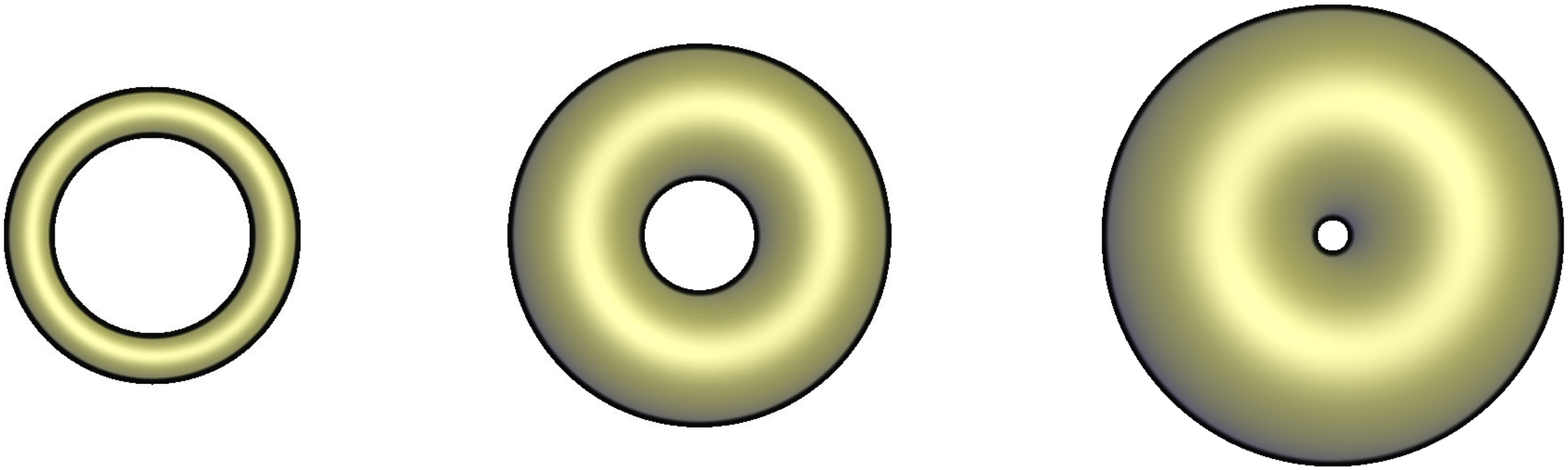
Problems of Implicit Surface Contours

Constant threshold on $|\mathbf{v} \cdot \mathbf{n}|$

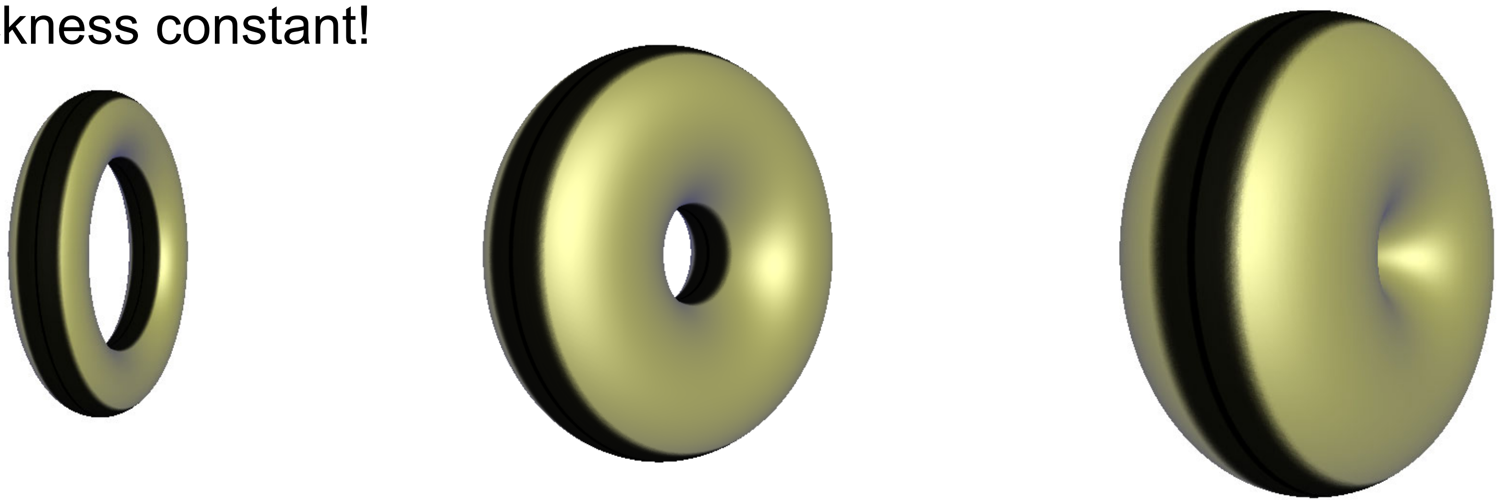


Curvature-Based Contour Threshold (1)

Threshold dependent on curvature in view direction

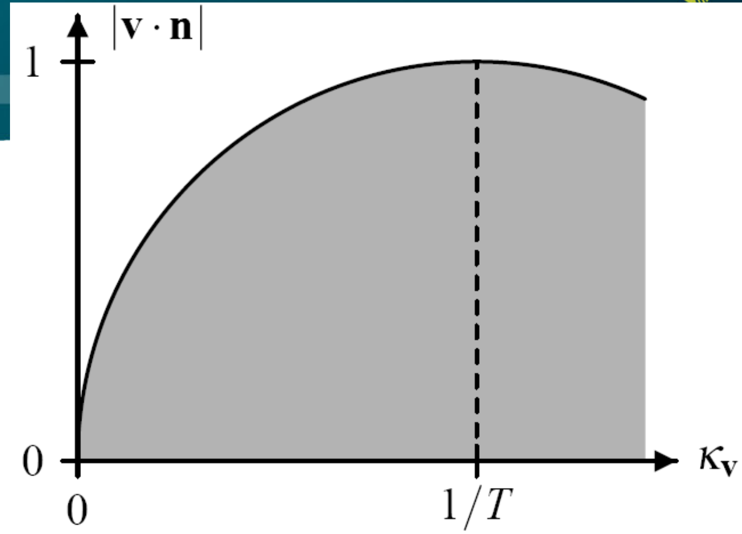


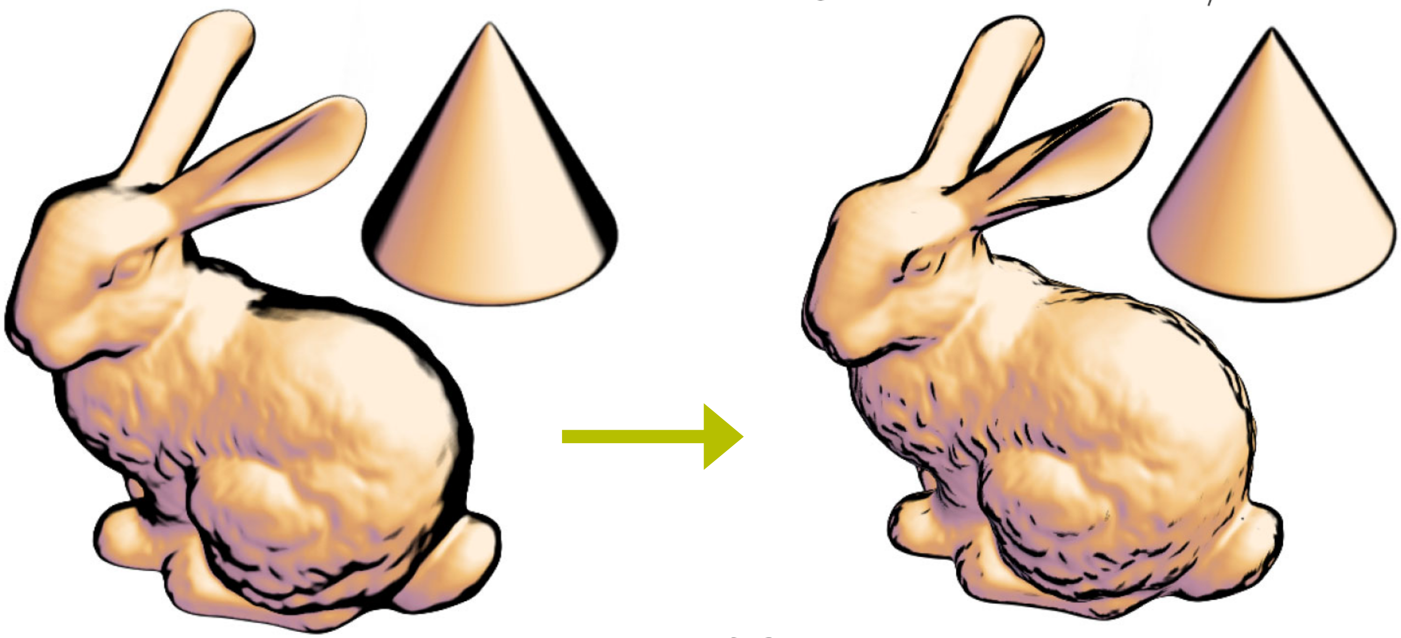
Thickness constant!



Curvature-Based Contour Threshold (2)

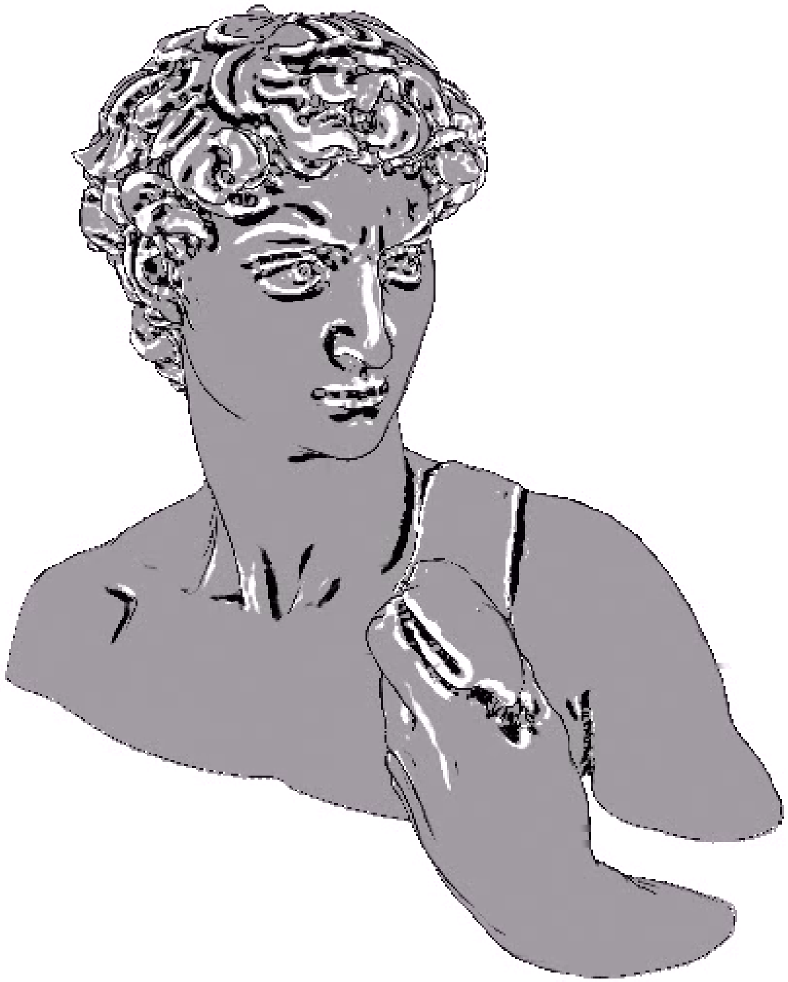
Higher curvature in view direction needs higher threshold





courtesy of Gordon Kindlmann

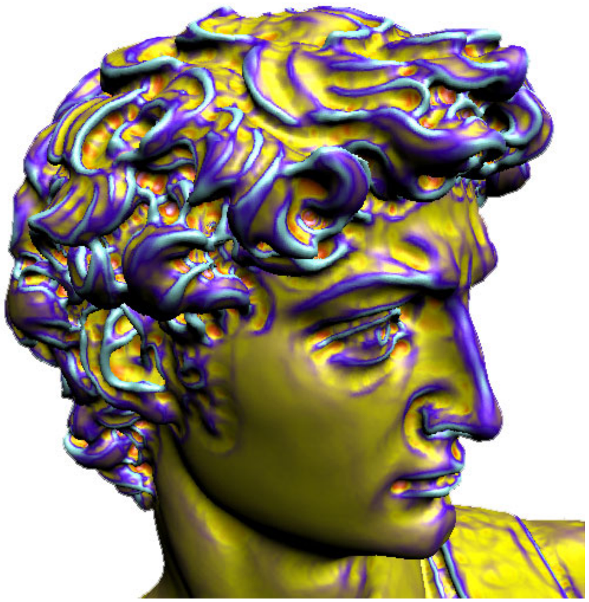
Example



Deferred Isosurface Shading

- Shading is expensive
- Compute surface intersection image from volume
- Compute derivatives and shading in image space

intersection image



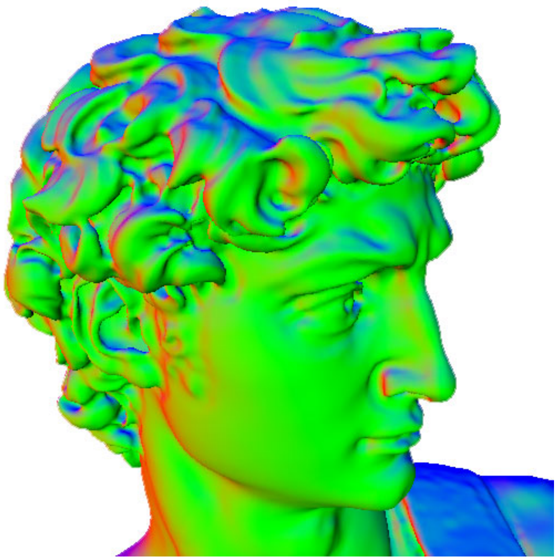
curvature color coding

ridges and valleys

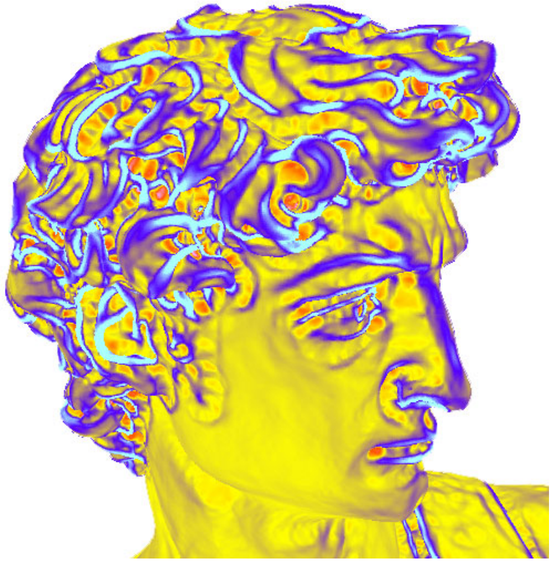
Implicit Curvature via Convolution

Computed from first and second derivatives

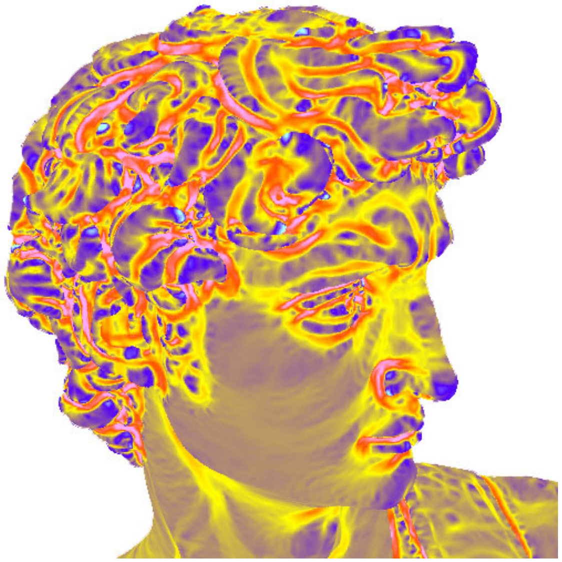
- Can use fast texture-based tri-cubic filters in shader
- Can use deferred computation and shading



first derivative



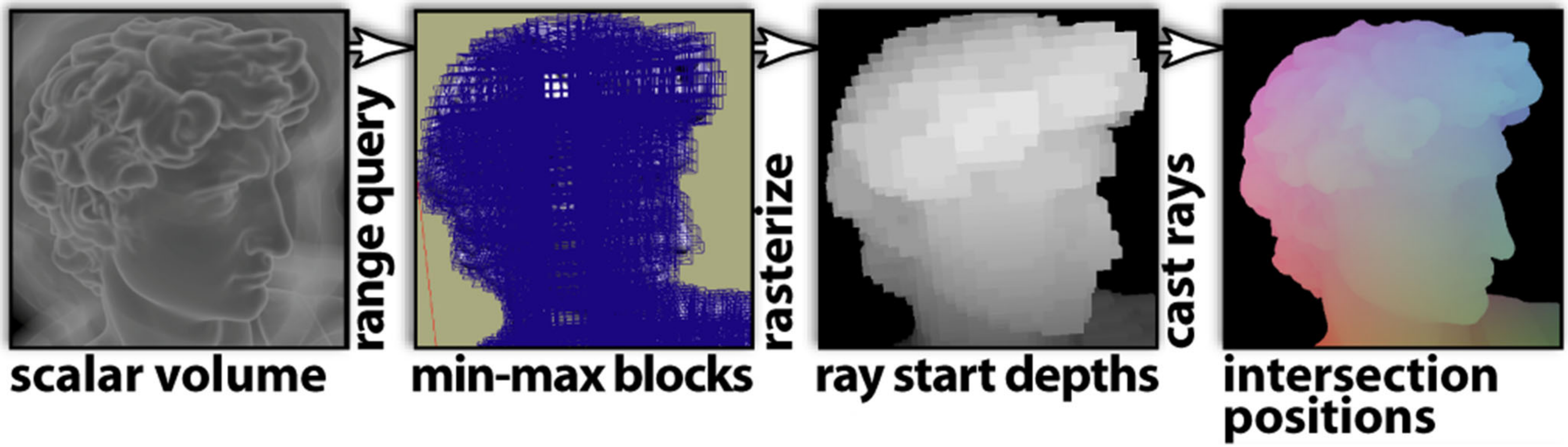
maximum curvature



minimum curvature

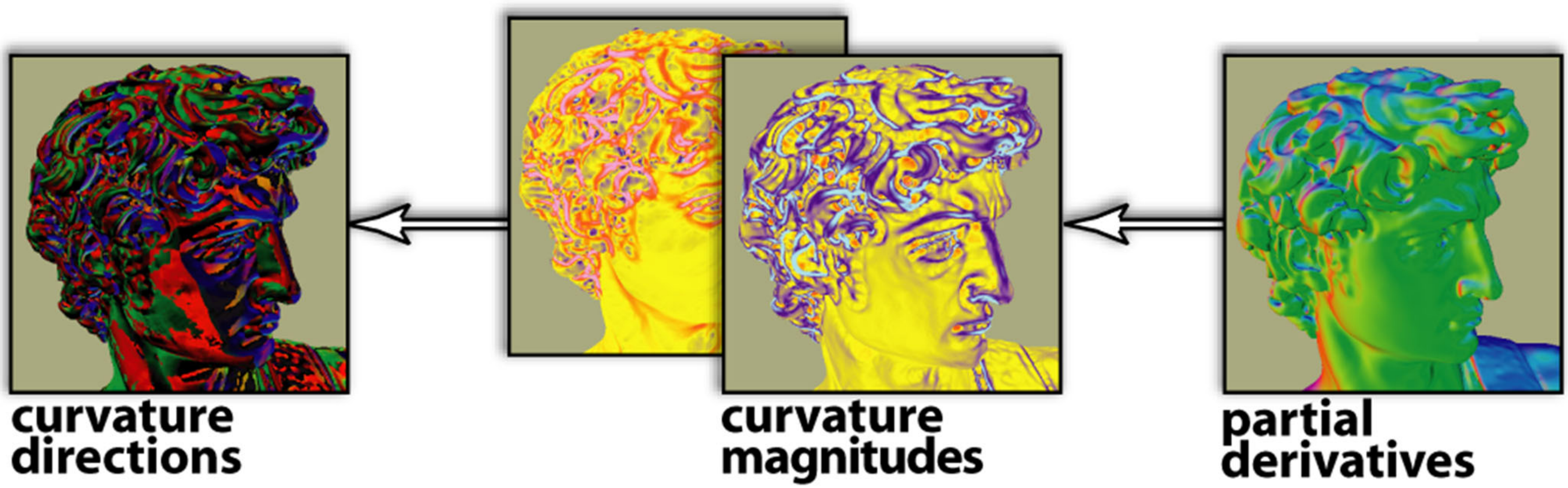
Pipeline Stage #1: Ray-Casting

- Rasterize faces of active min-max blocks
- Cast into the volume; stop when isosurface hit
- Refine isosurface hit positions (root search)



Pipeline Stage #2: Differential Properties

- Basis for visualization of surface shape
- First and second derivatives (gradient, Hessian)
- From these: curvature information, …



Pipeline Stage #3: Shading

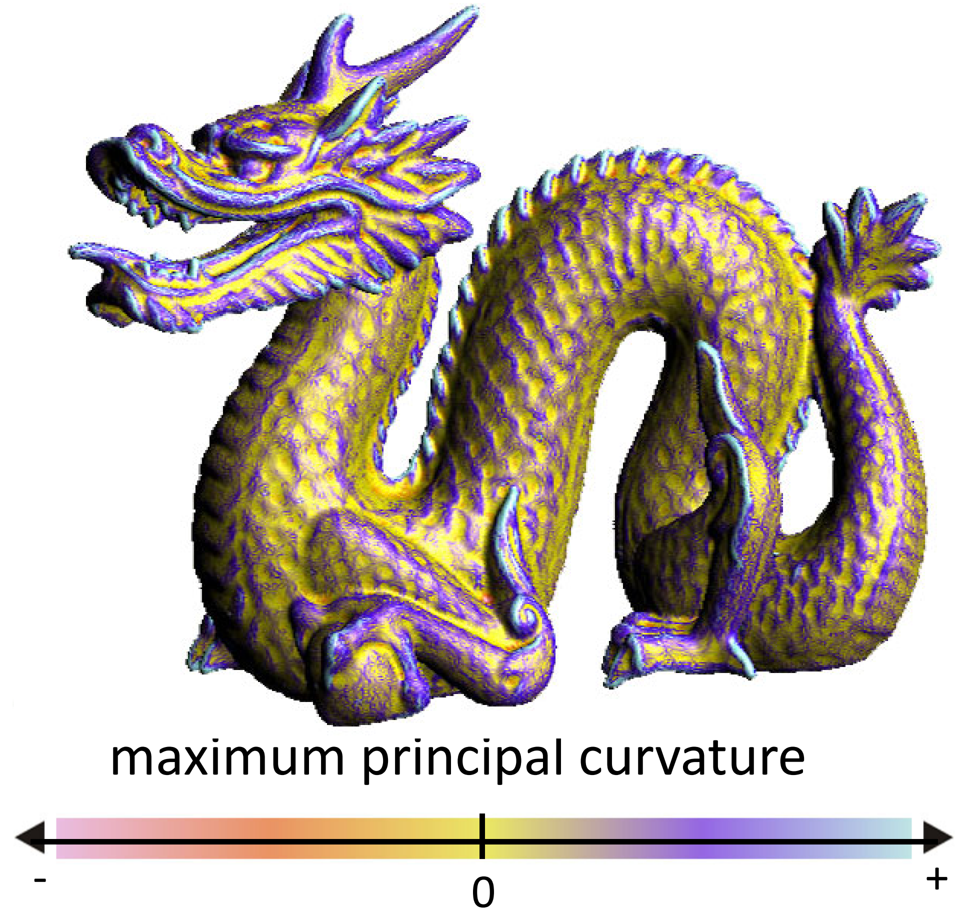
Build on previous images

- Position in object space
- Gradient
- Principal curvature magnitudes and directions

curvature flow | ridges+valleys | curvature mapping | tone shading

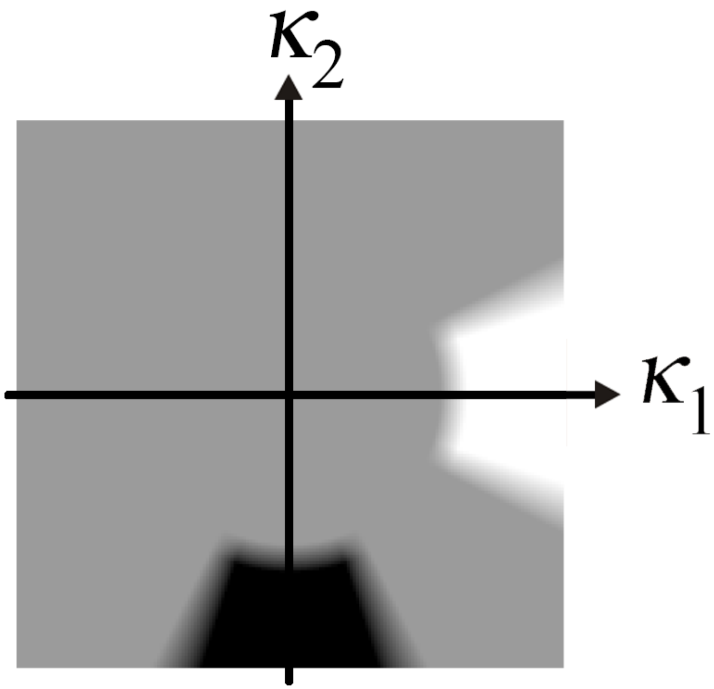
Color Coding Scalar Curvature Measures

- 1D color lookup table



2D Curvature Transfer Functions

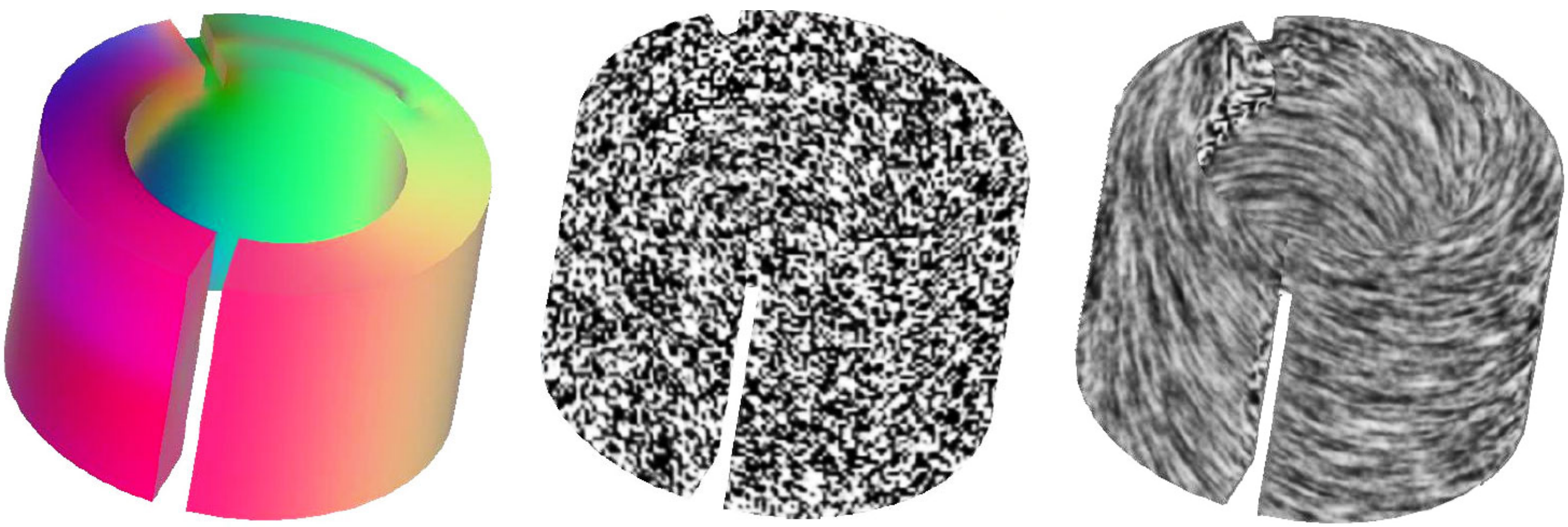
- 2D lookup table in domain of principal curvatures



ridges and valleys, plus contours:

Visualizing Curvature Directions (1)

- Use 3D vector field visualization on curved surfaces [van Wijk, 2003], [Laramee et al., 2003]
- Project 3D vectors to screen space
- Advect dense noise textures in screen space



Visualizing Curvature Directions (2)

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama