King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 12: Scalar Field Visualization, Pt. 6

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Reading Assignment #6 (until Mar 12)

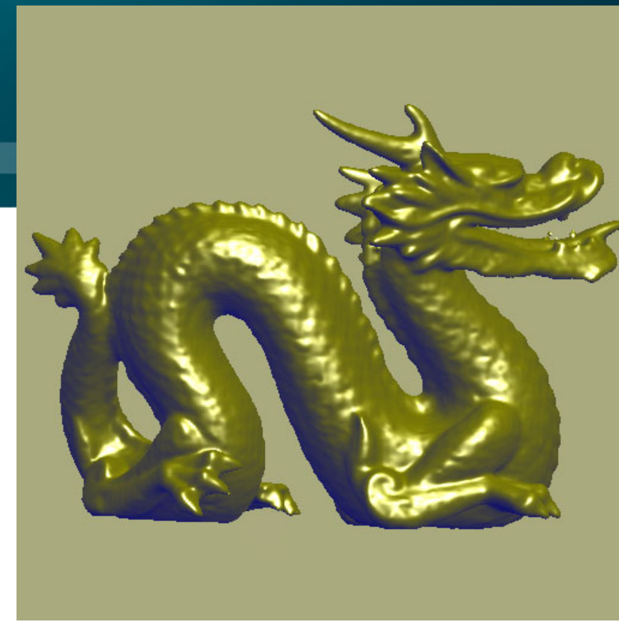
Read (required):

- Real-Time Volume Graphics, Chapter 2
  (*GPU Programming*)
- Real-Time Volume Graphics, Chapters 5.5 and 5.6 (you already had to read - 5.4)
  (*Local Volume Illumination*)
- Refresh your memory on eigenvectors and eigenvalues:
  `https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors`

Look at (optional):

- Riemannian Geometry for Scientific Visualization (notes and videos [part 1])
  `https://vccvisualization.org/RiemannianGeometryTutorial/`

What About Volume Illumination?



Crucial for perceiving shape and depth relationships

this is a scalar volume (3D distance field)!

Local Illumination in Volumes

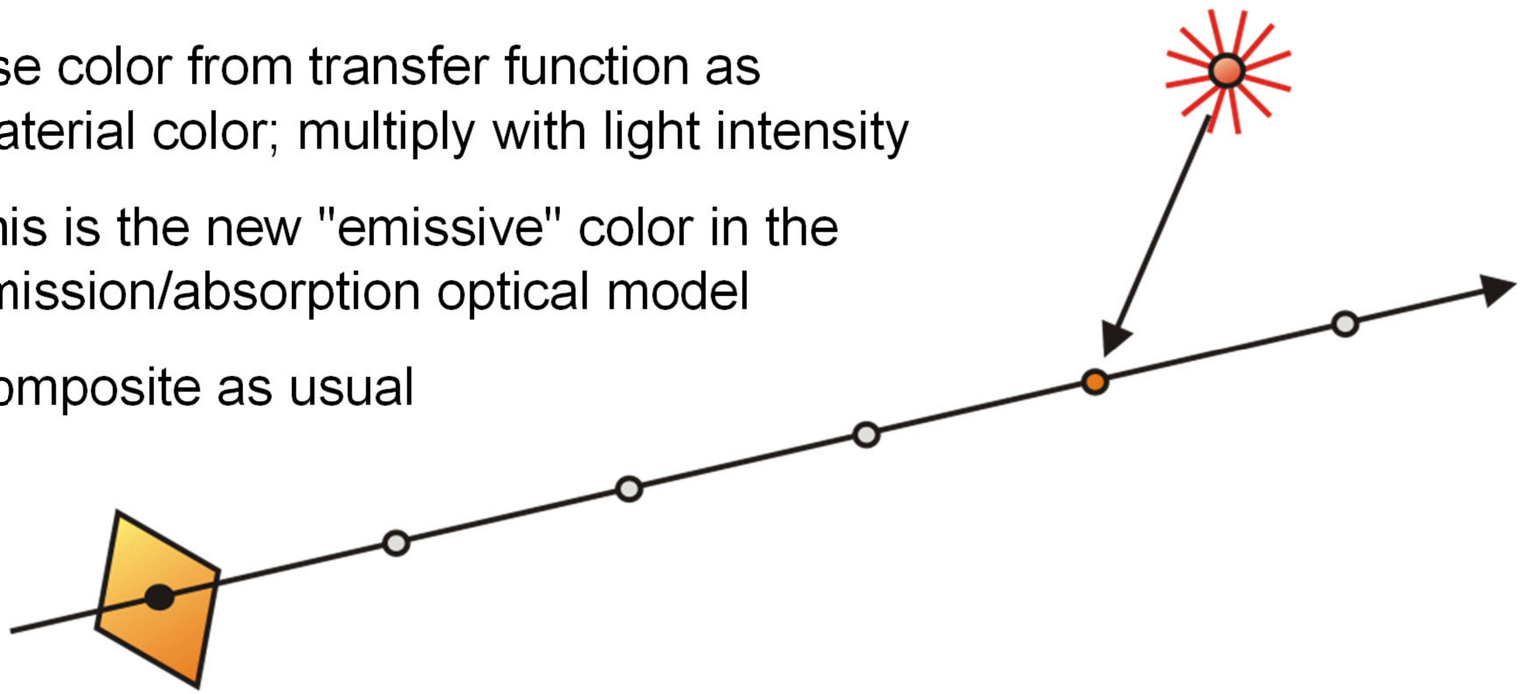
Interaction between light source and point in the volume

Local shading equation; evaluate at each point along a ray

Use color from transfer function as material color; multiply with light intensity

This is the new "emissive" color in the emission/absorption optical model

Composite as usual



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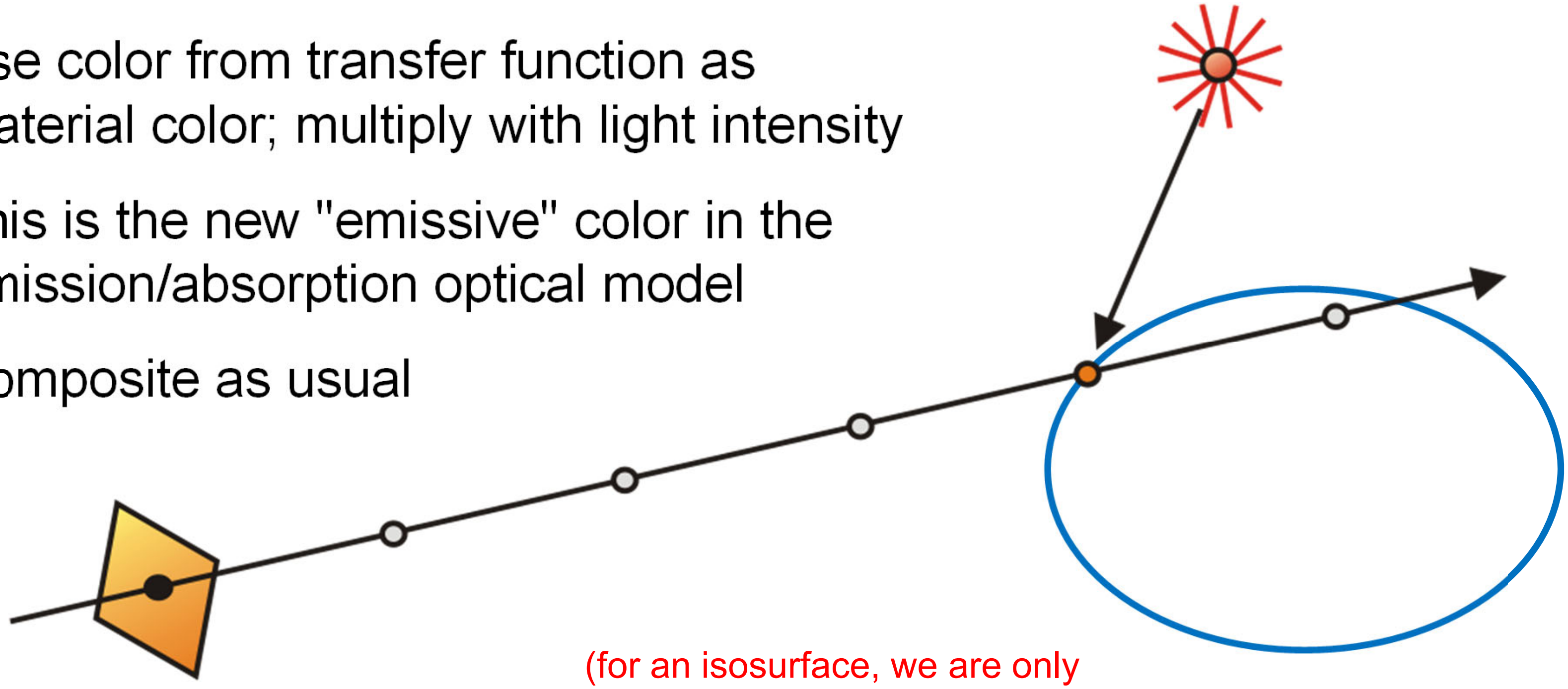
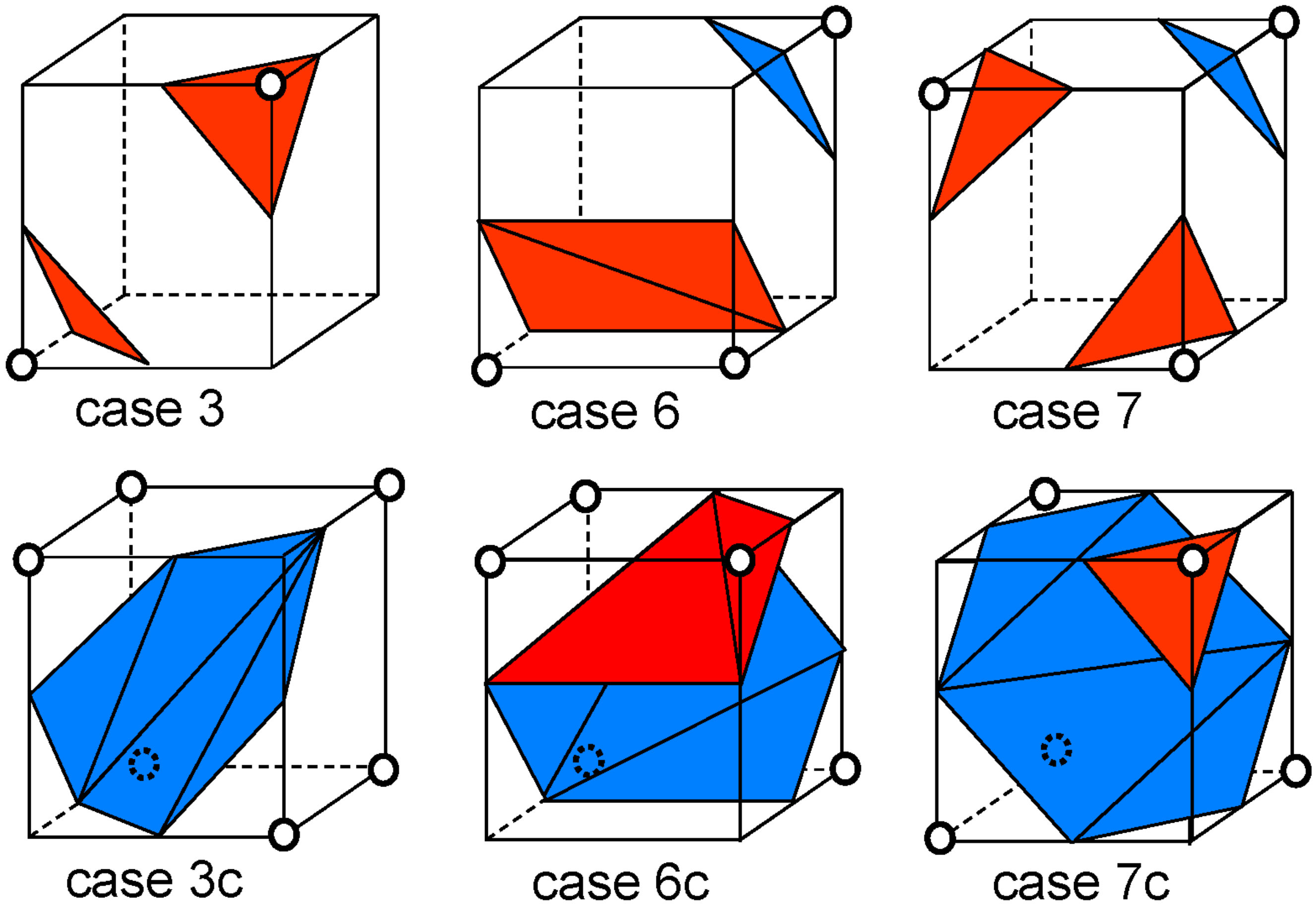


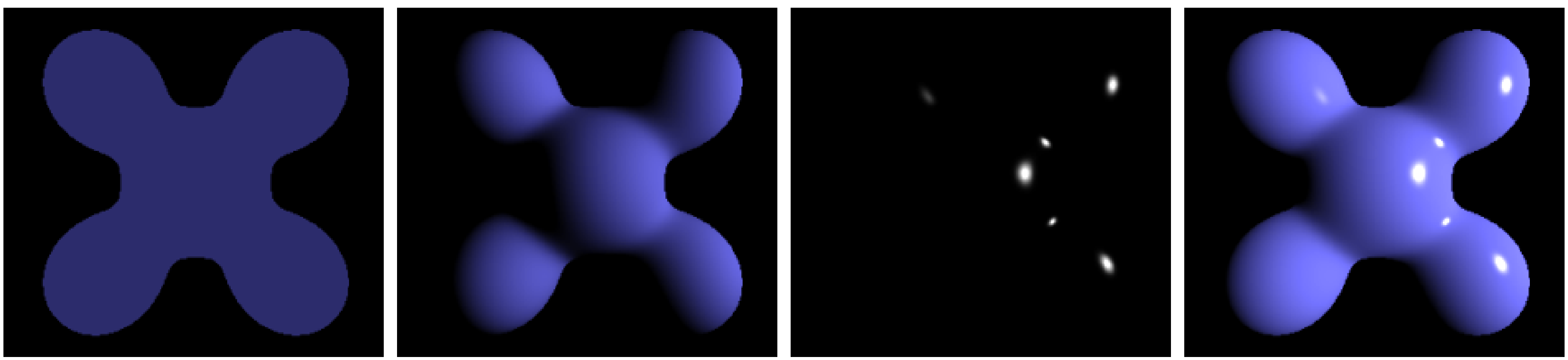
(for an isosurface, we are only interested in points *on* the surface; in marching cubes: the vertices)

The marching cubes algorithm



Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



Ambient + Diffuse + Specular = Phong Reflection

The Dot Product (Scalar / Inner Product)

Cosine of angle between two vectors times their lengths

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos\theta$$

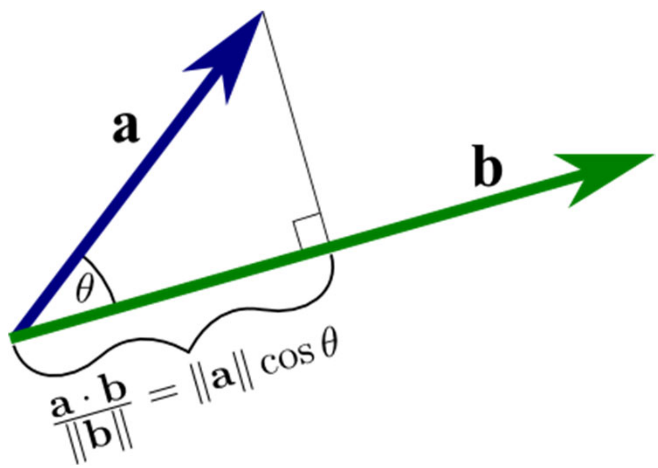
(geometric definition,
independent of coordinates)

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i$$

(standard inner product
in Cartesian coordinates)

Many uses:

- Project vector onto another vector
- Project into basis (using the dual basis, see later)
- Project into tangent plane



The Gradient as Normal Vector

Gradient of the scalar field gives direction+magnitude of fastest change

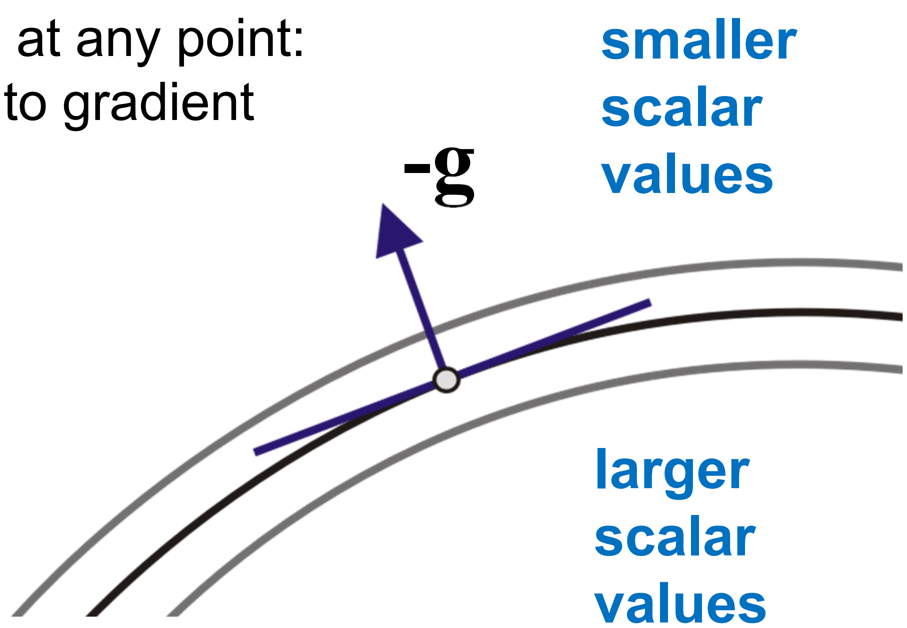
$$\mathbf{g} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)^{\mathbf{T}}$$

(only correct in Cartesian coordinates: see later)

Local approximation to isosurface at any point:
tangent plane = plane orthogonal to gradient

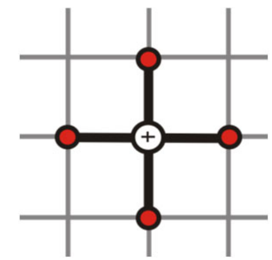
Normal of this isosurface:
normalized gradient vector
(negation is common convention)

$$\mathbf{n} = -\mathbf{g}/|\mathbf{g}|$$



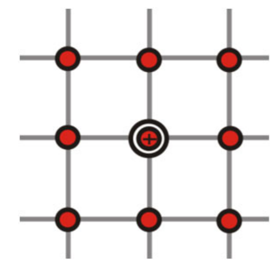
(Numerical) Gradient Reconstruction

We need to reconstruct the derivatives of a continuous function given as discrete samples



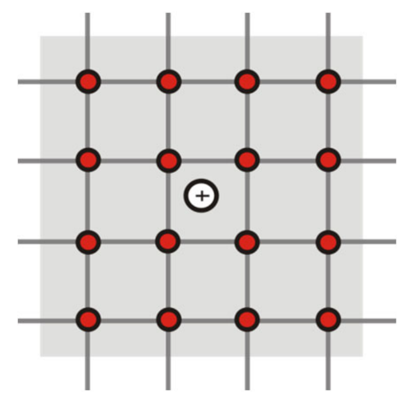
Central differences

- Cheap and quality often sufficient (2*3 neighbors in 3D)



Discrete convolution filters on grid

- Image processing filters; e.g. Sobel (3^3 neighbors in 3D)



Continuous convolution filters

- Derived continuous reconstruction filters
- E.g., the cubic B-spline and its derivatives (4^3 neighbors)

Finite Differences

Obtain first derivative from Taylor expansion

$$\begin{aligned} f(x_0 + h) &= f(x_0) + \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + \dots \\ &= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} h^n . \end{aligned}$$

Forward differences / backward differences

$$f(x_0)' = \frac{f(x_0 + h) - f(x_0)}{h} + o(h)$$

$$f(x_0)' = \frac{f(x_0) - f(x_0 - h)}{h} + o(h)$$

Finite Differences

Central differences

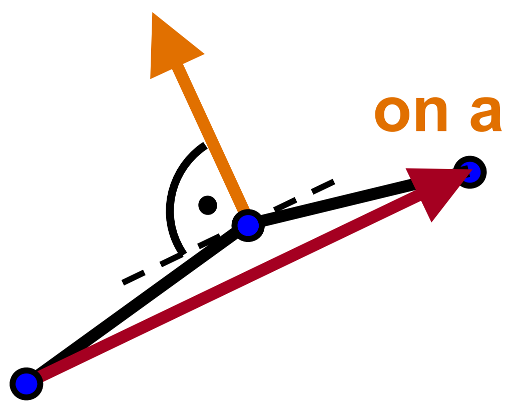
$$
\begin{aligned}
f(x_0 + h) &= f(x_0) + \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + o(h^3) \\
f(x_0 - h) &= f(x_0) - \frac{f'(x_0)}{1!} h + \frac{f''(x_0)}{2!} h^2 + o(h^3)
\end{aligned}
$$

$$
f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h} + o(h^2)
$$

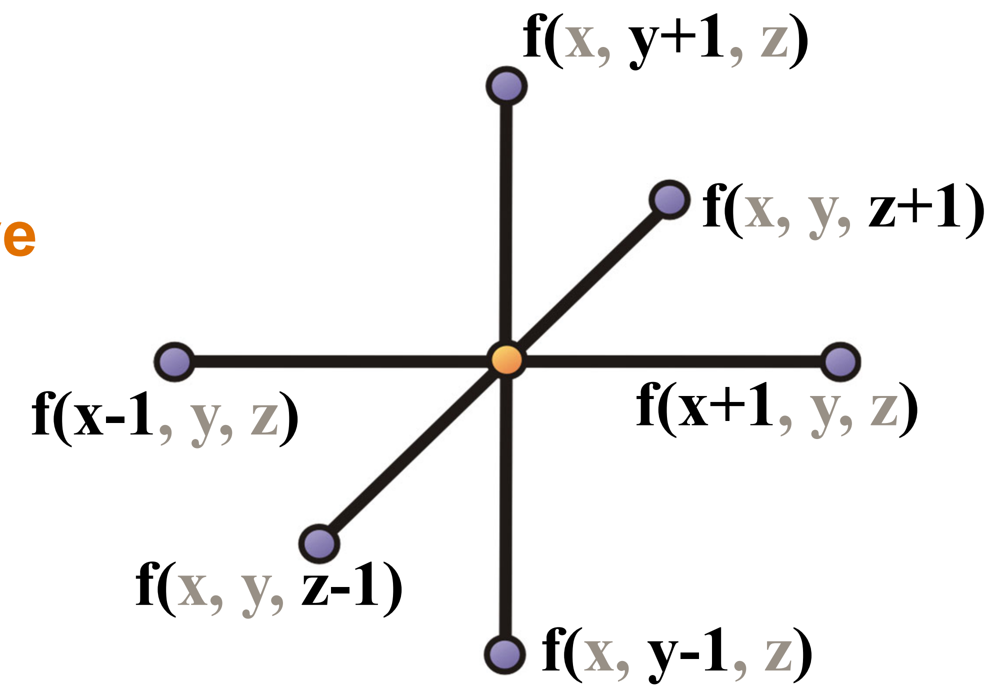
Central Differences

Need only two neighboring voxels per derivative

Most common method



on a curve



g_x = 0.5(f(x+1, y, z) − f(x−1, y, z))

g_y = 0.5(f(x, y+1, z) − f(x, y−1, z))

g_z = 0.5(f(x, y, z+1) − f(x, y, z−1))

in a volume

Gradient and Directional Derivative

Gradient $\nabla f(x,y,z)$ of scalar function $f(x,y,z)$:

$$\nabla f(x,y,z) = \left(\frac{\partial f(x,y,z)}{\partial x}, \frac{\partial f(x,y,z)}{\partial y}, \frac{\partial f(x,y,z)}{\partial z} \right)^T$$

(only correct in Cartesian coordinates: see later)

Directional derivative in direction $\mathbf{u}$:

$$D_{\mathbf{u}} f(x,y,z) = \nabla f(x,y,z) \cdot \mathbf{u}$$

And therefore also:

$$D_{\mathbf{u}} f(x,y,z) = ||\nabla f|| \; ||\mathbf{u}|| \; \cos\theta$$

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(Cartesian vector components; basis vectors not shown)

But: always need **basis vectors**! With Cartesian basis:

$$\nabla f(x,y,z) = \frac{\partial f(x,y,z)}{\partial x}\,\mathbf{i} + \frac{\partial f(x,y,z)}{\partial y}\,\mathbf{j} + \frac{\partial f(x,y,z)}{\partial z}\,\mathbf{k}$$

What about the Basis?

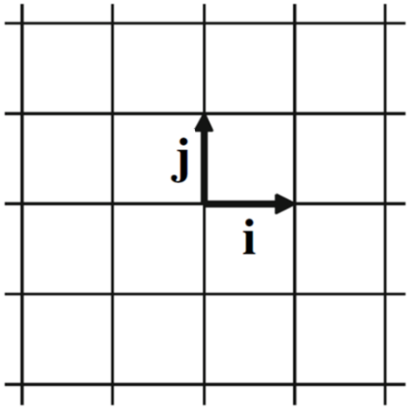
On the previous slide, this actually meant

$$\nabla f(x,y,z) = \frac{\partial f(x,y,z)}{\partial x}\,\mathbf{i}(x,y,z) + \frac{\partial f(x,y,z)}{\partial y}\,\mathbf{j}(x,y,z) + \frac{\partial f(x,y,z)}{\partial z}\,\mathbf{k}(x,y,z)$$

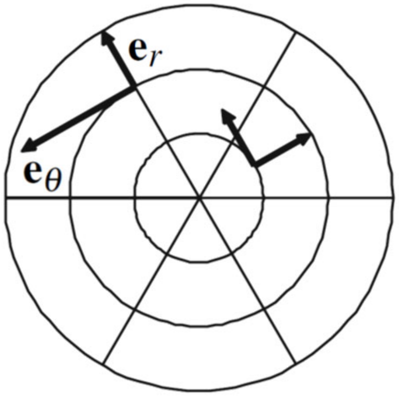
It's just that the Cartesian basis vectors are the same everywhere...

But this is not true for many other coordinate systems:

Cartesian coordinates



polar coordinates



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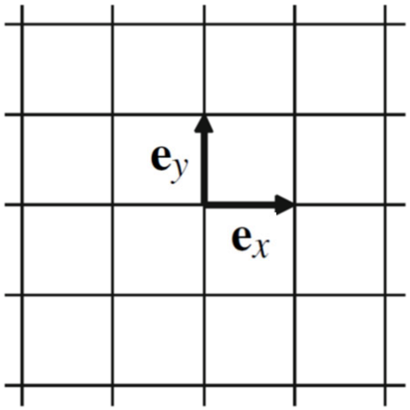

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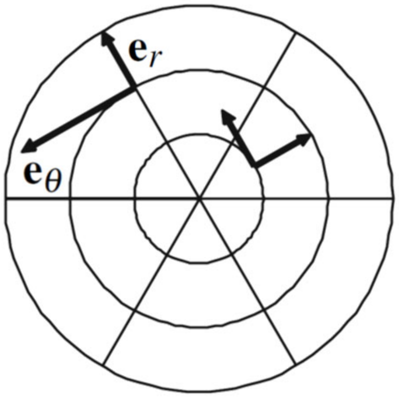

Gradients as Differential Forms (1-Forms)

The Gradient as a Differential Form

The gradient as a *differential* (differential 1-form) is the "primary" concept (also "total differential" or "total derivative")

$$df = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y}\, dy + \frac{\partial f}{\partial z}\, dz$$

A differential 1-form is a scalar-valued linear function that takes a (direction) vector as input, and gives a scalar as output

Each of the 1-forms df, dx, dy, dz takes direction vector as input, gives scalar output

In the expression of the gradient df above, all 1-forms on the right-hand side get the same vector as input

df is simply a linear combination of the coordinate differentials dx, dy, dz

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The directional derivative and the gradient vector

$$D_{\mathbf{u}} f = df(\mathbf{u})$$
$$df(\mathbf{u}) = \nabla f \cdot \mathbf{u}$$

The gradient vector is then *defined*, such that:

$$\nabla f \cdot \mathbf{u} := df(\mathbf{u})$$

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama