King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 8: Scalar Field Visualization, Pt. 2

Markus Hadwiger, KAUST

Reading Assignment #4 (until Feb 26)

Read (required):

- Real-Time Volume Graphics book, Chapter 5 until 5.4 inclusive (*Terminology, Types of Light Sources, Gradient-Based Illumination, Local Illumination Models*)
- Paper:
  Marching Cubes: A high resolution 3D surface construction algorithm,
  Bill Lorensen and Harvey Cline, ACM SIGGRAPH 1987
  [> 18,600 citations and counting...]

  `https://dl.acm.org/doi/10.1145/37402.37422`

Read (optional):

- Paper:
  Flying Edges, William Schroeder et al., IEEE LDAV 2015

  `https://ieeexplore.ieee.org/document/7348069`

Scalar Fields

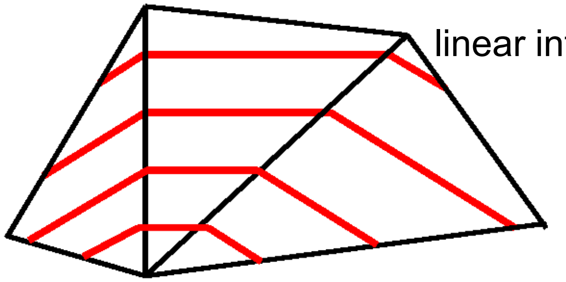
Contours

Set of points where the scalar field *s* has a given value *c*:

$$S(c) := f^{-1}(c) \qquad S(c) := \{x \in \mathbb{R}^n : f(x) = c\}$$

Common contouring algorithms

- 2D: marching squares, marching triangles
- 3D: marching cubes, marching tetrahedra



Implicit methods

- Point-on-contour test
- Isosurface ray-casting

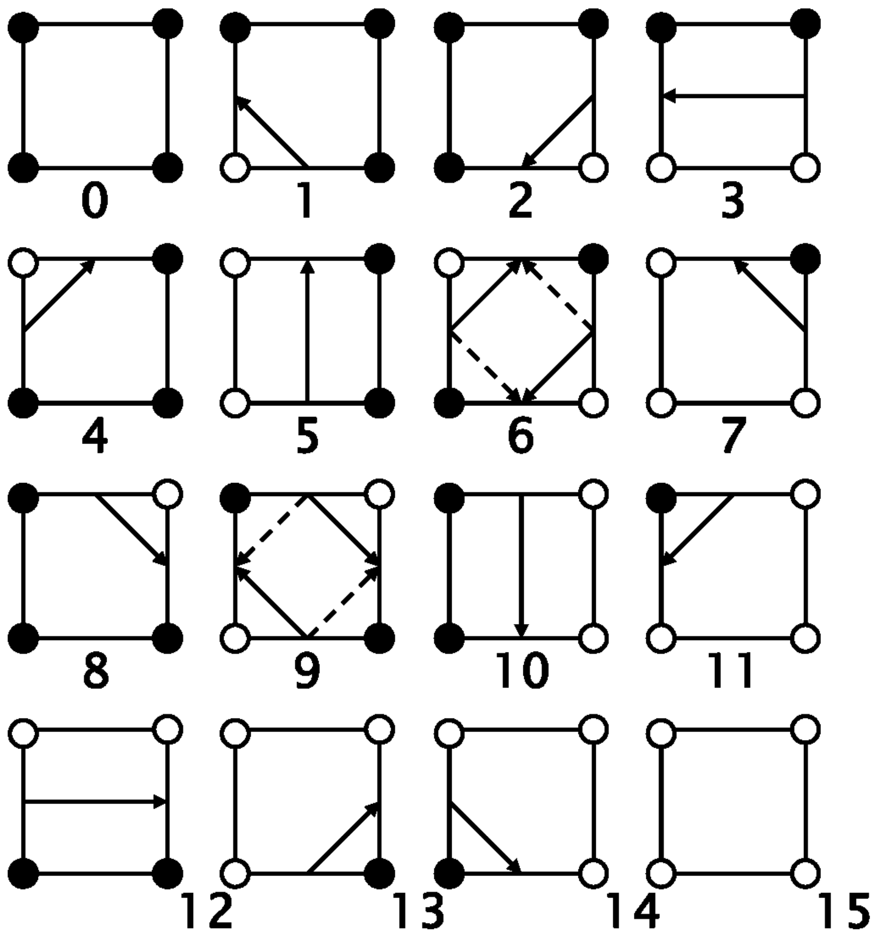
Basic contouring algorithms:

- cell-by-cell algorithms: simple structure, but generate disconnected segments, require post-processing
- contour propagation methods: more complicated, but generate connected contours

"Marching squares" algorithm (systematic cell-by-cell):

- process nodes in ccw order, denoted here as x_0, x_1, x_2, x_3
- compute at each node $\mathbf{x}_i$ the reduced field $\tilde{f}(x_i) = f(x_i) - (c - \varepsilon)$ (which is forced to be nonzero)
- take its sign as the i^{th} bit of a 4-bit integer
- use this as an index for lookup table containing the connectivity information:

Contours in a quadrangle cell



● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

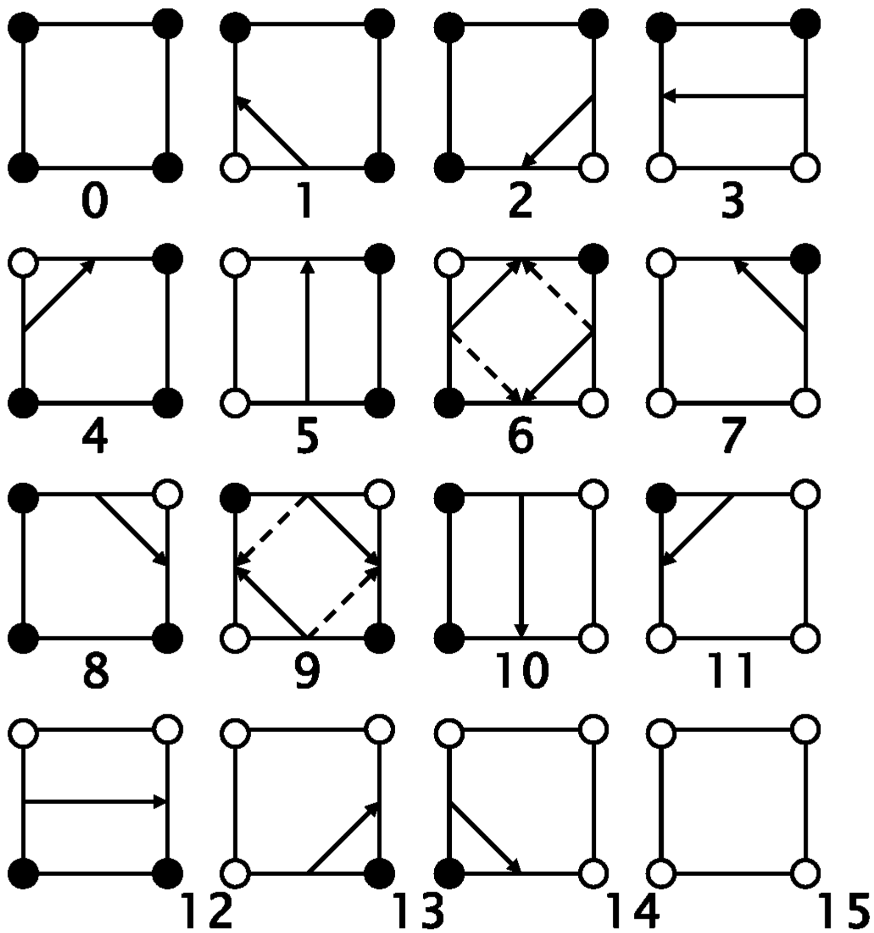
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



- ● $f(x_i) < c$
- ○ $f(x_i) \geq c$

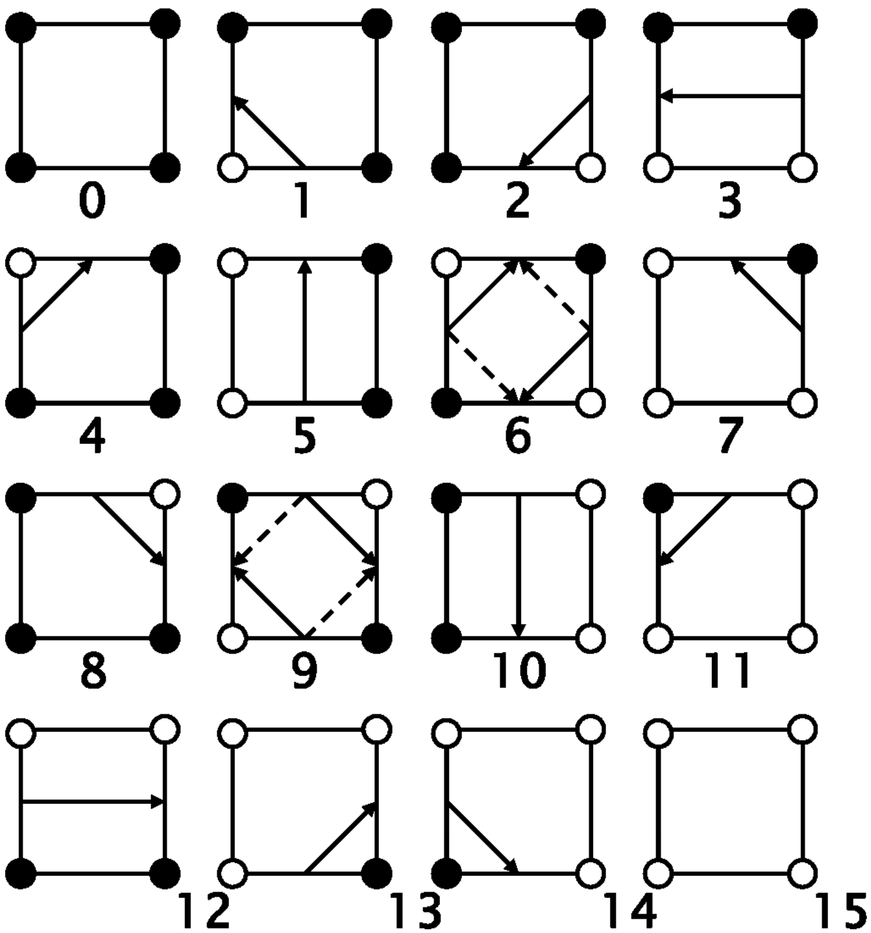
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



● $f(x_i) \leq c$

○ $f(x_i) > c$

Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Orientability (1-manifold embedded in 2D)

Orientability of 1-manifold:

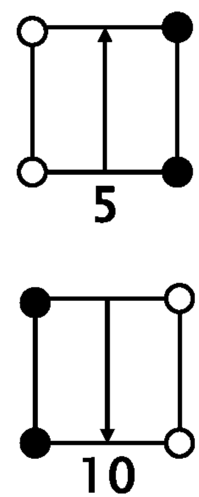
Possible to assign consistent left/right orientation

not orientable

Moebius strip
(only one side!)

Iso-contours

- Consistent side for scalar values…
  - greater than iso-value (e.g, *left* side)
  - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices
  (e.g., larger vertex index is “tip” of arrow;
  if (0,1) points “up”, “left” is left, …)



● $\tilde{f}(x_i) < 0$

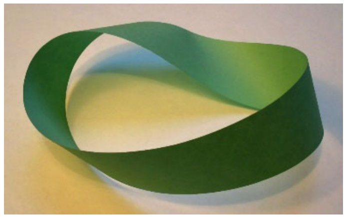
○ $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

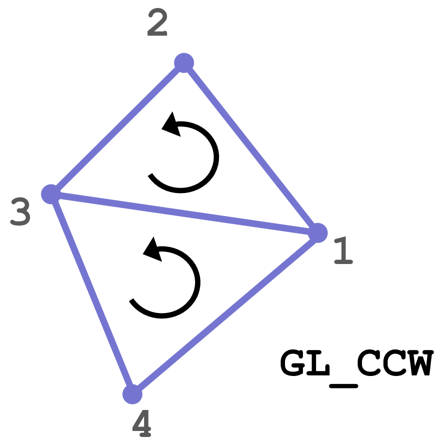
not orientable



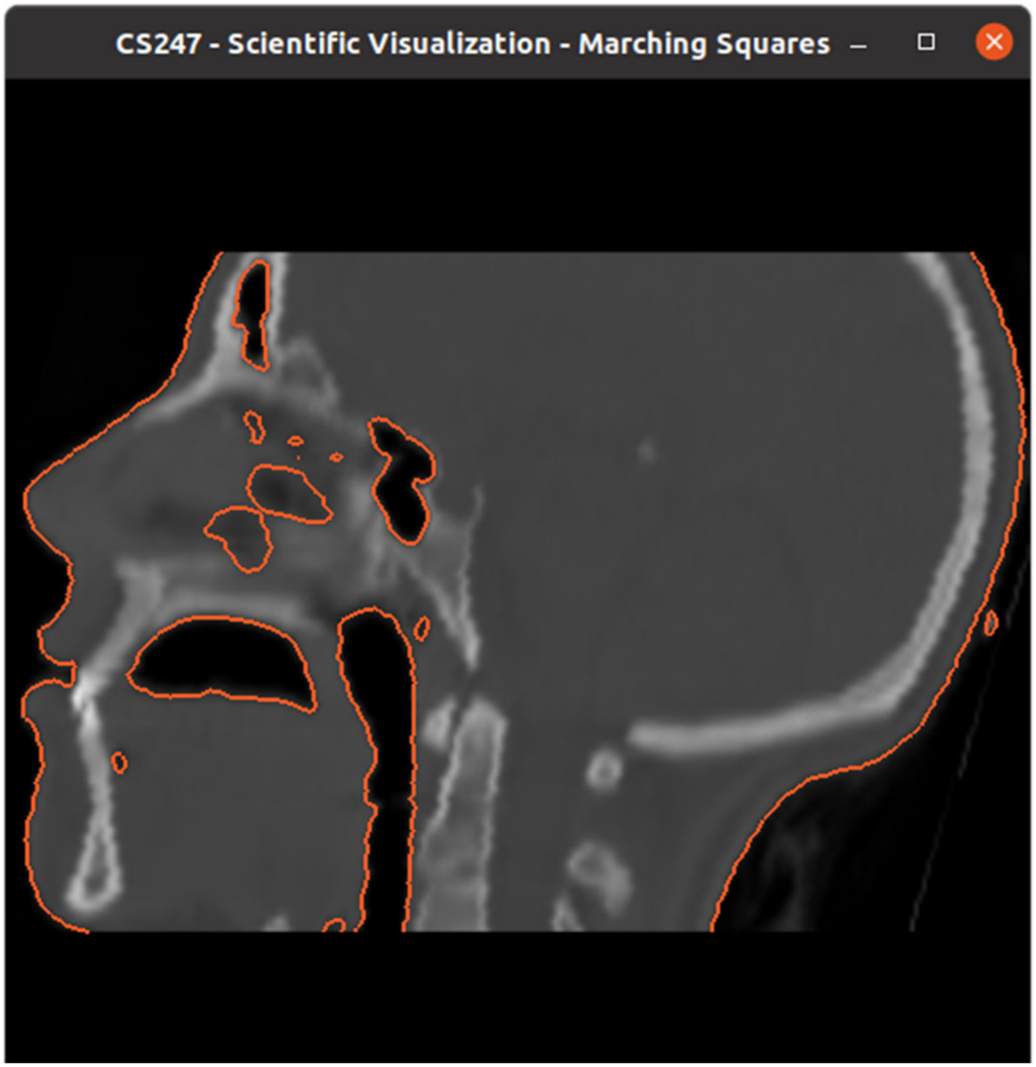
Moebius strip
(only one side!)

Triangle meshes

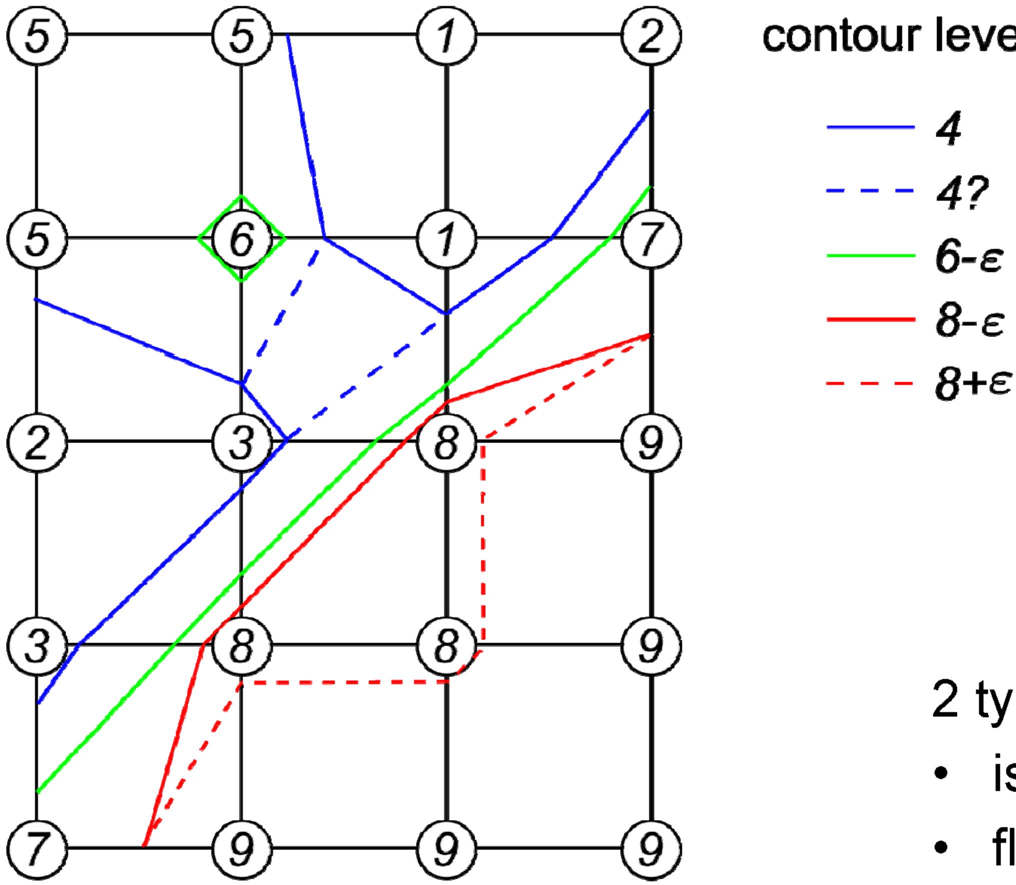
- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (3,1,2) on one side of edge, (1,3,4) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: "right-hand rule"



Marching Squares Example



Marching Squares Example



2 types of degeneracies:

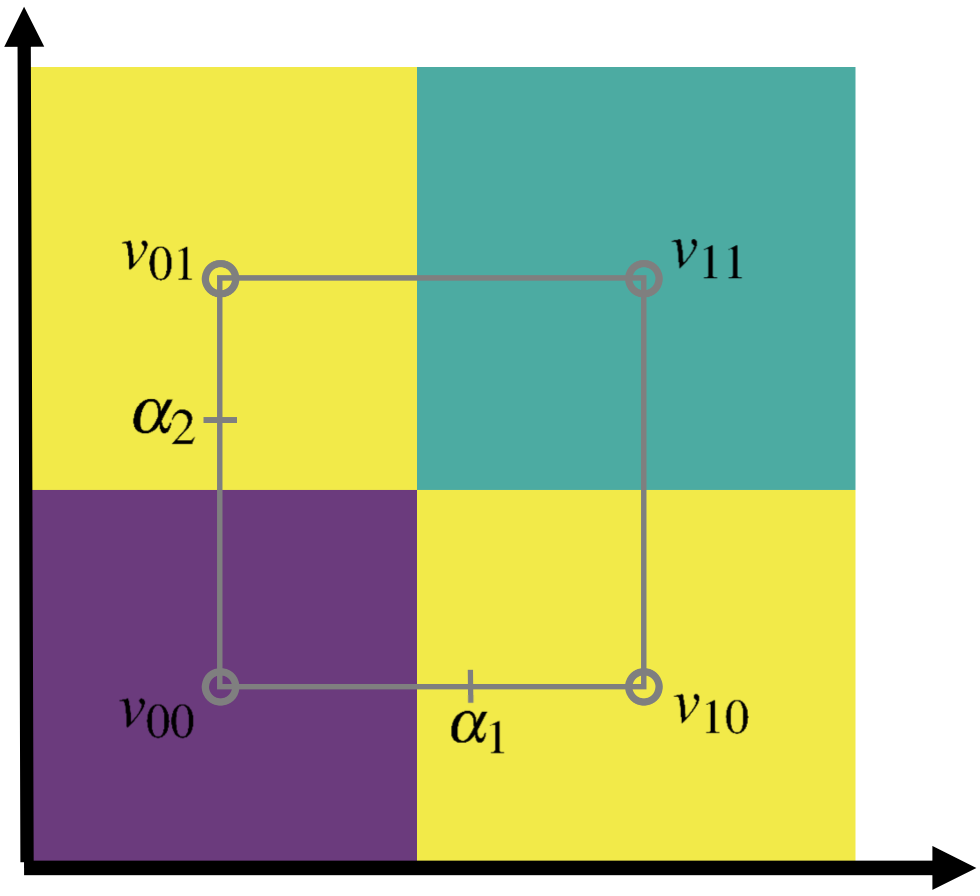
- isolated points (c=6)
- flat regions (c=8)

Sample Locations and Interpolation

Consider area between 2x2 adjacent samples (e.g., pixel centers):

Given any (fractional) position

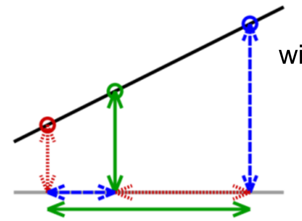
$$\alpha_1 := x_1 - \lfloor x_1 \rfloor \qquad \alpha_1 \in [0.0, 1.0)$$
$$\alpha_2 := x_2 - \lfloor x_2 \rfloor \qquad \alpha_2 \in [0.0, 1.0)$$



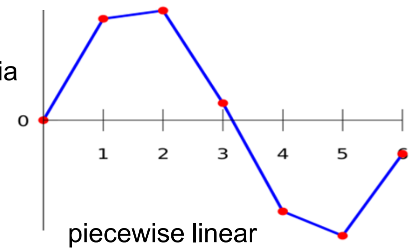
Linear Interpolation / Convex Combinations

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$



wikipedia



Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad f(\alpha) = v_1 + \alpha(v_2 - v_1)$$

$$\alpha_1 + \alpha_2 = 1 \qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

Compare to line parameterization with parameter t:

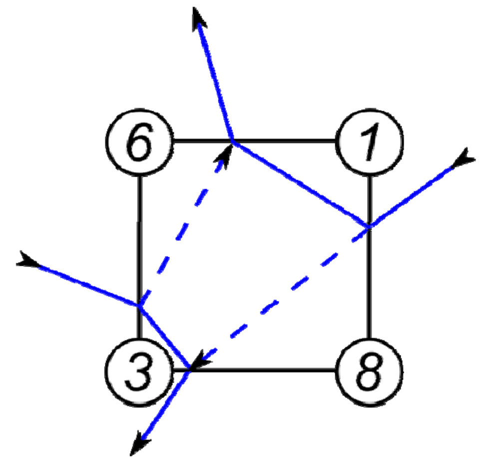
$$v(t) = v_1 + t(v_2 - v_1)$$

Ambiguities of contours

What is the correct contour of c=4?

Two possibilities, both are orientable:

- connect high values ———————
- connect low values - - - - - - - - - -



Answer: correctness depends on interior values of $f(x)$.

But: different interpolation schemes are possible.

Better question: What is the correct contour with respect to bilinear interpolation?

Sample Locations and Interpolation

Consider area between 2x2 adjacent samples (e.g., pixel centers):

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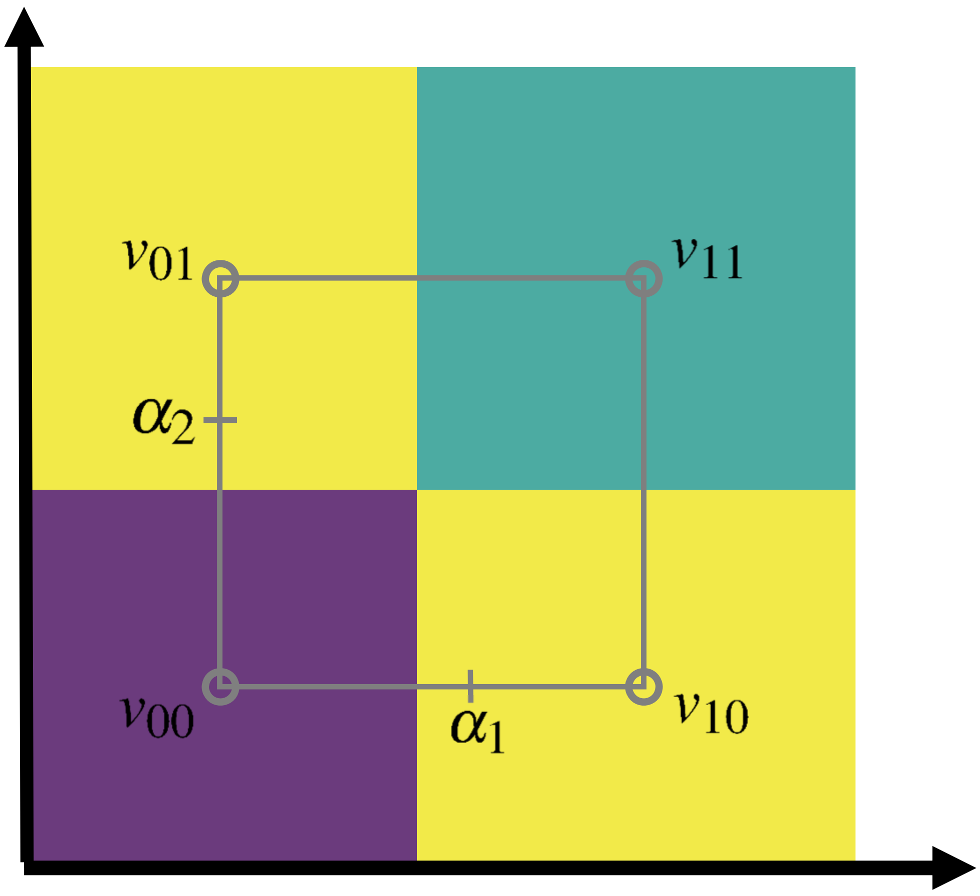

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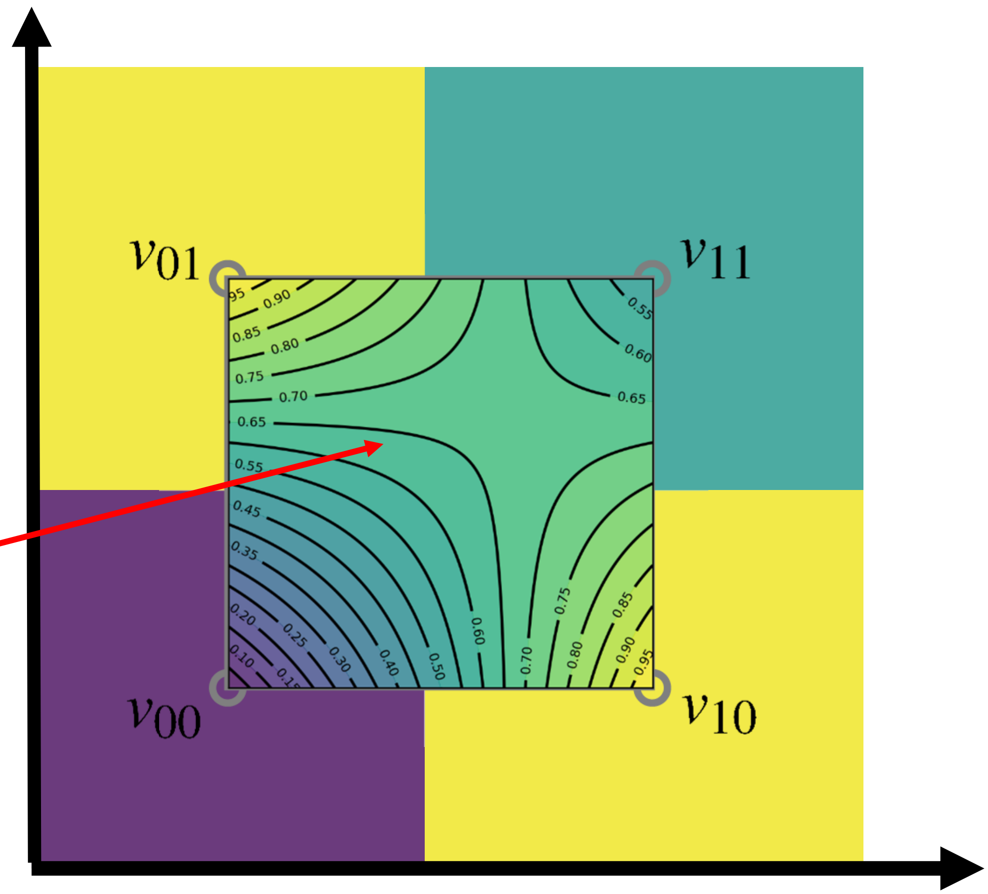
bilinear interpolation
(not marching squares!)



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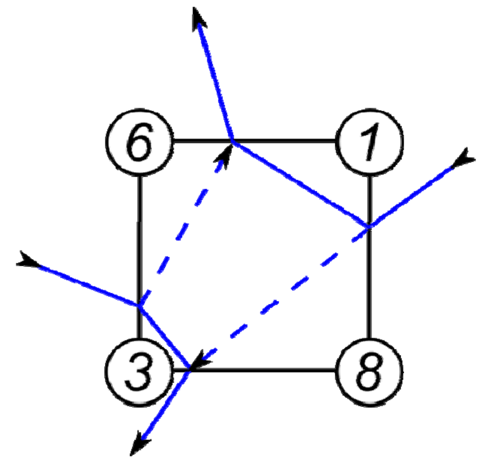


Answer: correctness depends on interior values of $f(x)$.

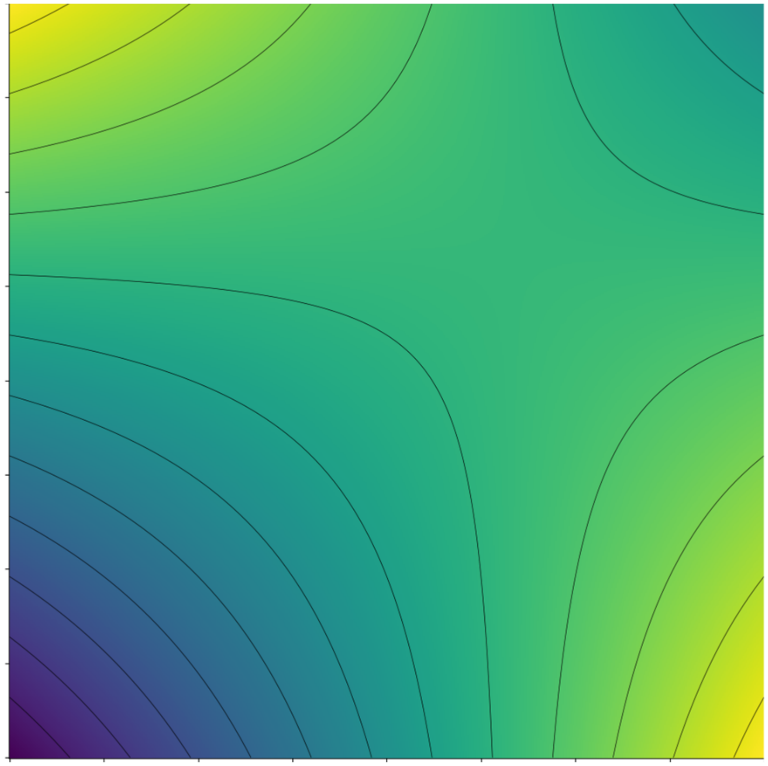
But: different interpolation schemes are possible.

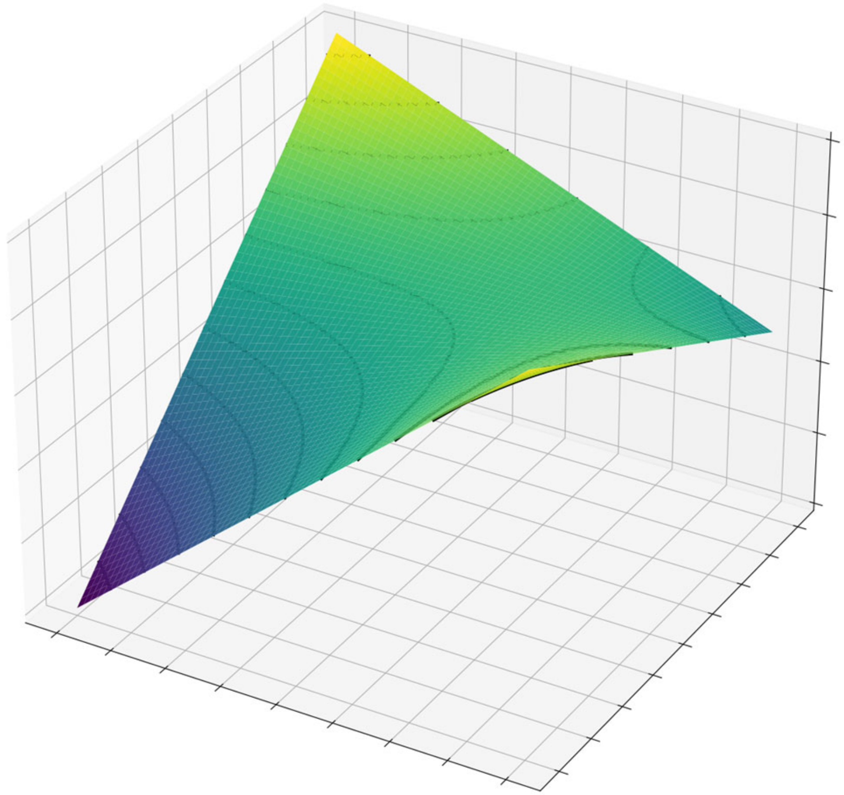
Better question: What is the correct contour with respect to bilinear interpolation?

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples

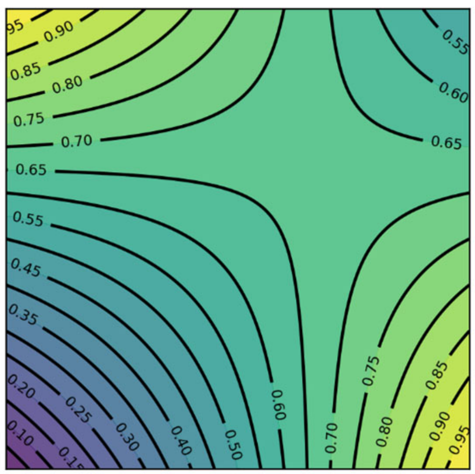
Example: 1.0 at top-left and bottom-right, 0.0 at bottom-left, 0.5 at top-right



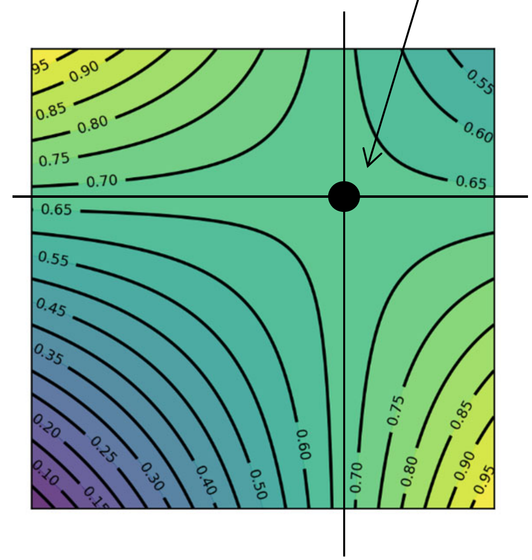


Bi-Linear Interpolation: Critical Points

Critical points are where the gradient vanishes (i.e., is the zero vector)



critical point
(saddle point)



here, the critical
value is 2/3=0.666…

“Asymptotic decider”: resolve ambiguous configurations (6 and 9) by
comparing specific iso-value with critical value (scalar value at critical point)

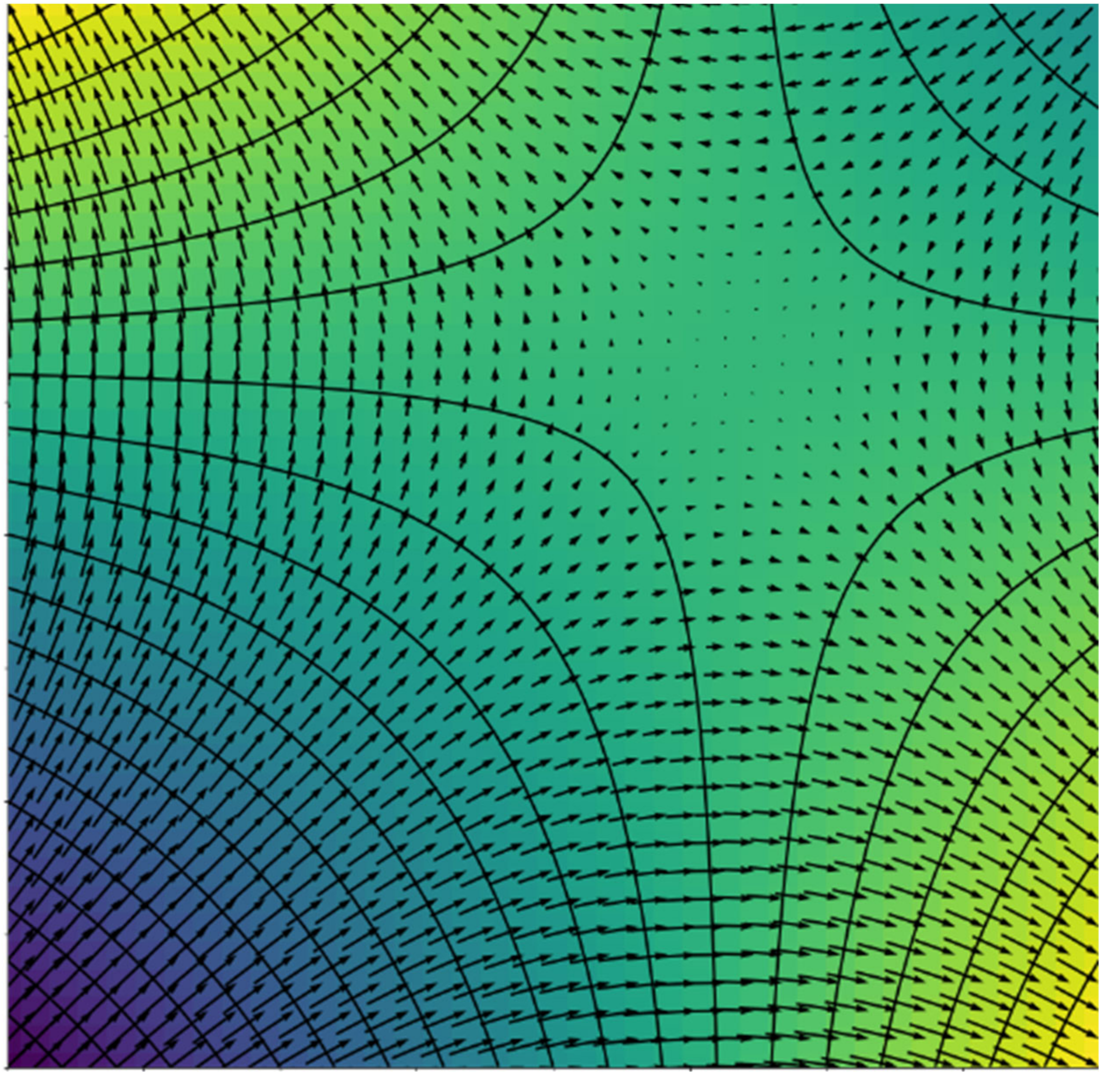
Bi-Linear Interpolation: Critical Points

Compute gradient

Note that isolines are
farther apart where
gradient is smaller

Note the horizontal and
vertical lines where
gradient becomes
vertical/horizontal

Note the critical point



Bi-Linear Interpolation: Critical Points

Compute gradient

Note that isolines are farther apart where gradient is smaller

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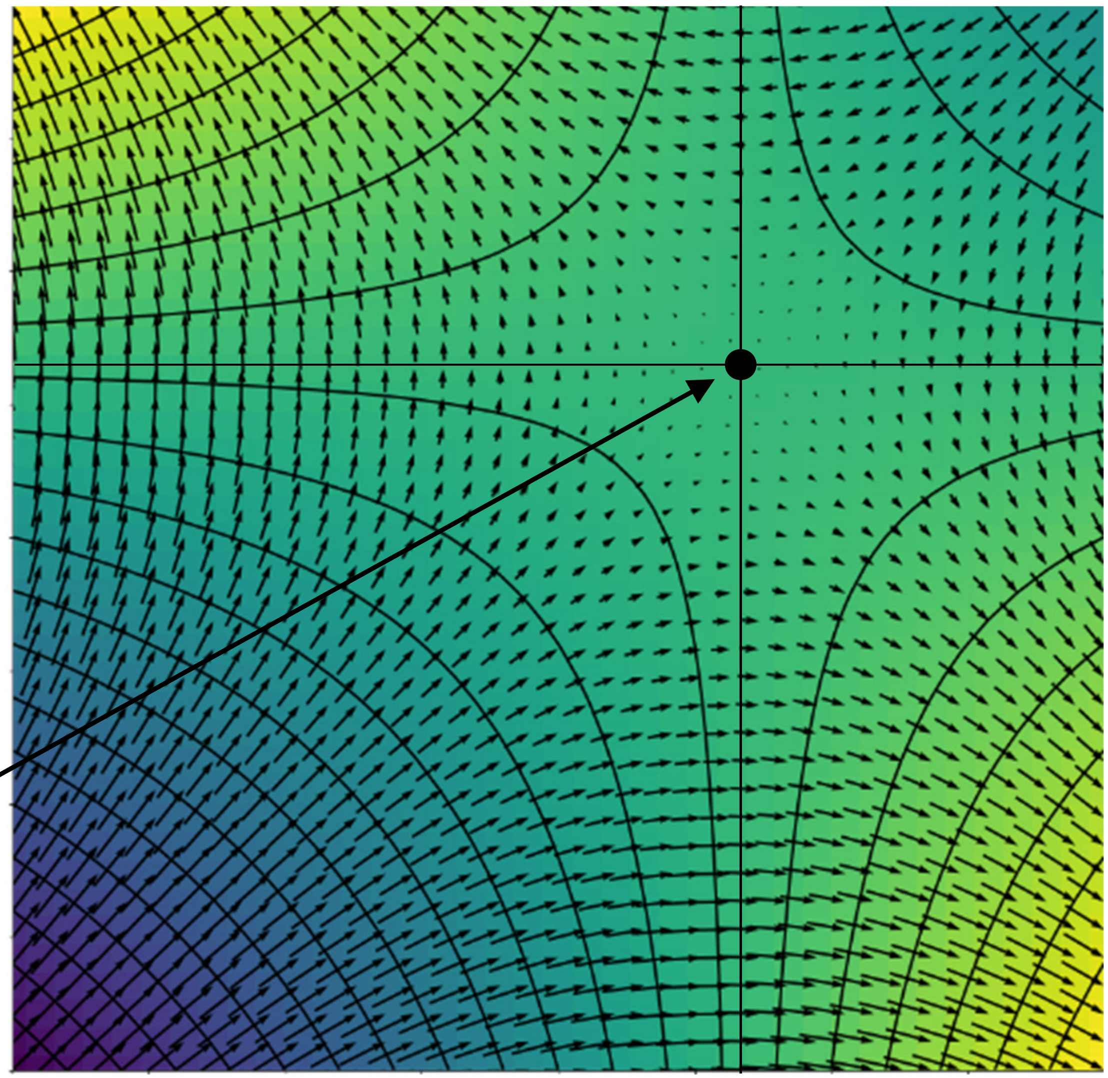

Interlude: Implicit Function Theorem

When can I write an implicit function in $\mathbb{R}^{n+m}$ such that it is the graph of a function $f: \mathbb{R}^n \to \mathbb{R}^m$ *at least locally?*

That is: is this implicitly described function an *n*-manifold embedded in $\mathbb{R}^{n+m}$? (with local coordinates in $\mathbb{R}^n$)

$$G(f) := \{(x, f(x)) | x \in \mathbb{R}^n\} \subset \mathbb{R}^n \times \mathbb{R}^m \simeq \mathbb{R}^{n+m}$$

Theorem: if *m* x *m* Jacobian matrix is invertible

(easier for scalar field: check if gradient of f is non-zero)

See **`https://en.wikipedia.org/wiki/Implicit_function_theorem`**

General result: *constant rank theorem*

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama