King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 6: Data Representation, Pt. 3: Structured and Unstructured Grids

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Reading Assignment #3 (until Feb 12)

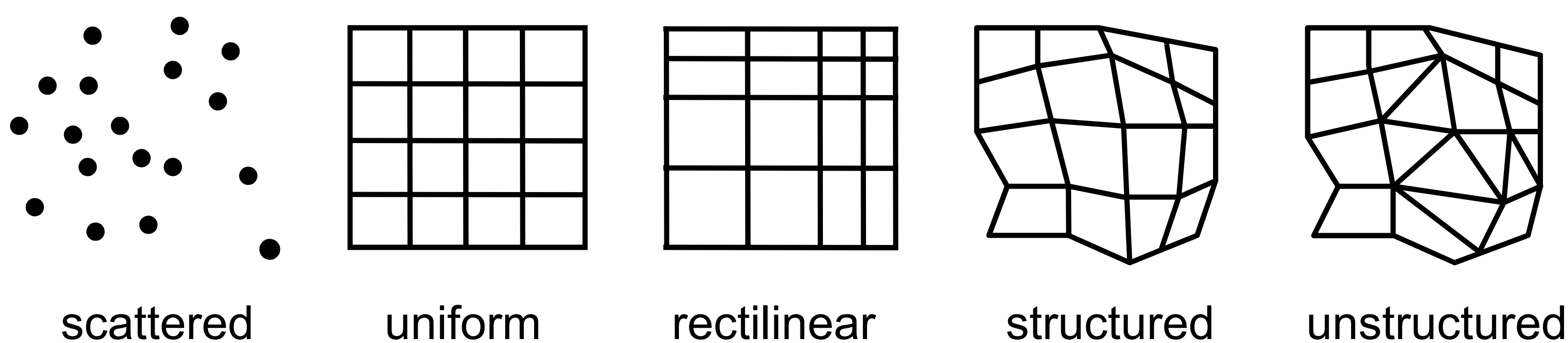
Read (required):

- Data Visualization book, finish Chapter 3 (read starting with 3.6)
- Data Visualization book, Chapter 5 until 5.3 (inclusive)

Data Representation

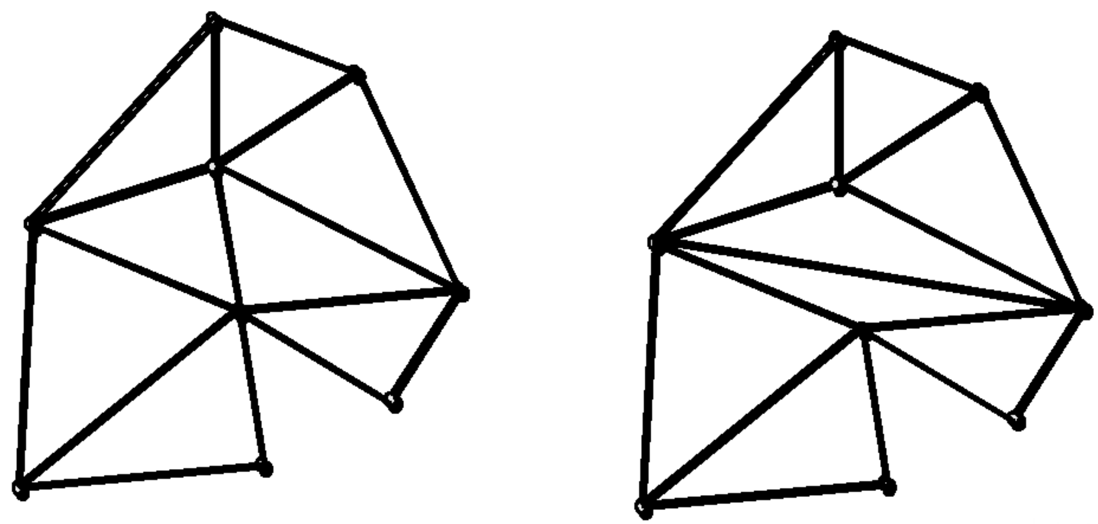
Data Structures

- Grid types
  - Grids differ substantially in the cells (basic building blocks) they are constructed from and in the way the topological information is given



Data Structures

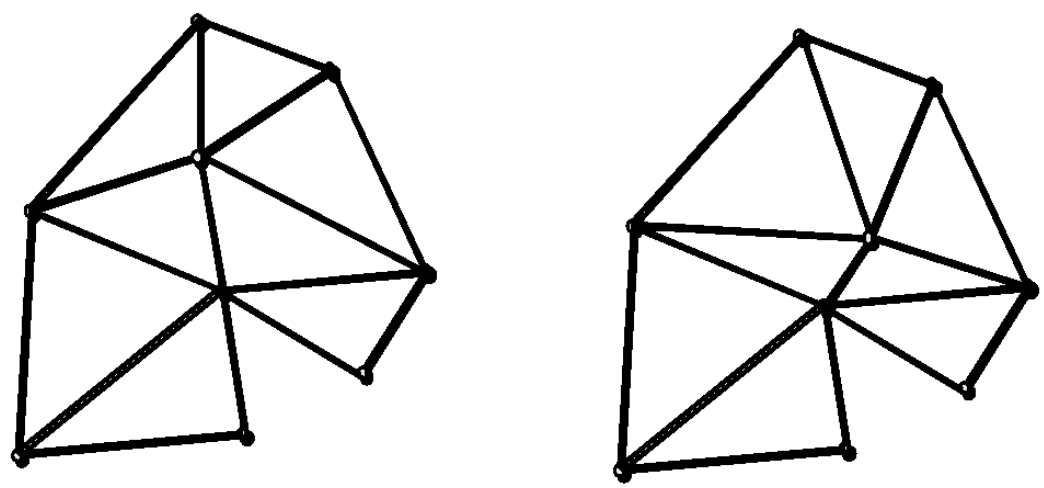
- Topology
  - Properties of geometric shapes that remain unchanged even when under distortion



Same geometry (vertex positions), different topology (connectivity)

Data Structures

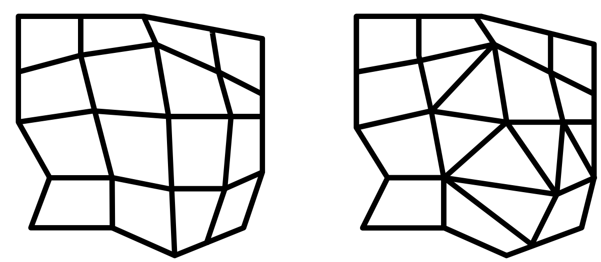
- Topologically equivalent
  - Things that can be transformed into each other by stretching and squeezing, without tearing or sticking together bits which were previously separated



topologically equivalent

Data Structures

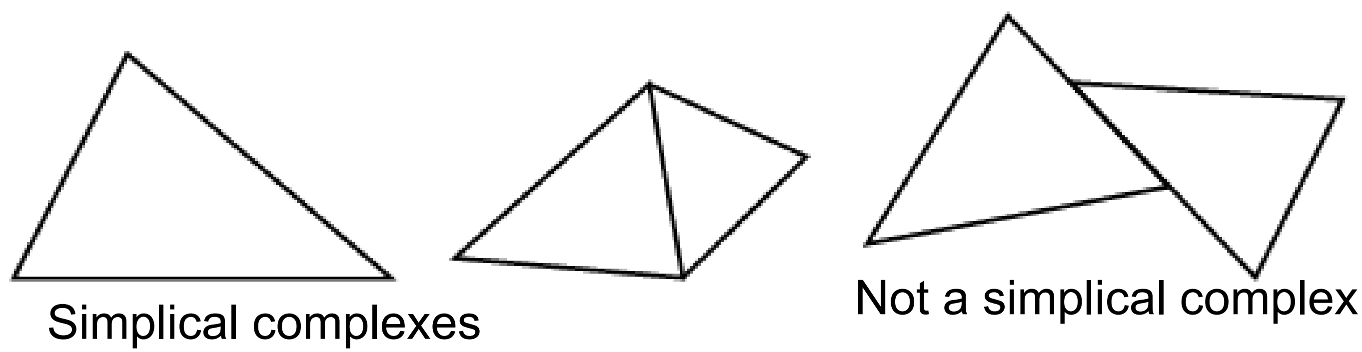
- Structured and unstructured grids can be distinguished by the way the elements or cells meet
- Structured grids
  - Have a regular topology and regular / irregular geometry
- Unstructured grids
  - Have irregular topology and geometry



structured unstructured

Data Structures

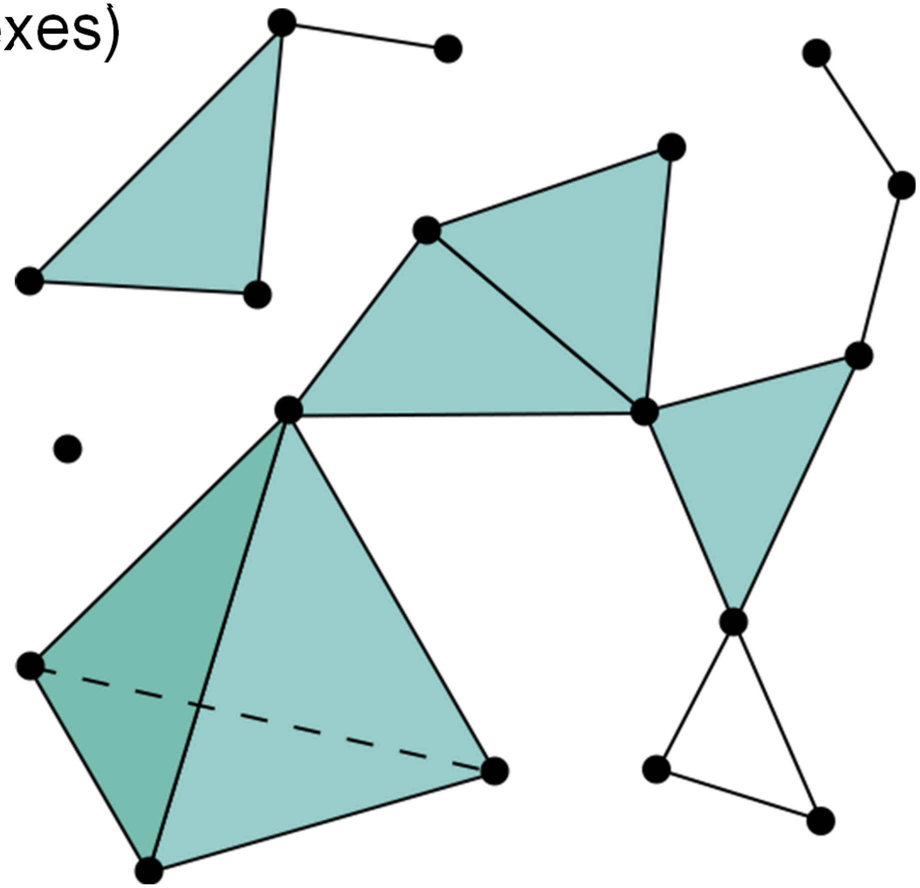
- An *n*-simplex
  - The convex hull of $n + 1$ affinely independent points
  - Lives in R^m , with $n \leq m$
  - 0: points, 1: lines, 2: triangles, 3: tetrahedra
- Partitions via simplices are called triangulations
- Simplical complex *C* is a collection of simplices with:
  - Every face of an element of *C* is also in *C*
  - The intersection of two elements of *C* is empty or it is a face of both elements
- Simplical complex is a space with a triangulation



Data Structures

- Simplicial complexes can be of mixed dimensions up to ≤ n
(except if “pure” complexes)

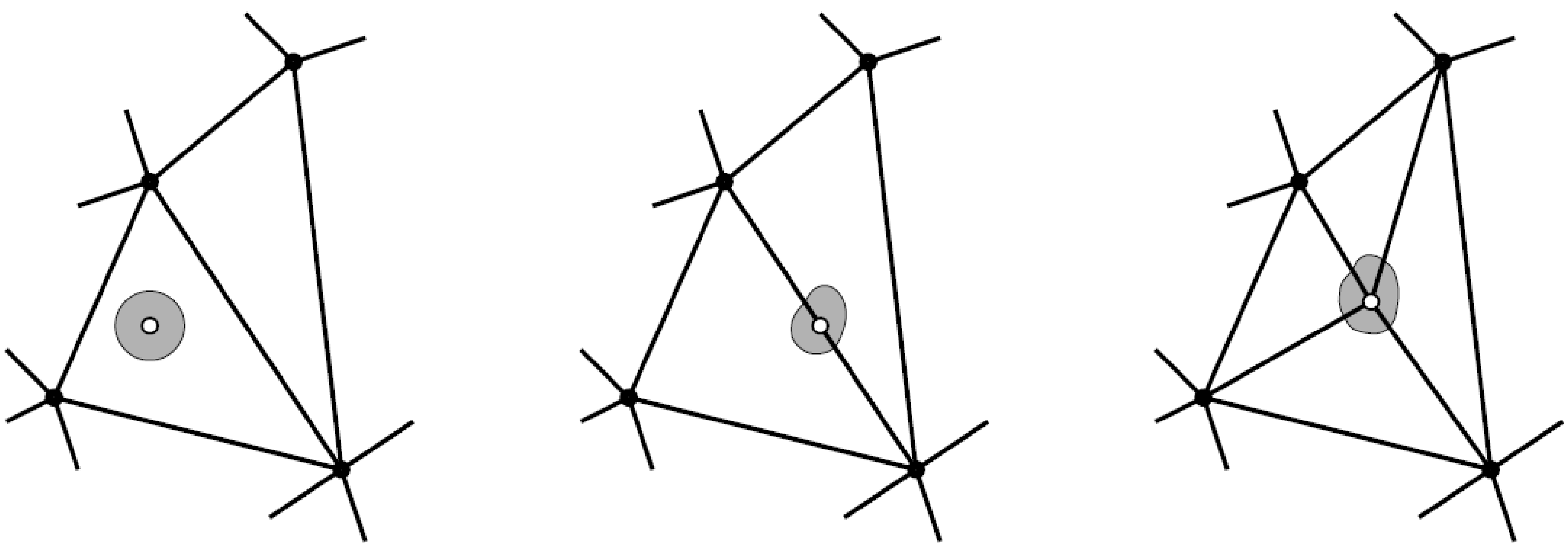
- Example:
Simplicial
3-complex



[Wikipedia.org]

Data Structures

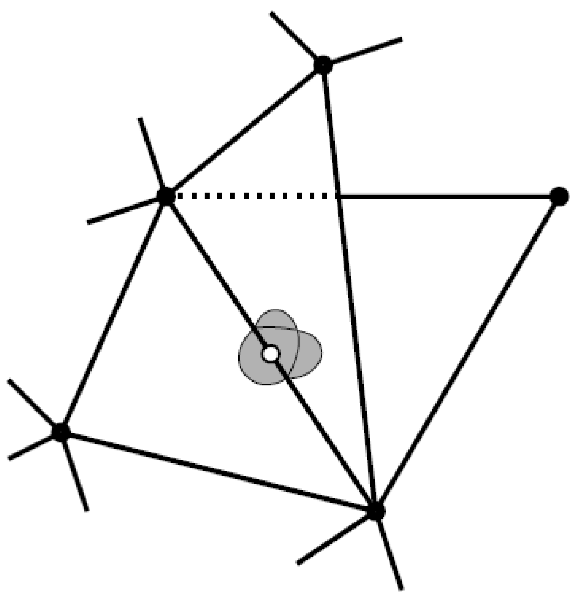
- 2-manifold meshes: neighborhood is 2-dimensional topological disc (or half disc for manifolds with boundary)



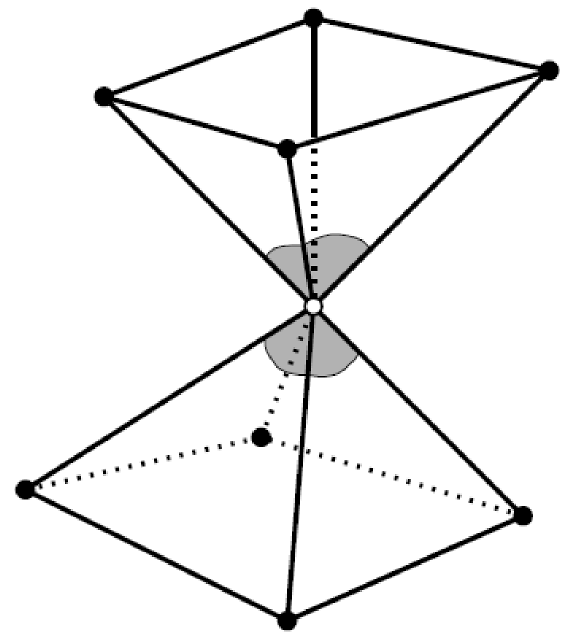
(a) (b) (c)

Data Structures

- Non-manifold meshes



(d)



(e)

Grid Types - Overview

struc-
tured
grids

ortho-
gonal
grids

equi-
dist.
grids

Cartesian
grids (dx=dy)

uniform (regular)
grids (dx≠dy)

rectilinear grids

curvi-linear grids

block-structured grids

unstructured grids

hybrid grids

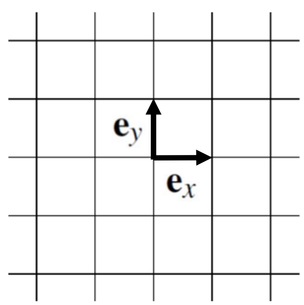
Interlude: Naming / Definition Caveats

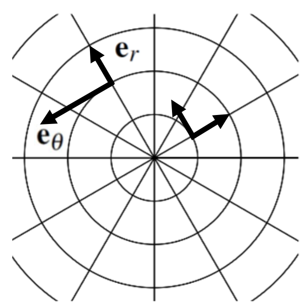
Beware of different naming conventions / different definitions

Example:

- On the previous slide, we used the term “orthogonal grid” in a simple, “global” way for the entire grid, i.e., different types of rectilinear grids, …
- In differential geometry, an orthogonal coordinate system is defined pointwise, i.e., a curvilinear grid with orthogonal basis vectors at each point is orthogonal

In differential geometry, both of these are orthogonal (in our context, the right one is not):

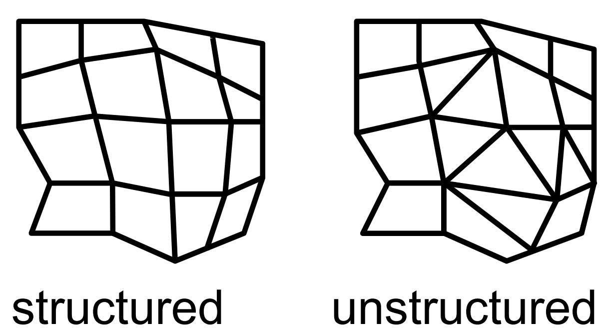




Structured Grids

Data Structures

- Characteristics of structured grids
  - Easier to compute with
  - Often composed of sets of connected parallelograms (hexahedra), with cells being equal or distorted with respect to (non-linear) transformations
  - May require more elements or badly shaped elements in order to precisely cover the underlying domain
  - Topology is represented implicitly by an *n*-vector of dimensions
  - Geometry is represented explicitly by an array of points
  - Every interior point has the same number of neighbors



Data Structures

- Characteristics of structured grids
  - Structured grids can be stored in a 2D / 3D array
  - Arbitrary samples can be directly accessed by indexing a particular entry in the array
  - Topological information is implicitly coded
    - Direct access to adjacent elements
  - Cartesian, uniform, and rectilinear grids are necessarily convex
  - Their visibility ordering of elements with respect to any viewing direction is given implicitly
  - Their rigid layout prohibits the geometric structure to adapt to local features
  - Curvilinear grids reveal a much more flexible alternative to model arbitrarily shaped objects
  - However, this flexibility in the design of the geometric shape makes the sorting of grid elements a more complex procedure

Data Structures

- Typical implementation of structured grids

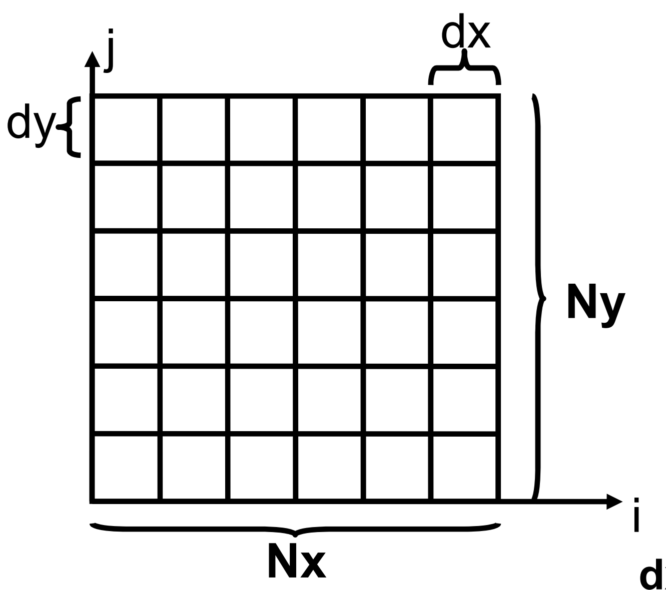
```
DataType *data = new DataType [Nx * Ny * Nz ];
val = data[ i + j * Nx + k * ( Nx * Ny ) ];
```

… code for geometry …

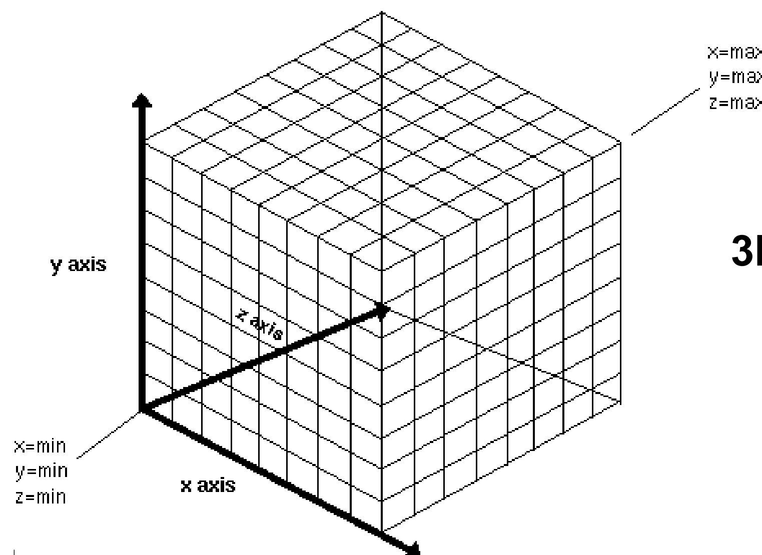
Data Structures

- Cartesian or equidistant grids
  - Structured grid
  - Cells and points are numbered sequentially with respect to increasing X, then Y, then Z, or vice versa
  - Number of points = Nx•Ny•Nz
  - Number of cells = (Nx-1)•(Ny-1)•(Nz-1)

2D



dx = dy = dz



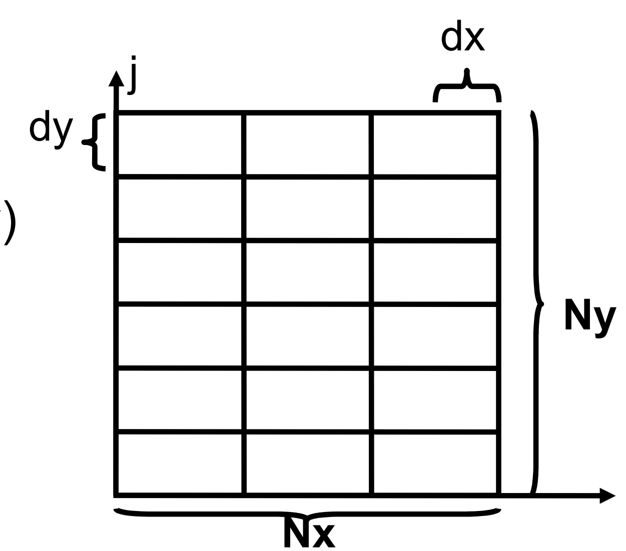
3D

Data Structures

- Cartesian grids
  - Vertex positions are given implicitly from [i,j,k]:
    - P[i,j,k].x = origin_x + i • dx
    - P[i,j,k].y = origin_y + j • dy
    - P[i,j,k].z = origin_z + k • dz
  - Global vertex index I[i,j,k] = k•Ny•Nx + j•Nx + i
    - k = I / (Ny•Nx)
    - j = (I % (Ny•Nx)) / Nx
    - i = (I % (Ny•Nx)) % Nx
  - Global index allows for linear storage scheme
    - Wrong access pattern might destroy cache coherence

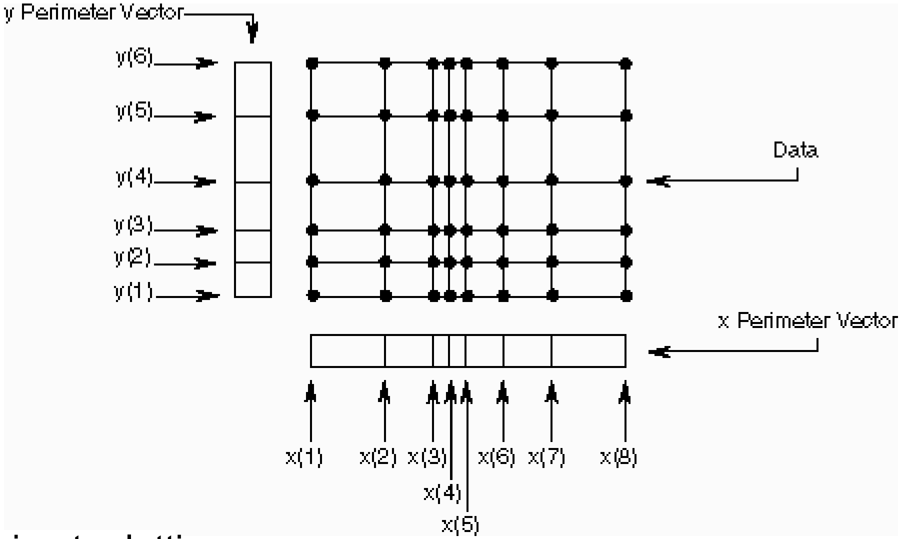
Data Structures

- Uniform grids
  – Similar to Cartesian grids
  – Consist of equal cells but with different resolution in at least one dimension (dx ≠ dy (≠ dz))
  – Spacing between grid points is constant in each dimension → same indexing scheme as for Cartesian grids
  – Most likely to occur in applications where the data is generated by a 3D imaging device providing different sampling rates in each dimension
  – Typical example: medical volume data consisting of slice images
    - Slice images with square pixels (dx = dy)
    - Larger slice distance (dz > dx = dy)



Data Structures

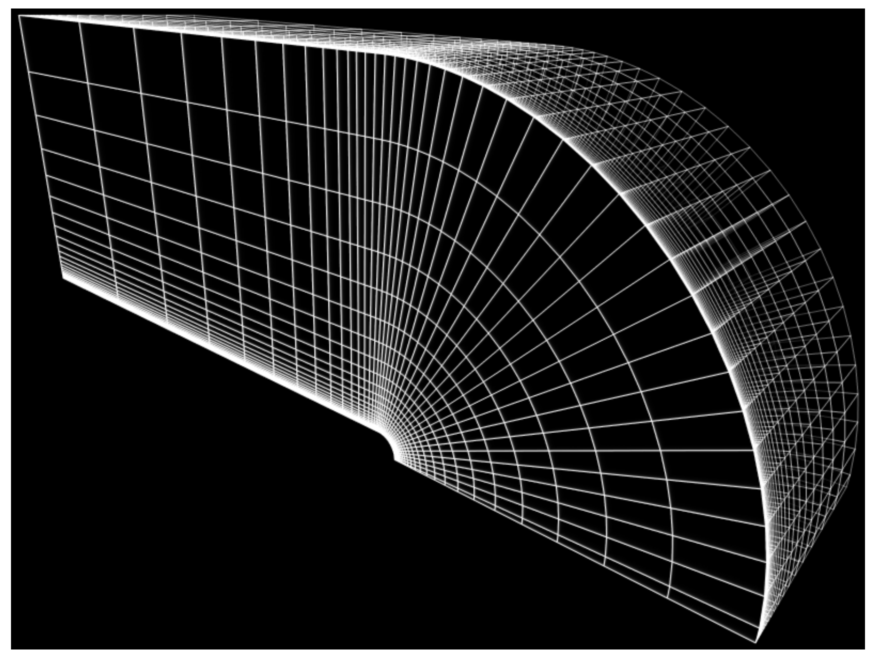
- Rectilinear grids
  - Topology is still regular but irregular spacing between grid points
    - Non-linear scaling of positions along either axis
    - Spacing, x_coord[L], y_coord[M], z_coord[N], must be stored explicitly
  - Topology is still implicit



(2D perimeter lattice:
rectilinear grid in IRIS Explorer)

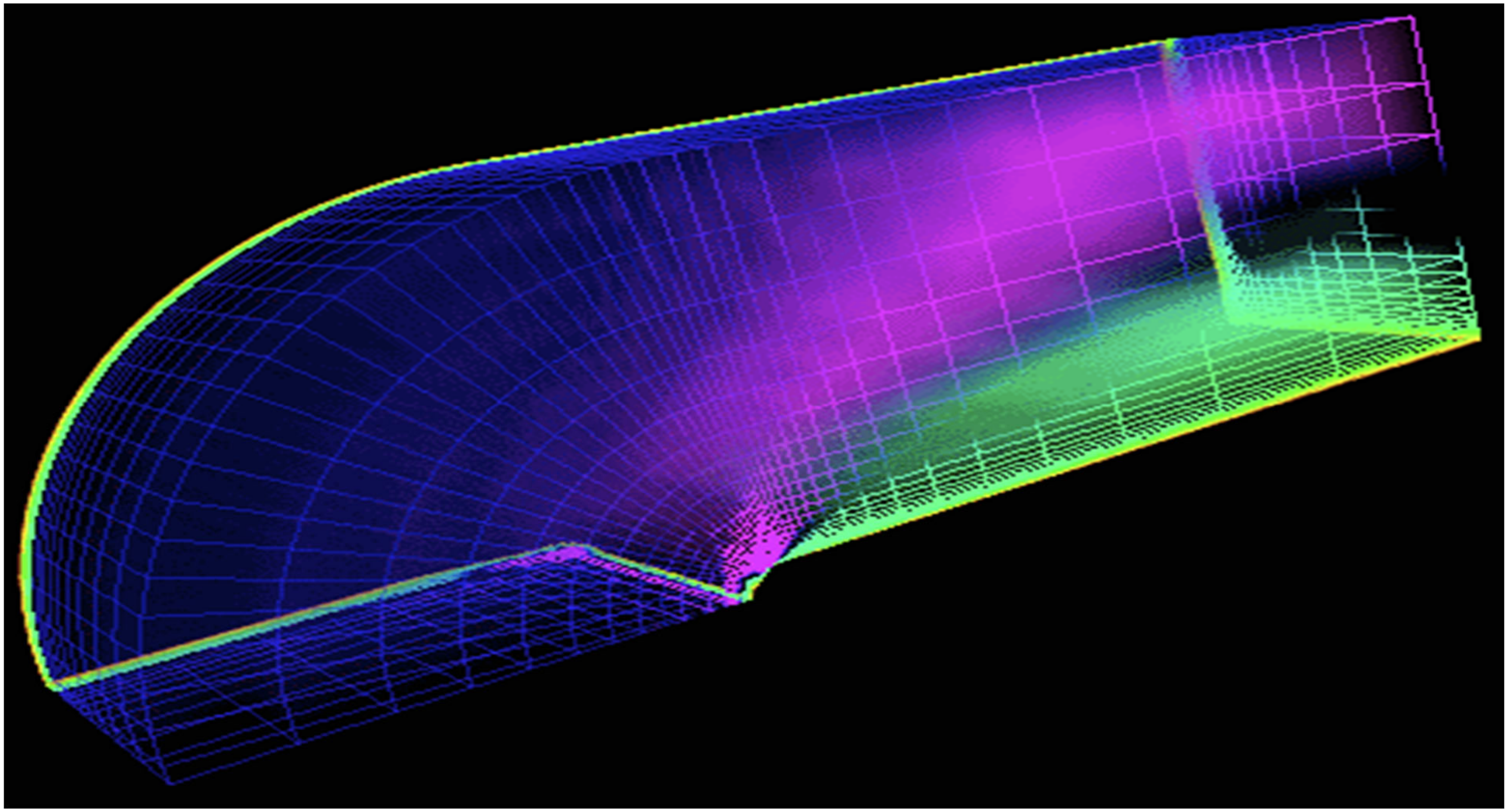
Data Structures

- Curvilinear grids
  - Topology is still regular but irregular spacing between grid points
    - Positions are non-linearly transformed
  - Topology is still implicit, but vertex positions are explicitly stored
    - x_coord[L,M,N]
    - y_coord[L,M,N]
    - z_coord[L,M,N]
  - Geometric structure might result in concave grids



Data Structures

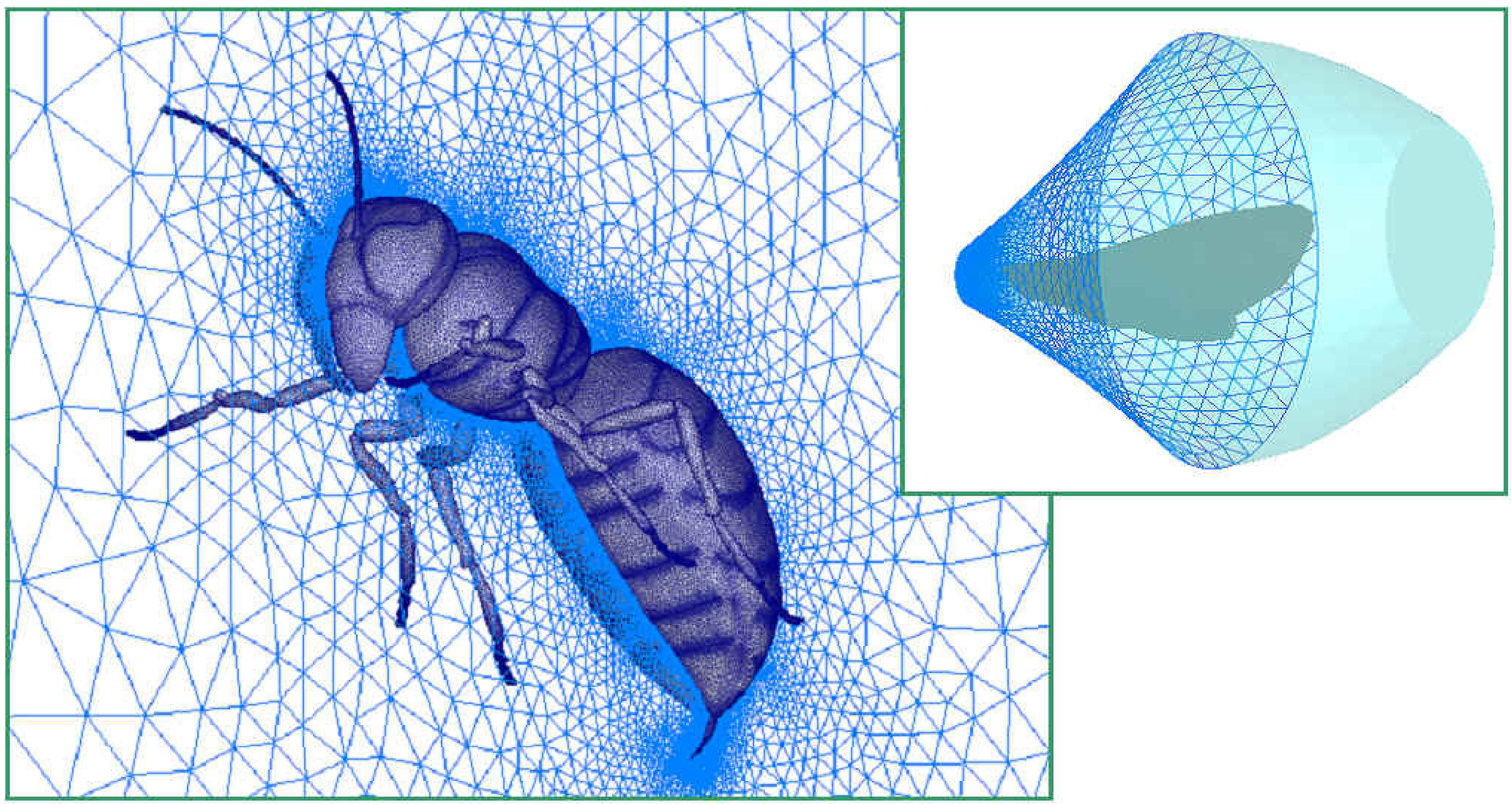
- Curvilinear grids



Unstructured Grids

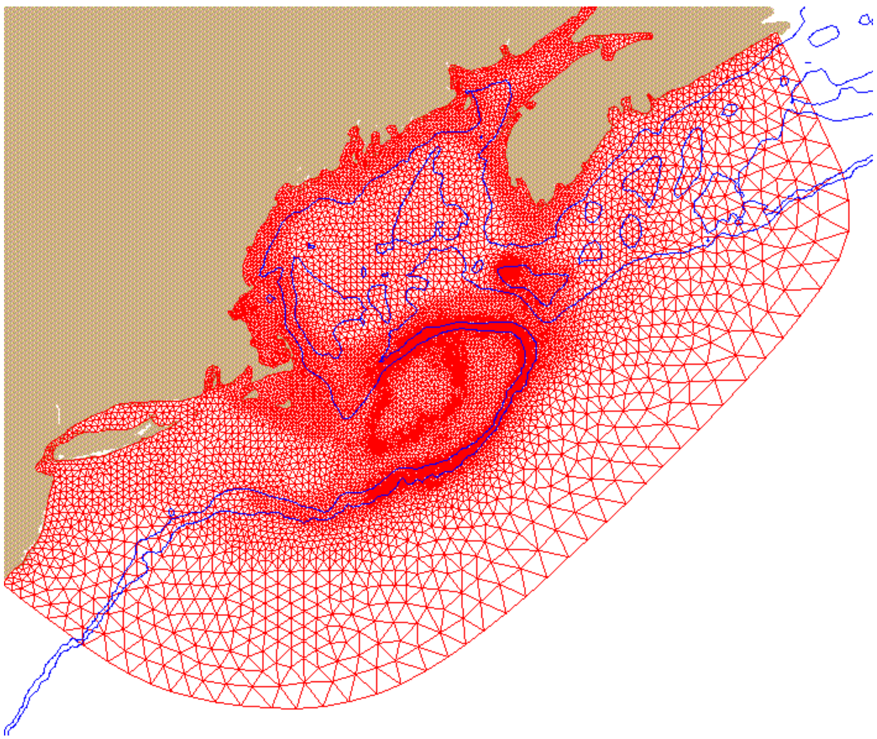
Data Structures

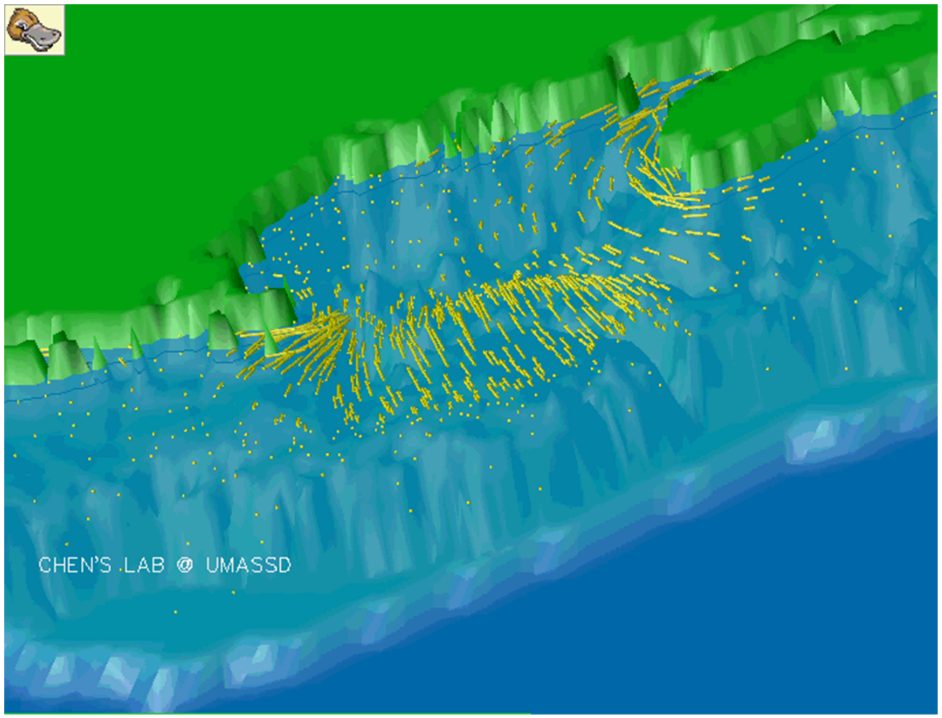
- Unstructured grids
  - Can be adapted to local features



Data Structures

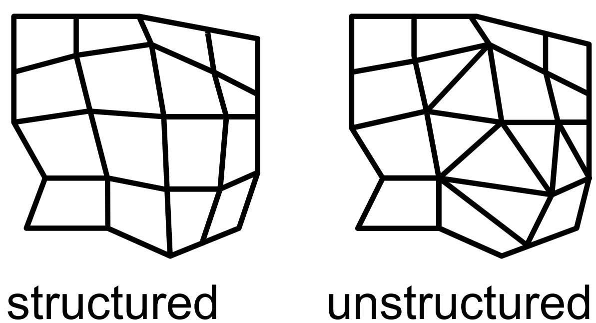
- Unstructured grids
  - Can be adapted to local features





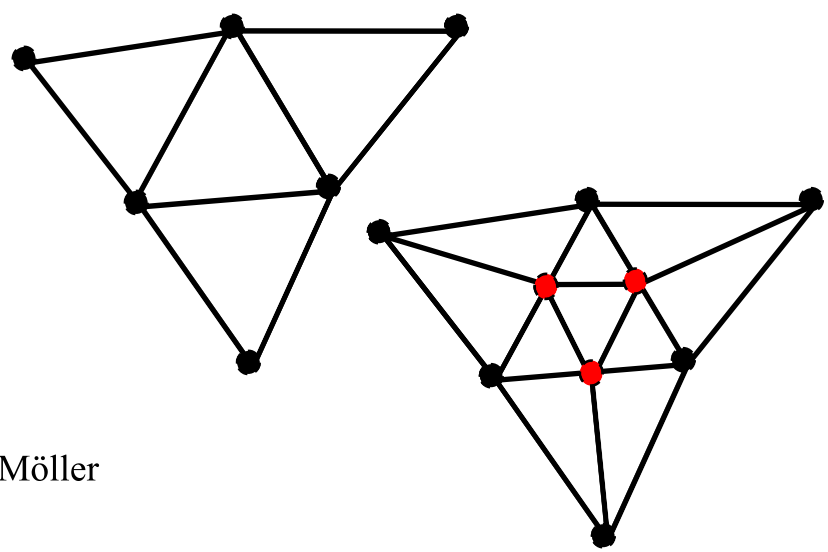
Data Structures

- If no implicit topological (connectivity) information is given, the grids are called unstructured grids
  - Unstructured grids are often computed using quadtrees (recursive domain partitioning for data clustering), or by triangulation of point sets
  - The task is often to create a grid from scattered points
- Characteristics of unstructured grids
  - Grid point geometry **and** connectivity must be stored
  - Dedicated data structures needed to allow for efficient traversal and thus data retrieval
  - Often composed of triangles or tetrahedra
  - Typically, fewer elements are needed to cover the domain



Data Structures

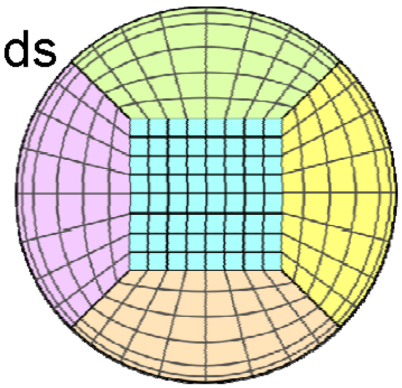
- Unstructured grids
  - Composed of arbitrarily positioned and connected elements
  - Can be composed of one unique element type or they can be hybrid (tetrahedra, hexas, prisms)
  - Triangle meshes in 2D and tetrahedral grids in 3D are most common
  - Can adapt to local features (small vs. large cells)
  - Can be refined adaptively
  - Simple linear interpolation in simplices



Data discretizations

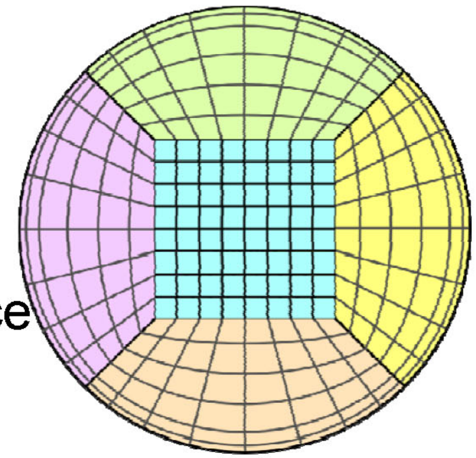
Types of data sources have typical types of discretizations:

- Measurement data:
  - typically scattered (no grid)
- Numerical simulation data:
  - structured, block-structured, unstructured grids
  - adaptively refined meshes
  - multi-zone grids with relative motion
  - etc.



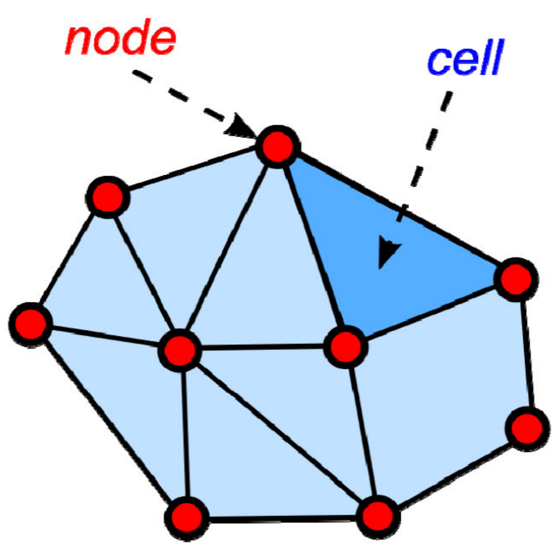
- Imaging methods:
  - uniform grids
- Mathematical functions:
  - uniform/adaptive sampling on demand

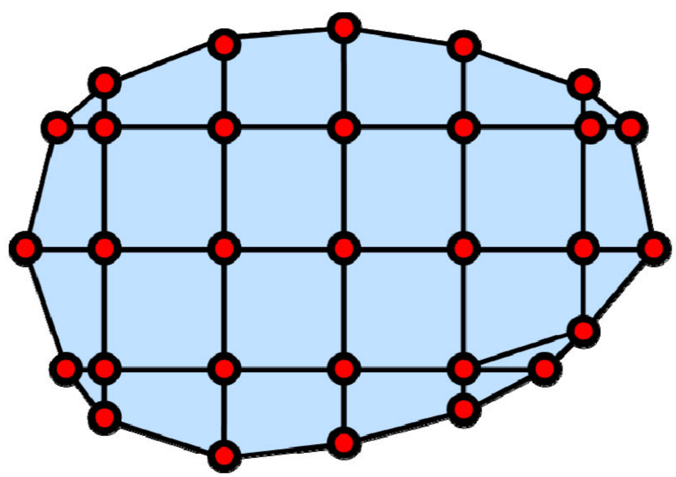
Unstructured grids



2D unstructured grids:

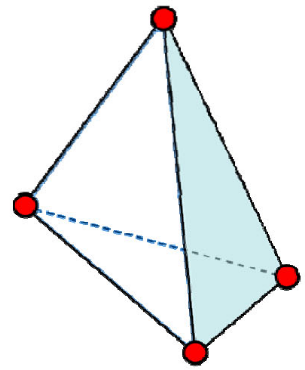
- cells are triangles and/or quadrangles
- domain can be a surface embedded in 3-space
  (distinguish n-dimensional from n-space)

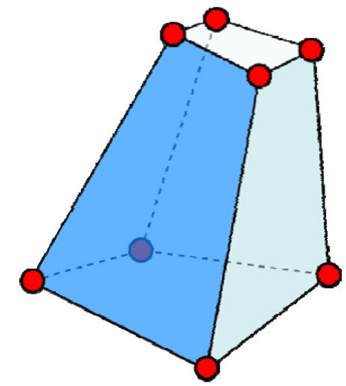




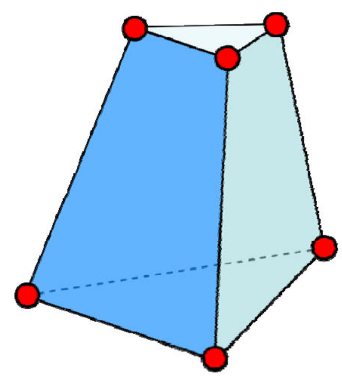
3D unstructured grids:

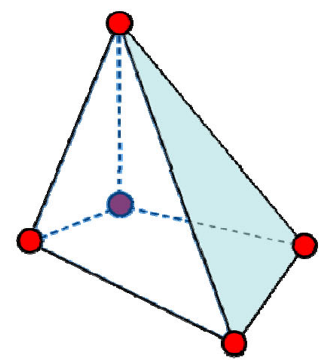
- cells are tetrahedra or hexahedra





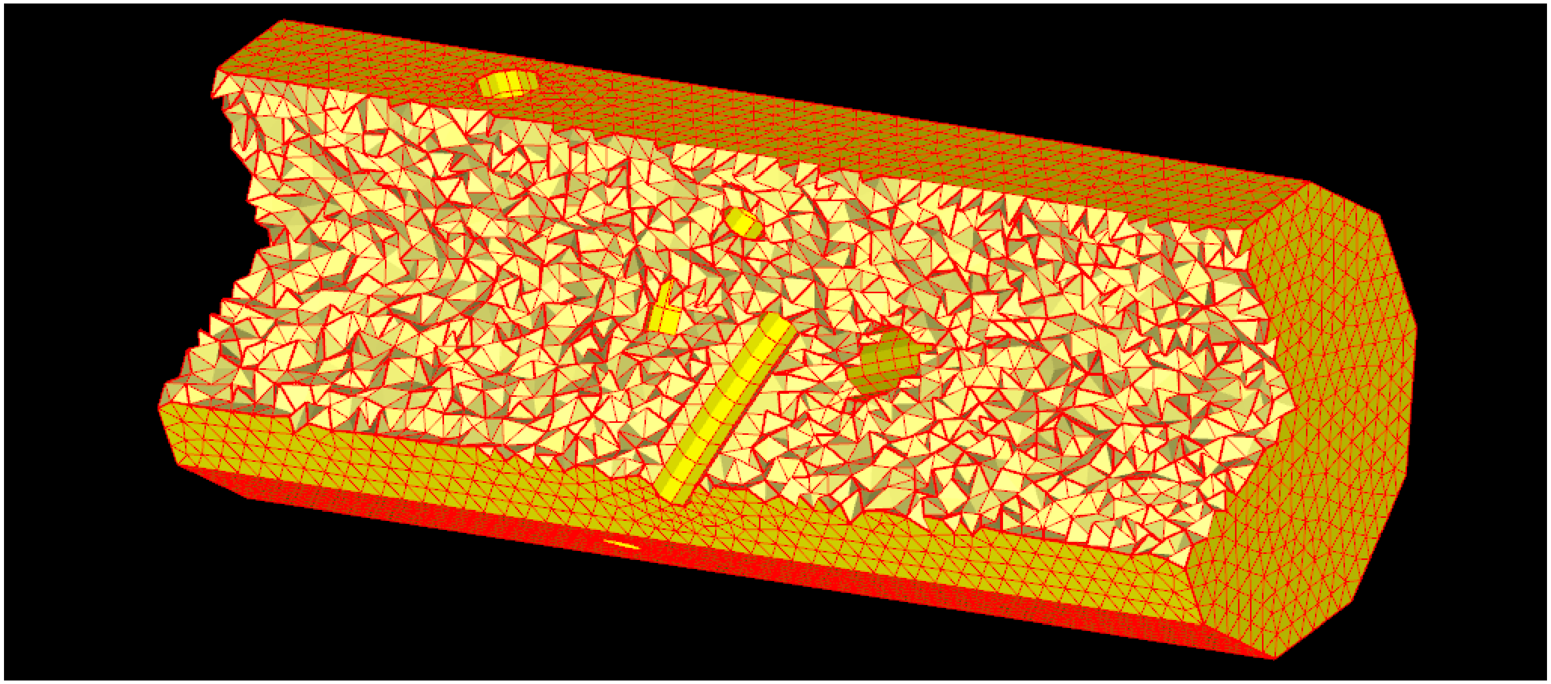
- mixed grids ("zoo meshes") require additional types:
  wedge (3-sided prism), and pyramid (4-sided)





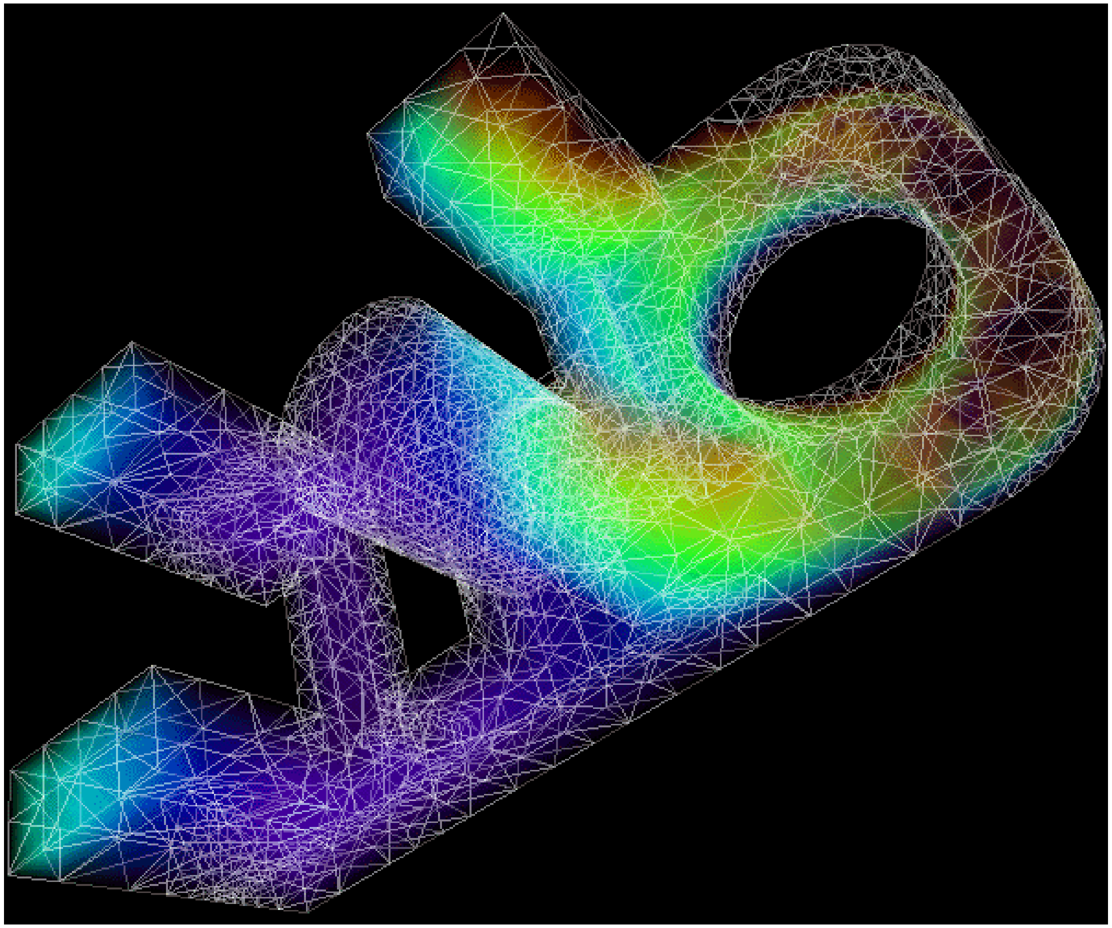
Common Unstructured Grid Types (1)

- Simplest: purely tetrahedral



Grid Structures

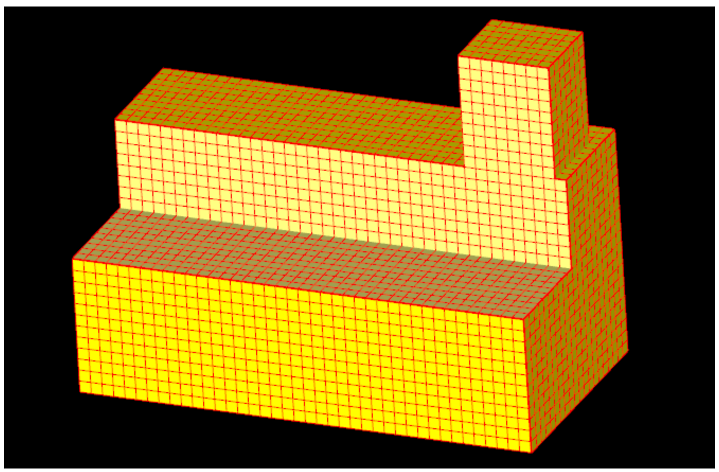
Tet grid example

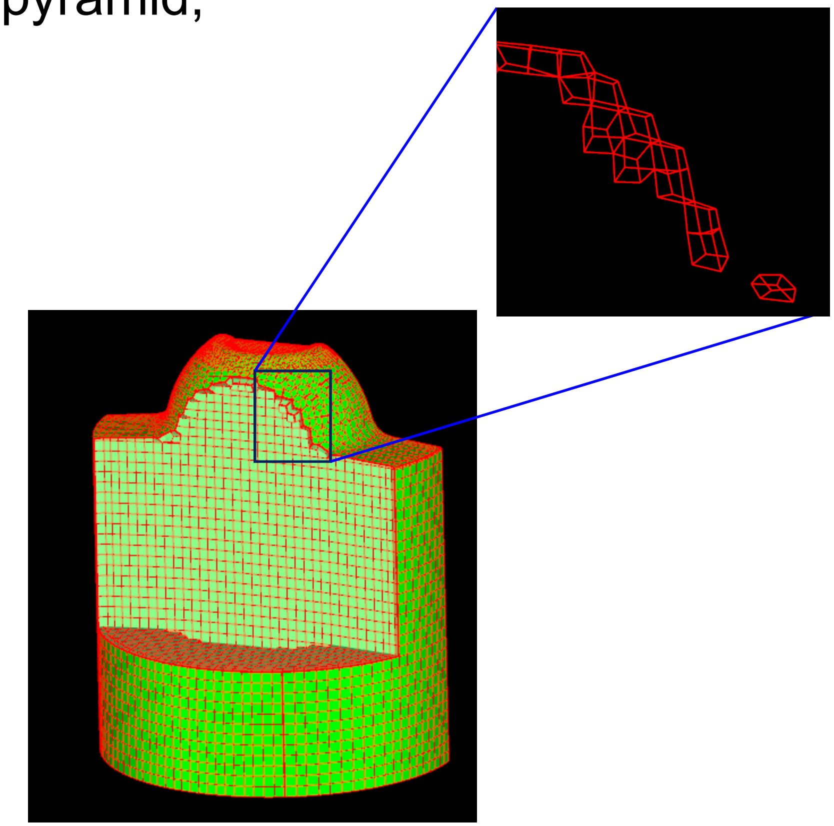


Common Unstructured Grid Types (2)

Pre-defined cell types
(tetrahedron, triangular prism, quad pyramid, hexahedron, octahedron)

- Only triangle / quad faces
- Planar / non-planar faces

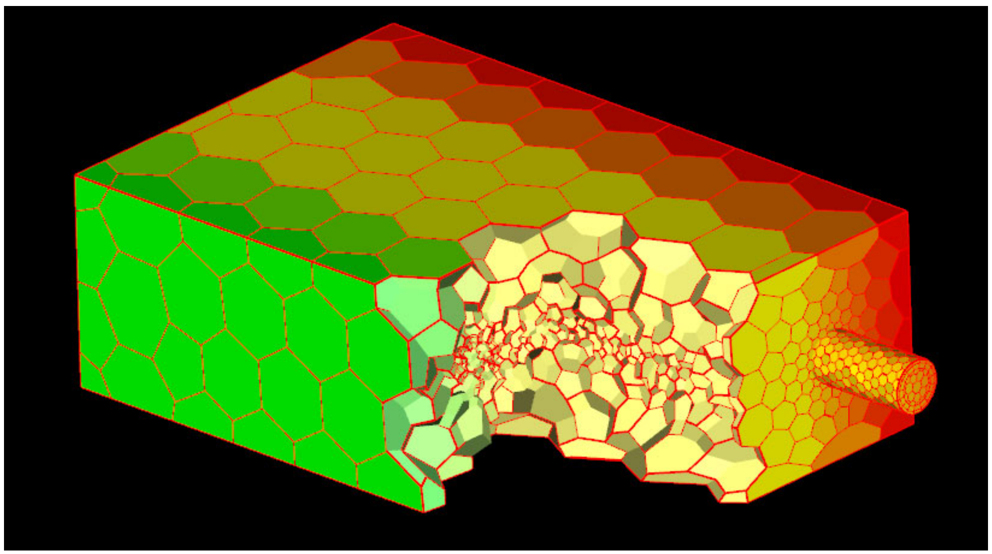


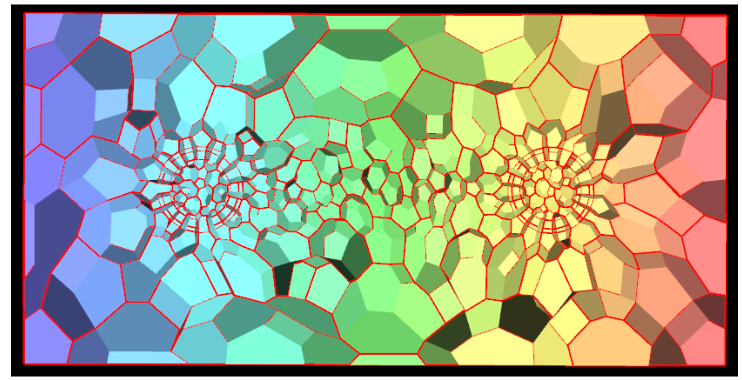


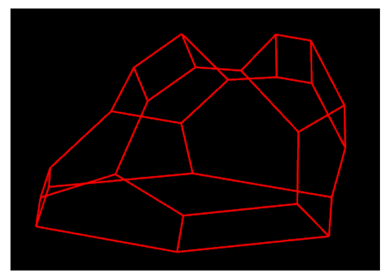
Common Unstructured Grid Types (3)

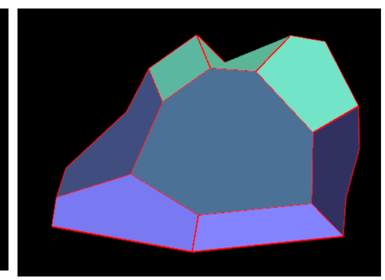
(Nearly) arbitrary polyhedra

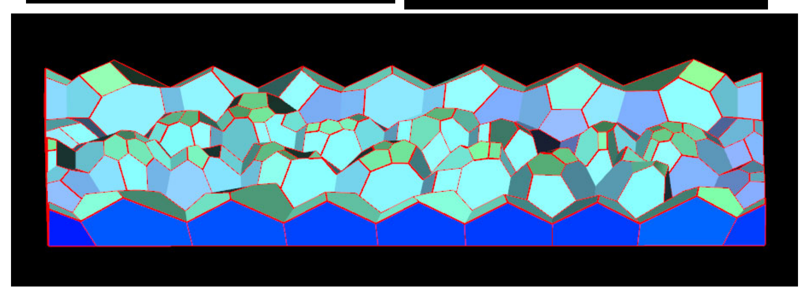
- Possibly non-planar faces







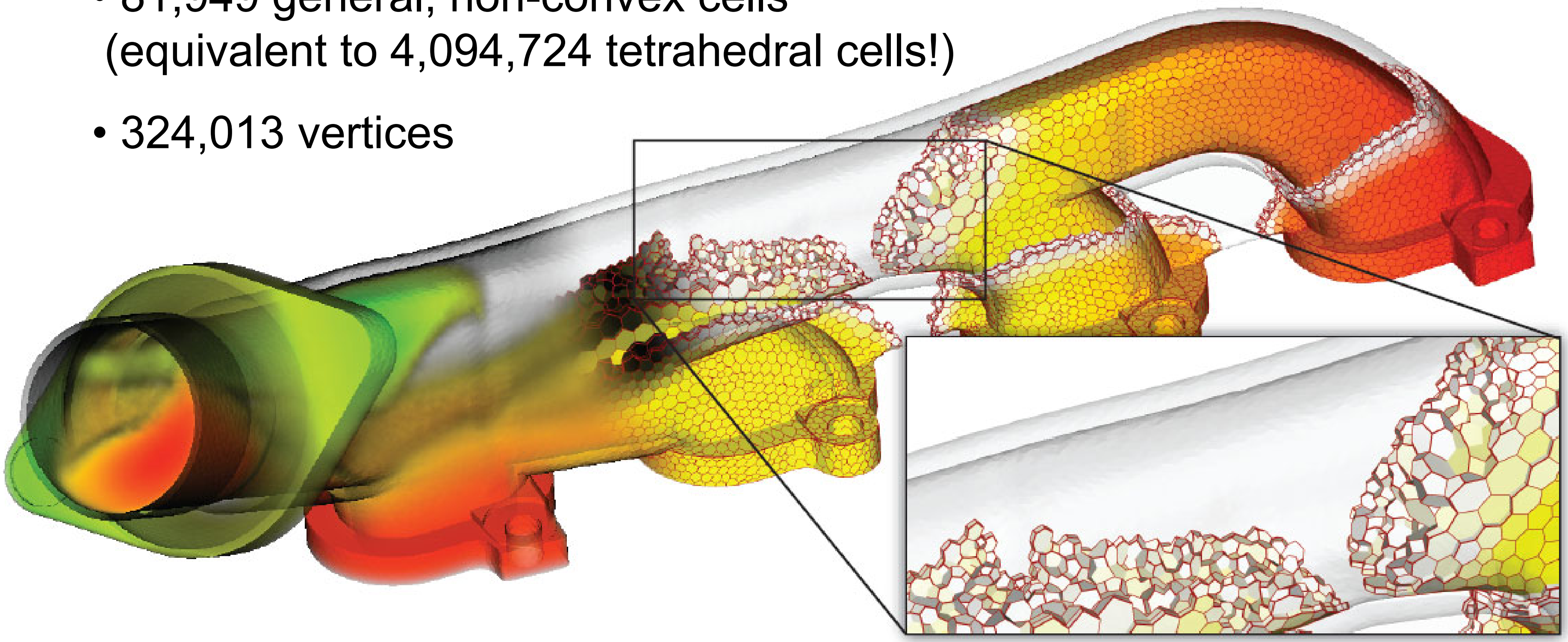




Example: General Polyhedral Cells

Exhaust manifold

- 81,949 general, non-convex cells
  (equivalent to 4,094,724 tetrahedral cells!)
- 324,013 vertices

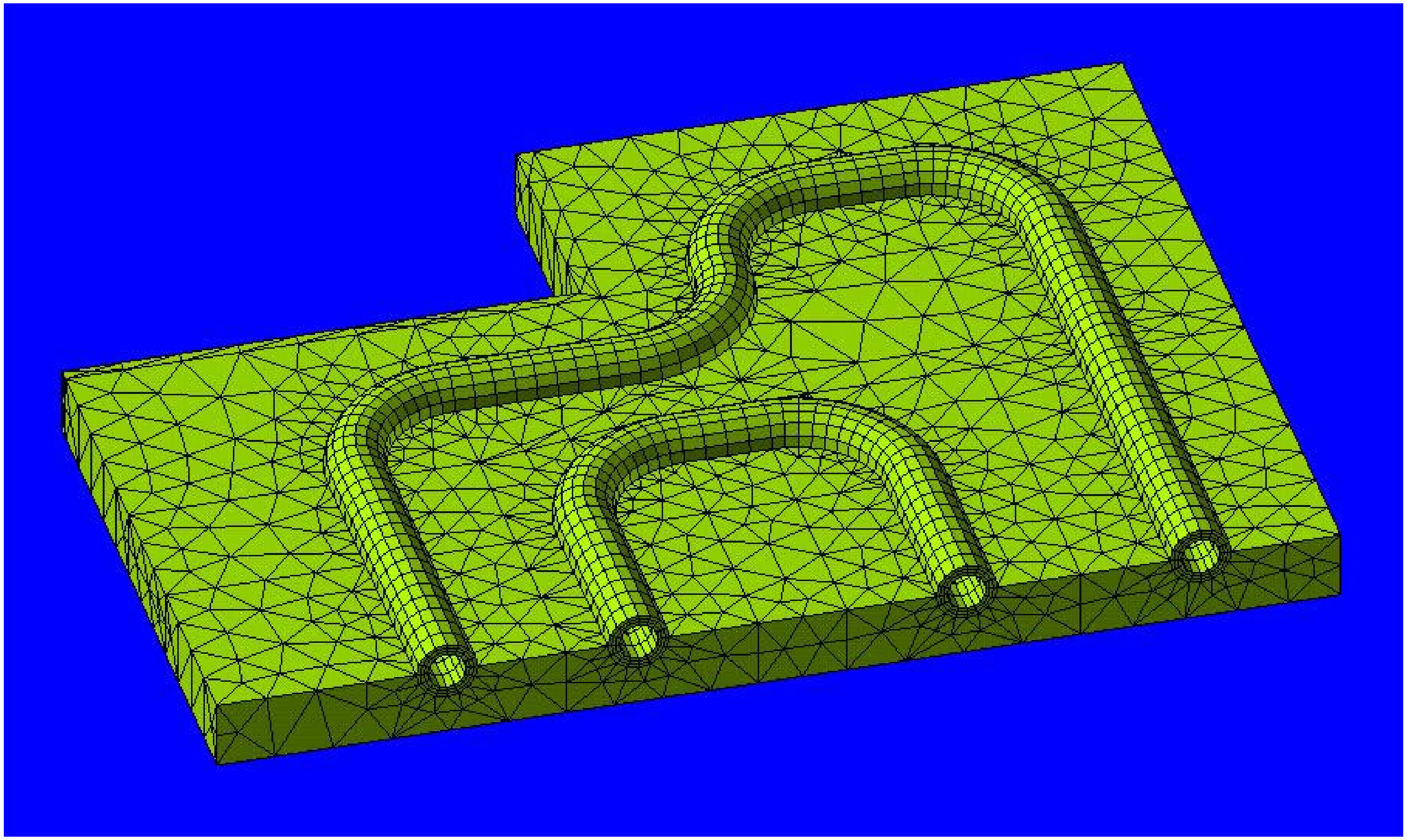


- Color coding: temperature distribution

Hybrid Grids

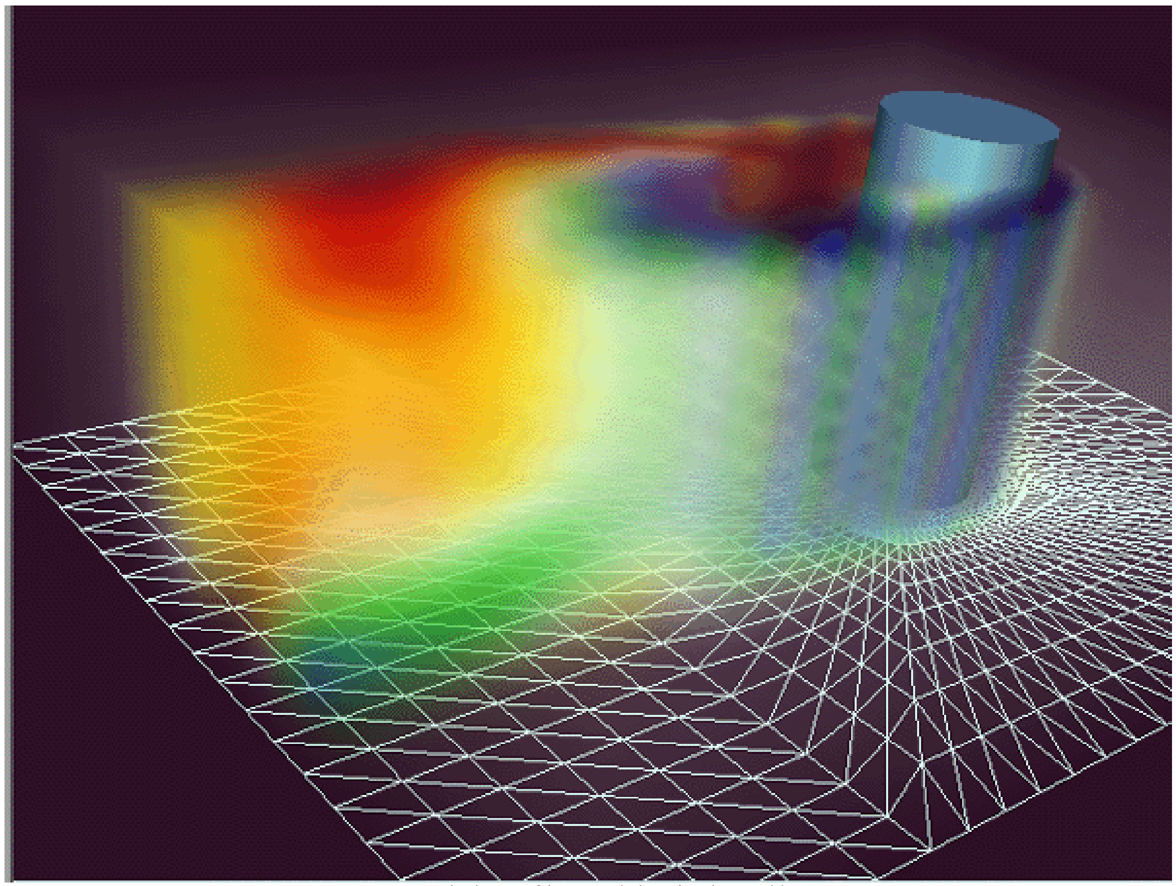
Data Structures

- Hybrid grids
  - Combination of different grid types



Data Structures

Hybrid grid example



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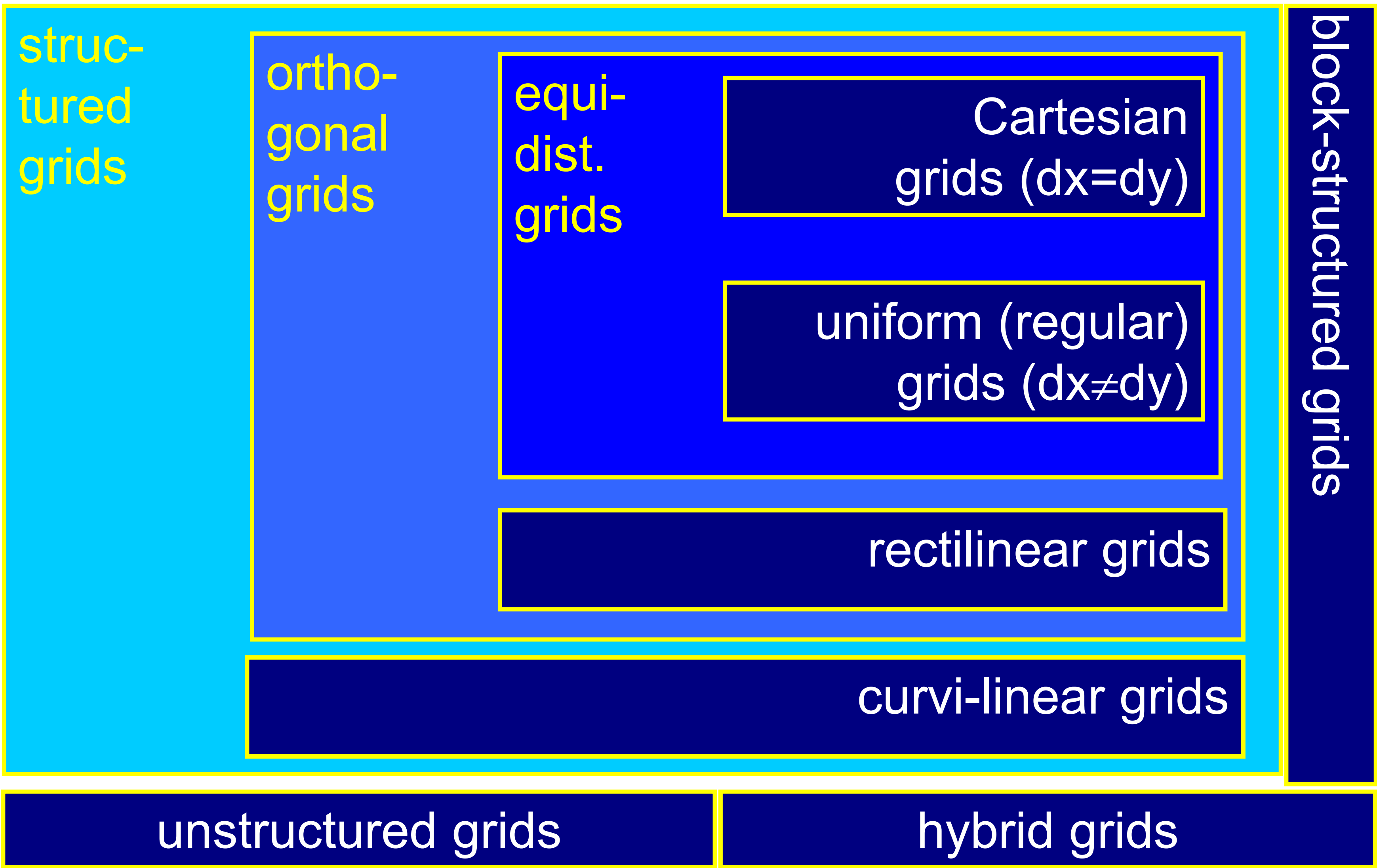
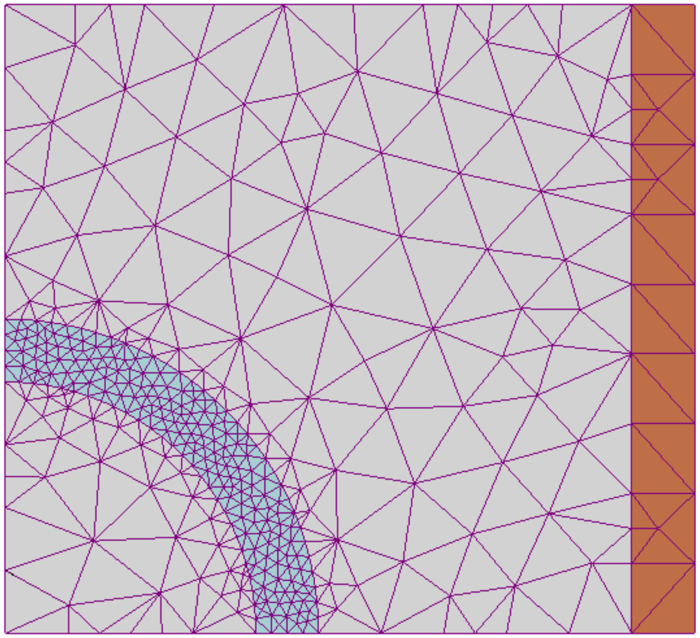
Grid Types - Overview

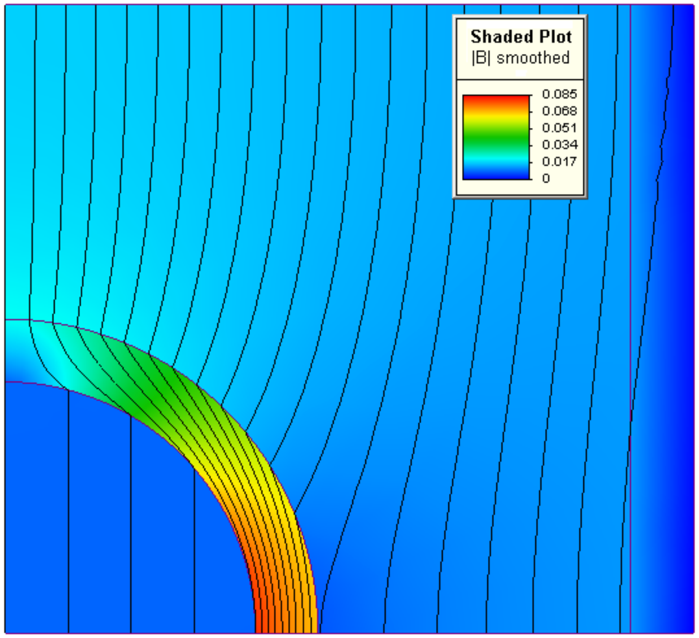


Grids vs. Data on Grids

grid



scalar field on grid



wikipedia

Unstructured Grid (Mesh) Data Structures

Unstructured 2D Grid: Direct Storage

Store list of vertices; vertices shared by triangles are replicated

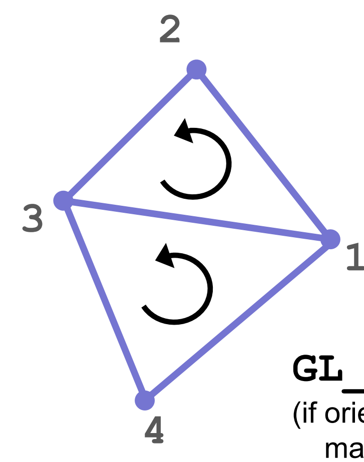
Render, e.g., with OpenGL immediate mode, …

coords for vertex 1

```
x1,y1,z1   ] face 1
x2,y2,z2
x3,y3,z3
x1,y1,z1   ] face 2
x3,y3,z3
x4,y4,z4

...
```

```
struct face
  float verts[3][3]
  DataType val;
```



GL_CCW

(if orientable manifold)

Redundant, large storage size, cannot modify shared vertices easily

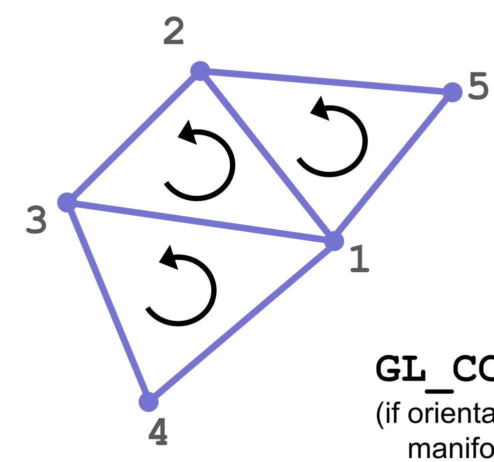
Store data values per face, or separately

Unstructured 2D Grid: Indirect Storage

Indexed face set: store list of vertices; store triangles as indexes

Render using separate vertex and index arrays / buffers

coords for vertex 1	vertex list	face list
	`x1,y1,(z1)`	`1,2,3`
	`x2,y2,(z2)`	`1,3,4`
	`x3,y3,(z3)`	`2,1,5`
	`x4,y4,(z4)`	
	`...`	`...`



`GL_CCW`
(if orientable manifold)

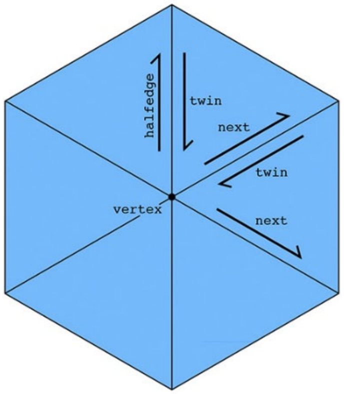
Less redundancy, more efficient in terms of memory

Easy to change vertex positions; still have to do (global) search for shared edges (local information)

Unstructured 2D Grids: Connectivity/Incidence

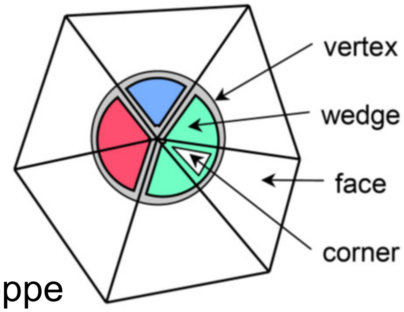
Half-edge (doubly-connected edge list) data structure

- Pointer to half-edge (twin) in neighboring face
  (mesh needs to be orientable 2-manifold)
- Pointer to next half-edge in same face
- Half-edge associated with one vertex, edge, face



Modifications: attributes, mesh simplification, …

- Vertices, corners, wedges, faces
- Express attribute continuity vs. discontinuity



Hugues Hoppe

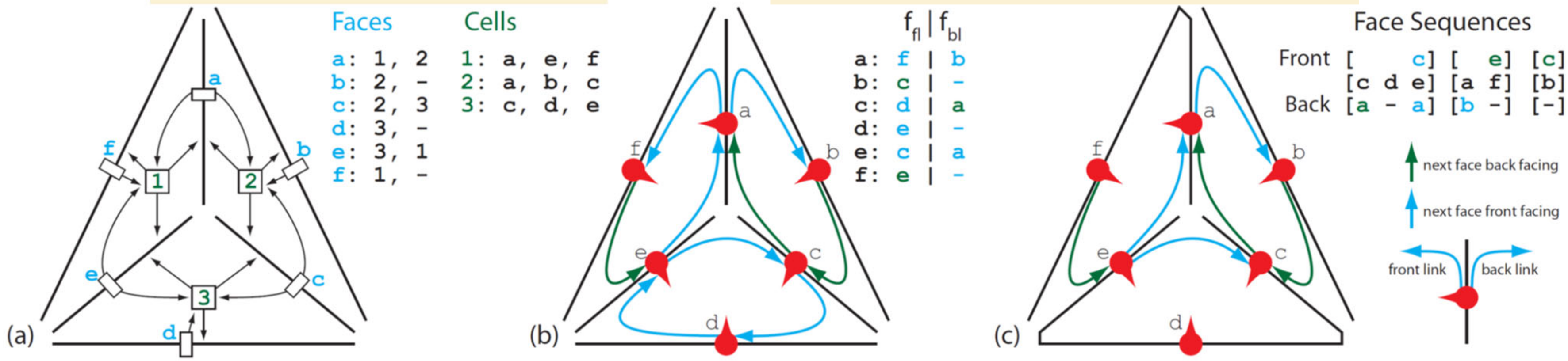
Visualization often needs volumetric version of these ideas
(tet meshes, polyhedral meshes, …)

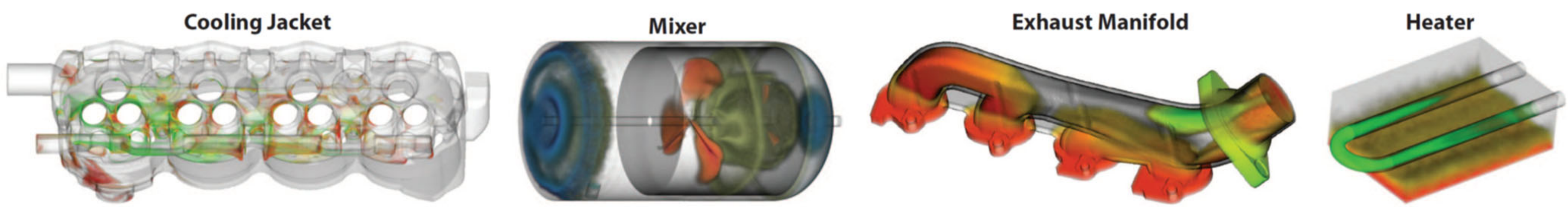
3D Grids: Two-Sided Face Sequence Lists

General polyhedral grids (arbitrary polyhedral cells); example: TSFSL (Muigg et al., 2011)

standard face/cell incidence

two-sided face (sequence) lists





	Cooling Jacket	Mixer	Exhaust Manifold	Heater
Cells/Vertices/Faces:	1,538K / 1,631K / 4,707K	1,362K / 7,432K / 8,869K	82K / 324K / 441K	17K / 68K / 91K
Tetrahedra:	17,044K (~8.5 byte/tet)	89,417K (~7.5 byte/tet)	4,095K (~7.0 byte/tet)	851K (~7.0 byte/tet)
Celltypes:	tets/pyramids/wedges/hexas	general (non-convex) polyhedra	general (non-convex) polyhedra	general (non-conv.) polyh.
Bricks/Cell Overhead:	4/1.7%	10/8.6%	1/0%	1/0%
TSFSL Creation Time:	4.0s	9.0s	1.7s	1.0s

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
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- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama