CS 247 – Scientific Visualization
Lecture 22: Vector / Flow Visualization, Pt. 1

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Reading Assignment #12 (until Apr 18)

Read (required):

- Data Visualization book
  - Chapter 6 (Vector Visualization)
    - Beginning (before 6.1)
    - Chapters 6.2, 6.3, 6.5
- More general vector field basics (the book is not very precise on the basics)

  `https://en.wikipedia.org/wiki/Vector_field`

Read (optional):

- Paper:
  Bruno Jobard and Wilfrid Lefer
  Creating Evenly-Spaced Streamlines of Arbitrary Density,

  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.9498`

Online Demos and Info

Numerical ODE integration methods (Euler vs. Runge Kutta, etc.)

https://demonstrations.wolfram.com/NumericalMethodsForDifferentialEquations/

Flow visualization concepts

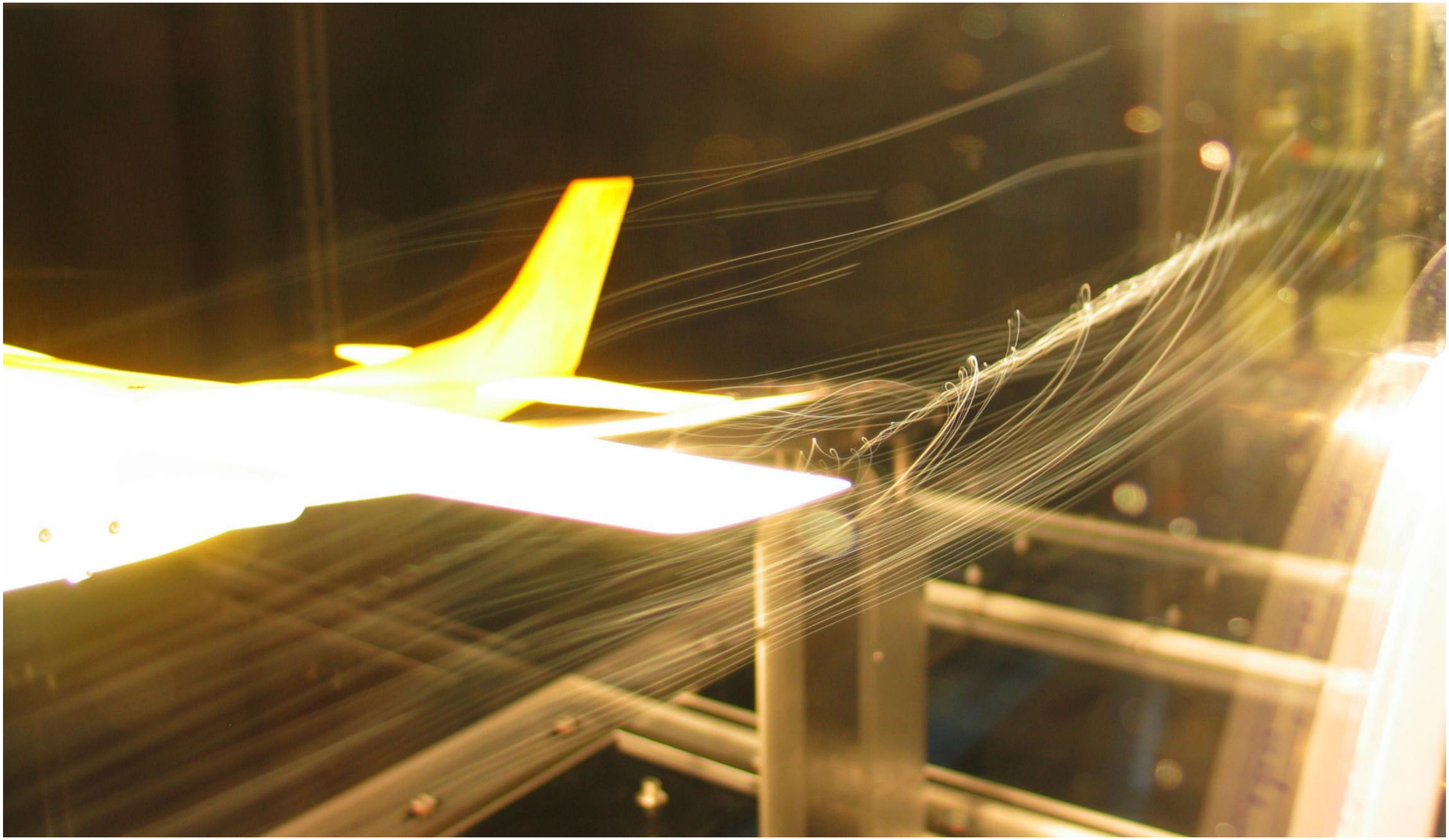
https://www3.nd.edu/~cwang11/flowvis.html

Vector Fields: Motivation

Smoke angel

A C-17 Globemaster III from the 14th Airlift Squadron, Charleston Air Force Base, S.C. flies off after releasing flares over the Atlantic Ocean near Charleston, S.C., during a training mission on Tuesday, May 16, 2006. The "smoke angel" is caused by the vortex from the engines.

(U.S. Air Force photo/Tech. Sgt. Russell E. Cooley IV)



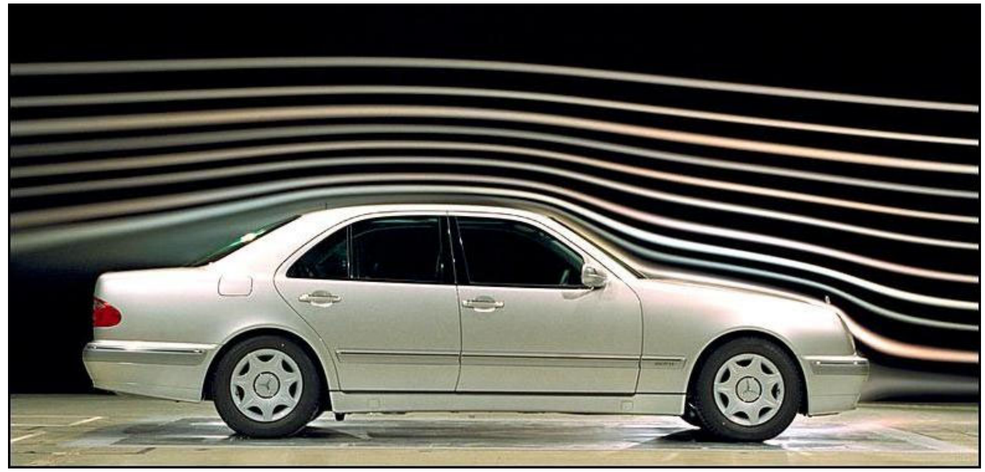
A wind tunnel model of a Cessna 182 showing a wingtip vortex.
Tested in the RPI (Rensselaer Polytechnic Institute) Subsonic Wind Tunnel.
By Ben FrantzDale (2007).


http://autospeed.com/cms/A_108677/article.html

wool tufts


http://autospeed.com/cms/A_108677/article.html

smoke injection



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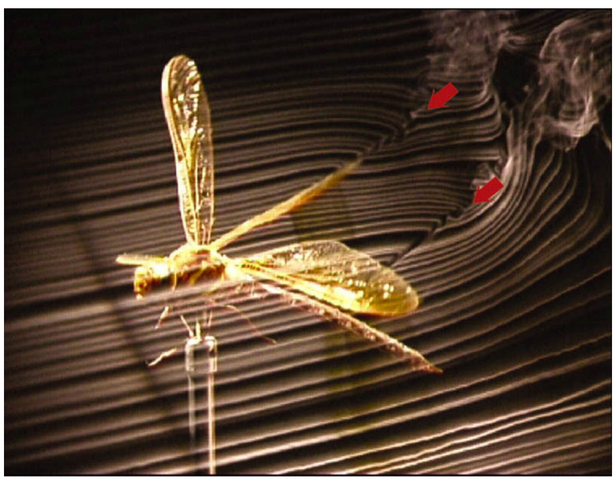
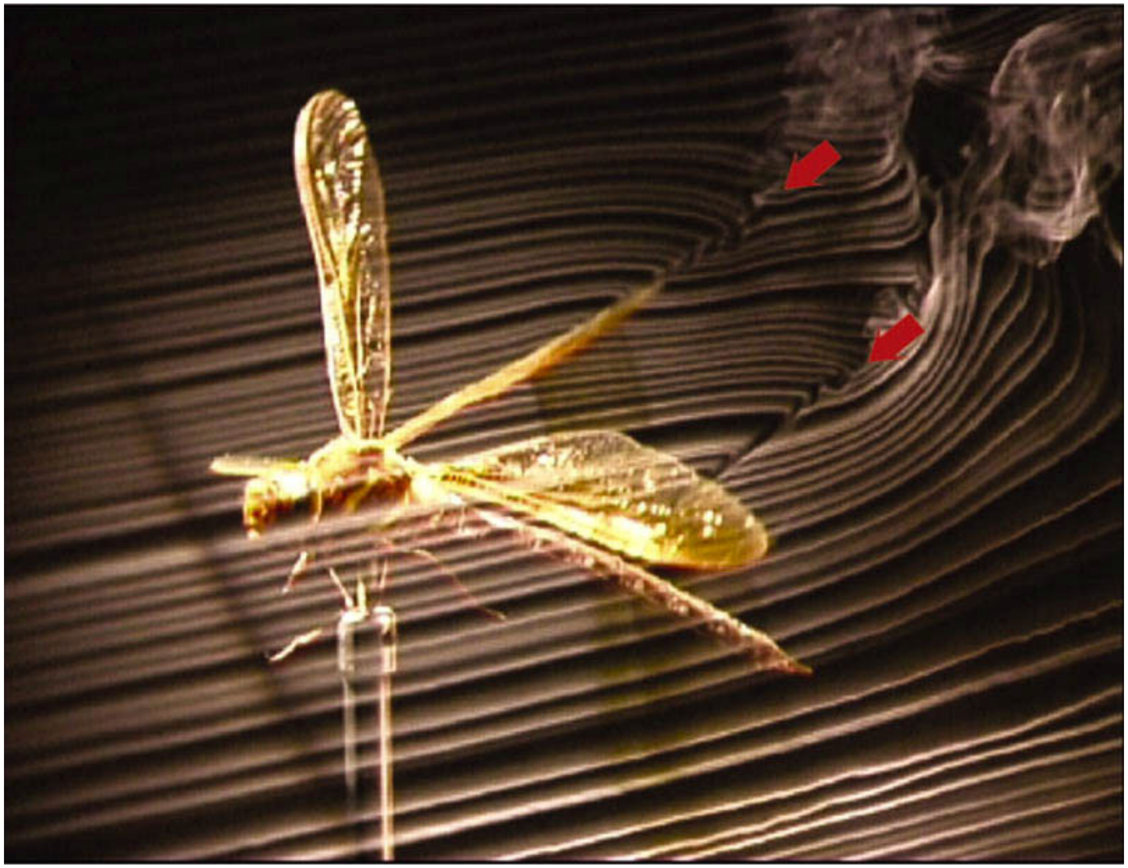


[NASA, J. Exp. Biol.]

http://autospeed.com/cms/A_108677/article.html smoke nozzles

Smoke injection

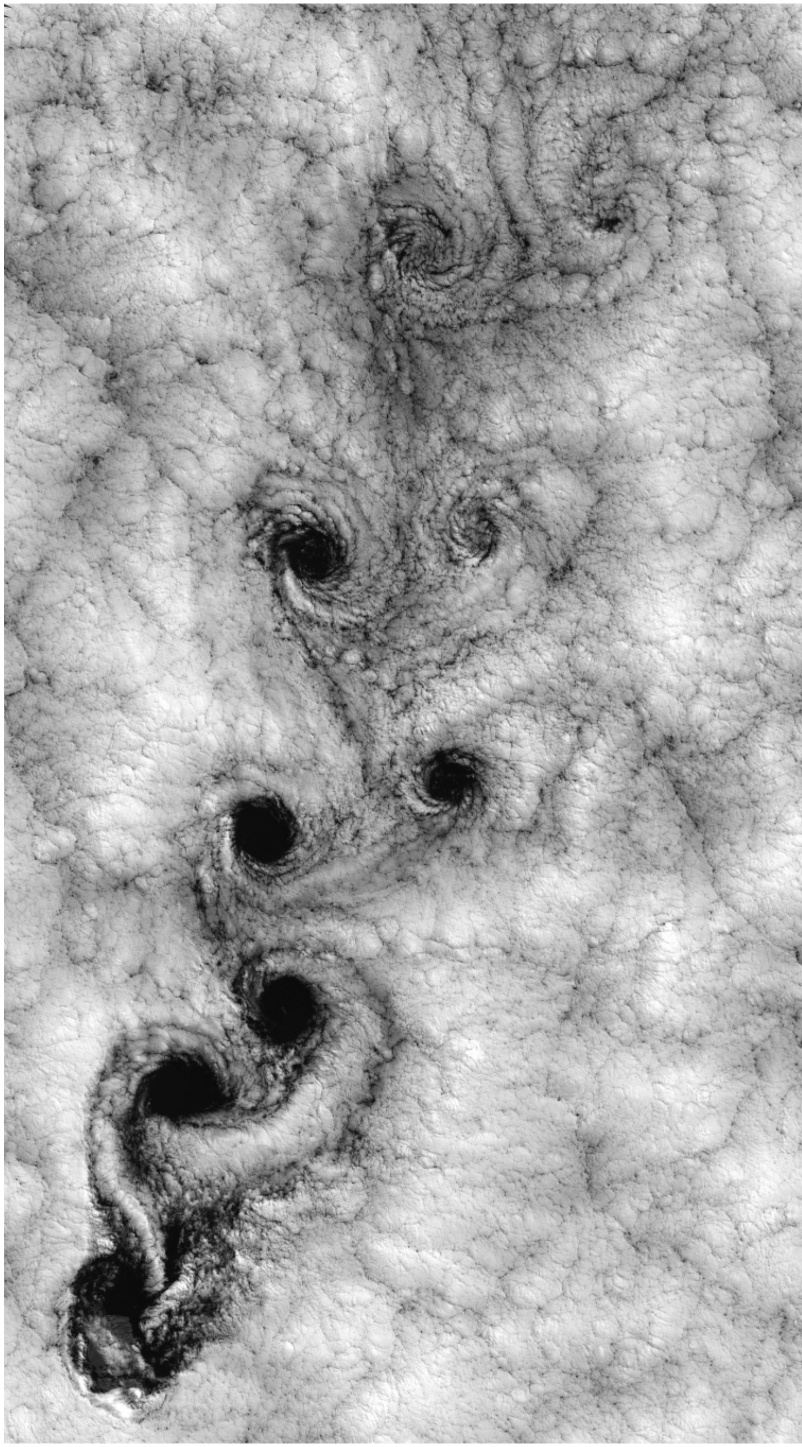


A. L. R. Thomas, G. K. Taylor, R. B. Srygley, R. L. Nudds, and R. J. Bomphrey. Dragonfly flight: free-flight and tethered flow visualizations reveal a diverse array of unsteady lift-generating mechanisms, controlled primarily via angle of attack. J Exp Biol, 207(24):4299–4323, 2004.

http://de.wikipedia.org/wiki/Bild:Airplane_vortex_edit.jpg

Smoke injection

http://www-me.ccny.cuny.edu/research/aerolab/facilities/images/wt2.jpg



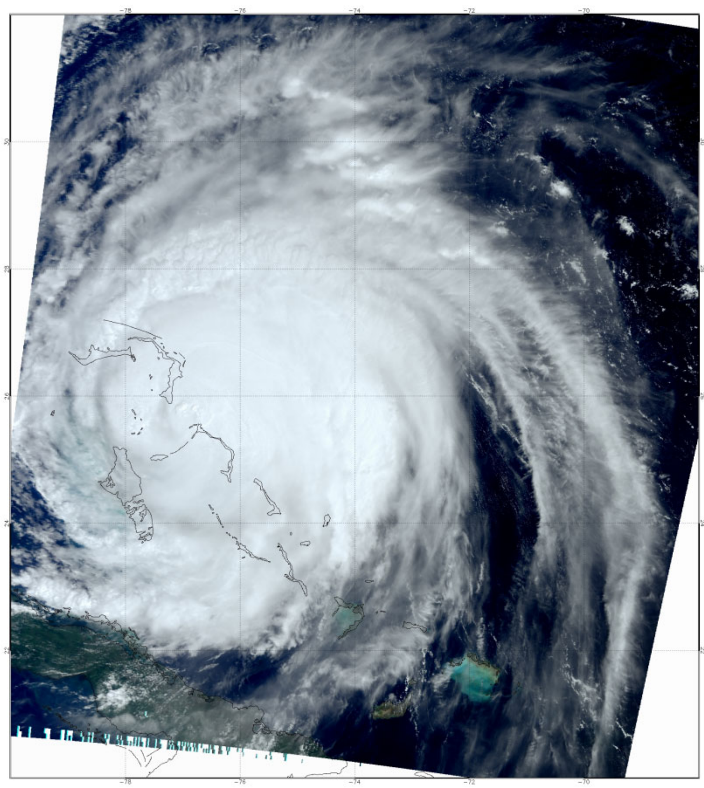
Clouds (satellite image)

Juan Fernandez Islands

http://de.wikipedia.org/wiki/Bild:Vortex-street-1.jpg

Clouds (satellite image)

http://daac.gsfc.nasa.gov/gallery/frances/

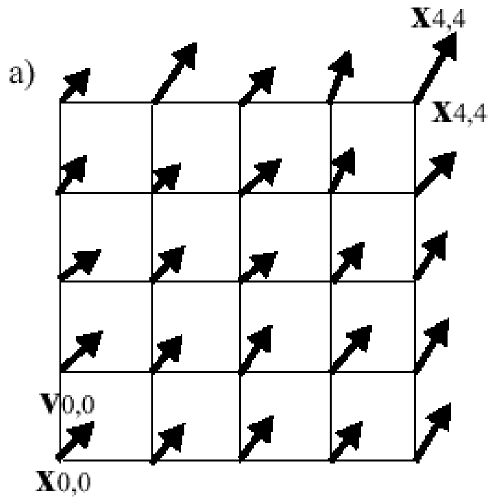


- **Vortex/ Vortex core lines**
  - There is no exact definition of vortices
  - capturing some swirling behavior

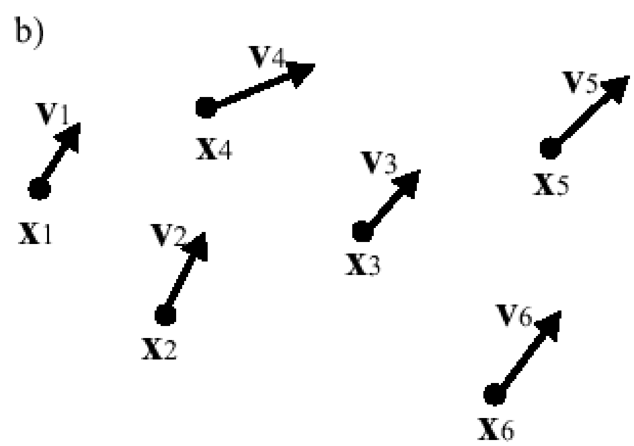
Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)



vectors given at grid points



vectors given at particle positions

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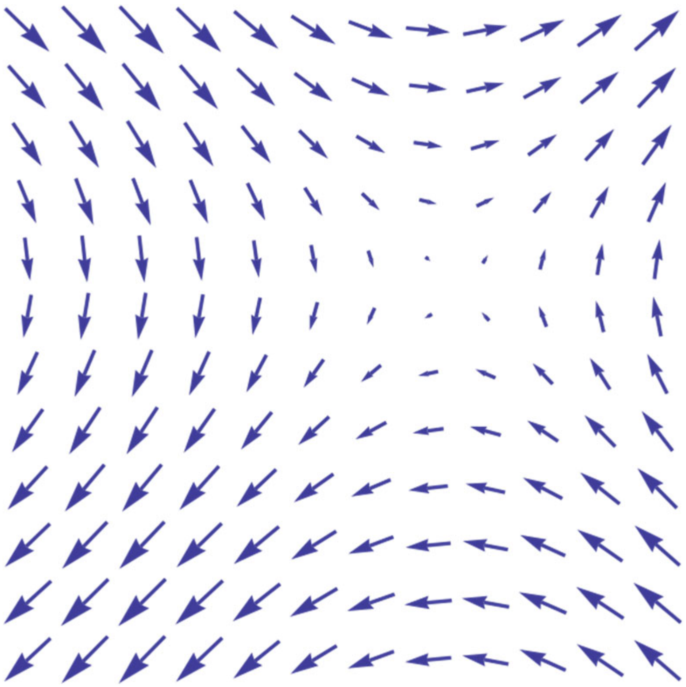
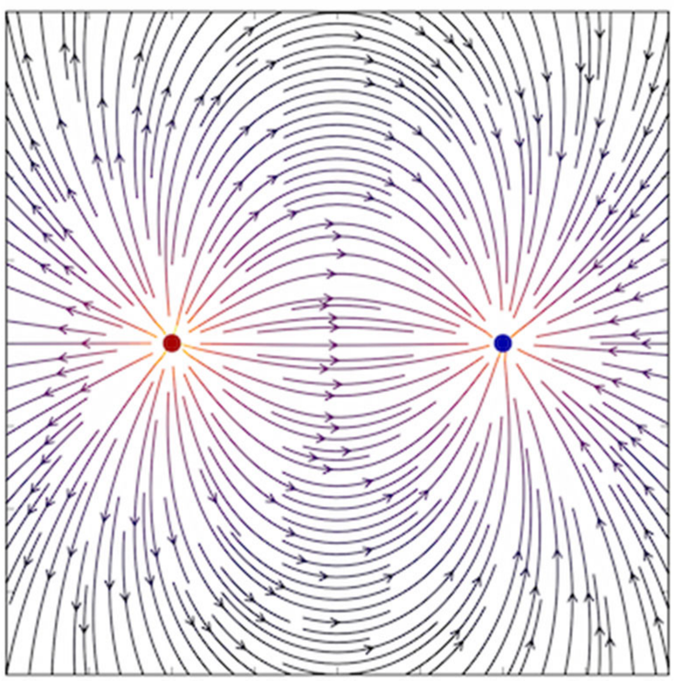
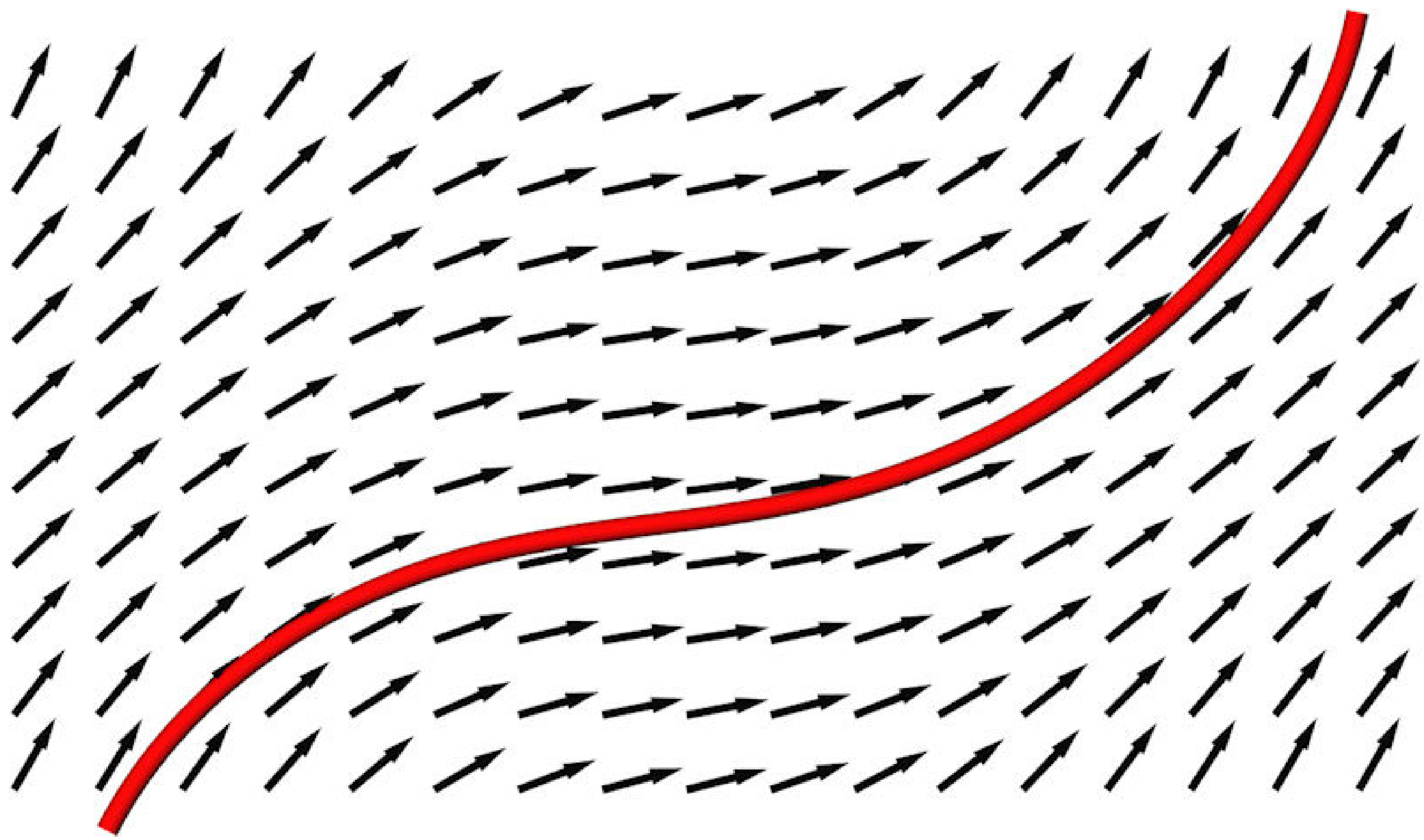




images from wikipedia

Integral Curves / Stream Objects

Integrating velocity over time yields spatial motion



Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
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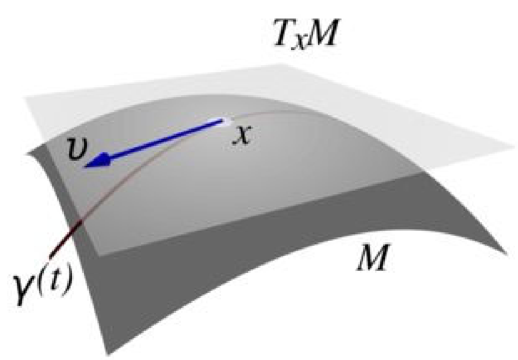
Each vector in a vector field
lives in the **tangent space**
of the manifold at that point:

Each vector is a **tangent vector**



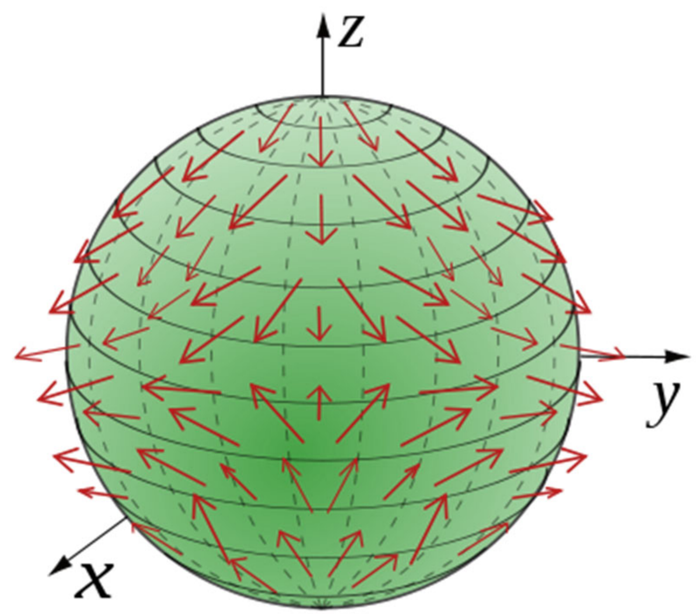


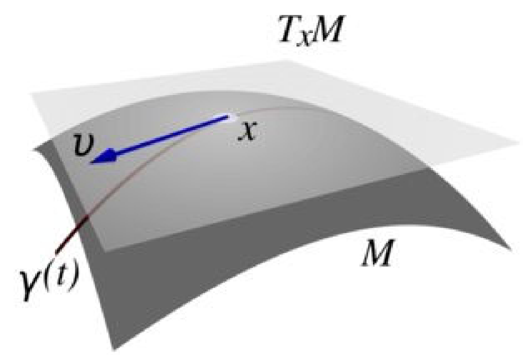
image from wikipedia

Vector Fields

Vector fields on general manifolds M (not just Euclidean space)

Tangent space at a point $x \in M$:

$$T_xM$$

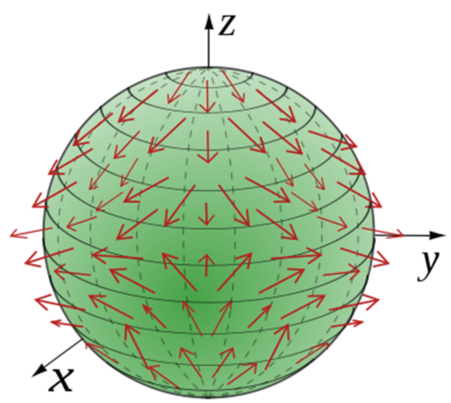


Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \to M$$

Vector field: *Section* of tangent bundle

$$s : M \to TM, \quad x \mapsto s(x). \qquad \pi\big(s(x)\big) = x$$



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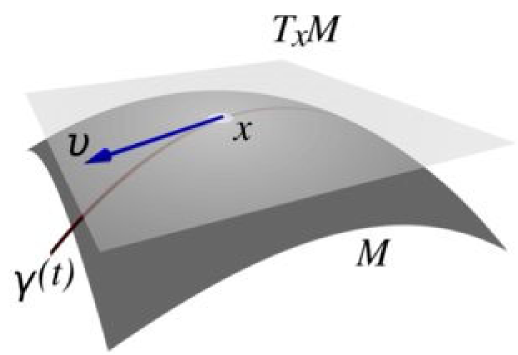

Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \to M$$

Vector field: *Section* of tangent bundle

$$\mathbf{v} : M \to TM,$$
$$x \mapsto \mathbf{v}(x).$$

$$\mathbf{v}(x) \in T_x M$$

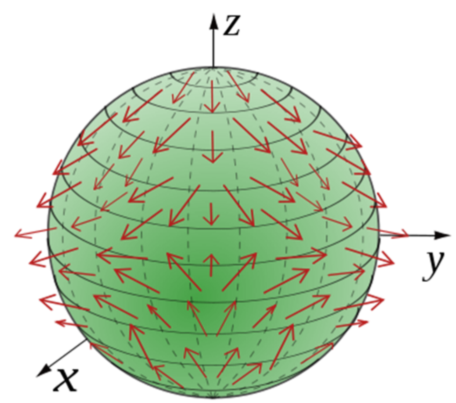

image from wikipedia

Interlude: Coordinate Charts

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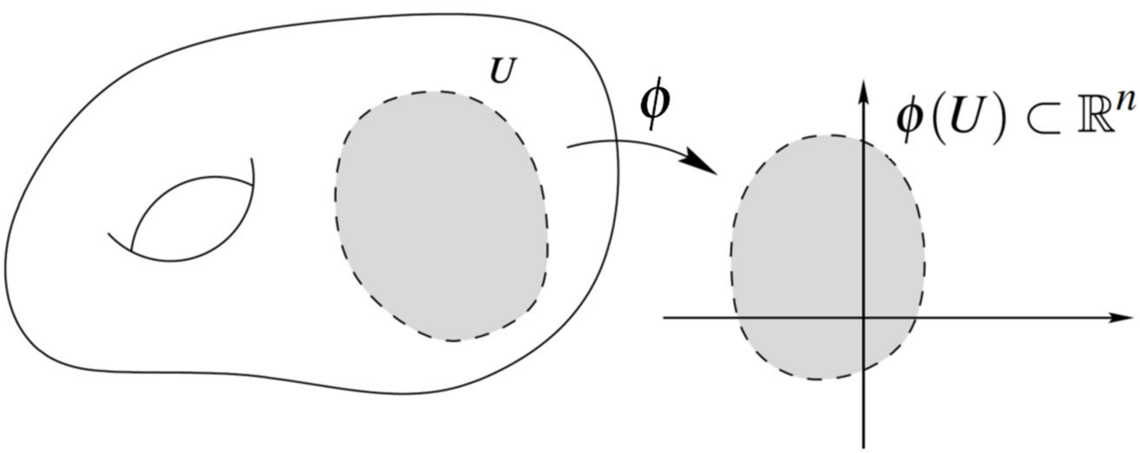
Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$



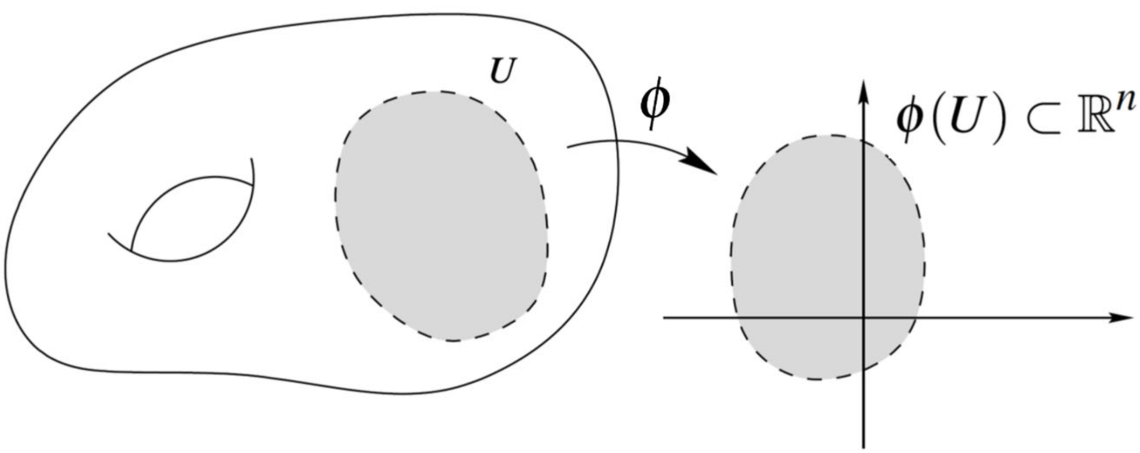
Interlude: Coordinate Charts

Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Coordinate functions

$$x^i : U \subset M \to \mathbb{R},$$
$$x \mapsto x^i(x).$$



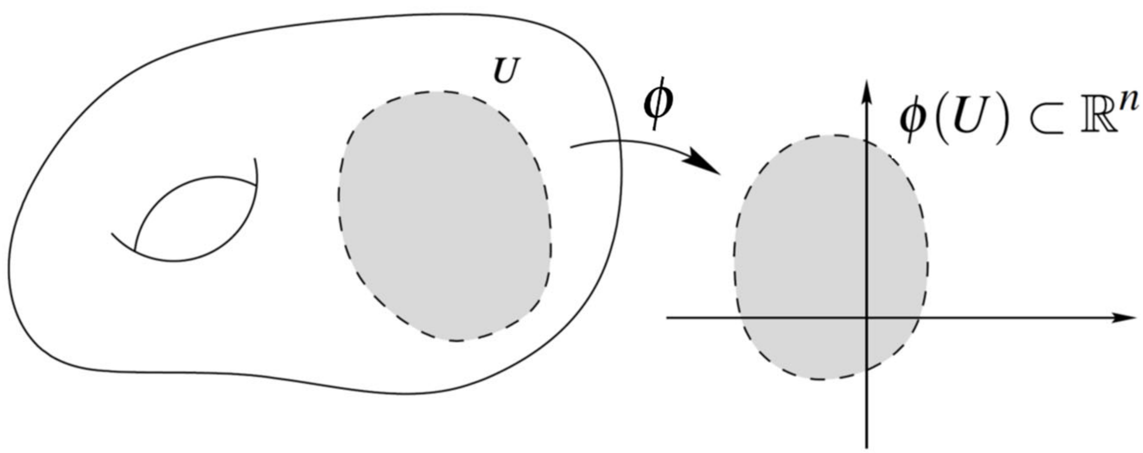
Interlude: Coordinate Charts

Coordinate charts

$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Atlas

$$\{(U_\alpha, \phi_\alpha)\}_{\alpha \in I}$$



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Atlas

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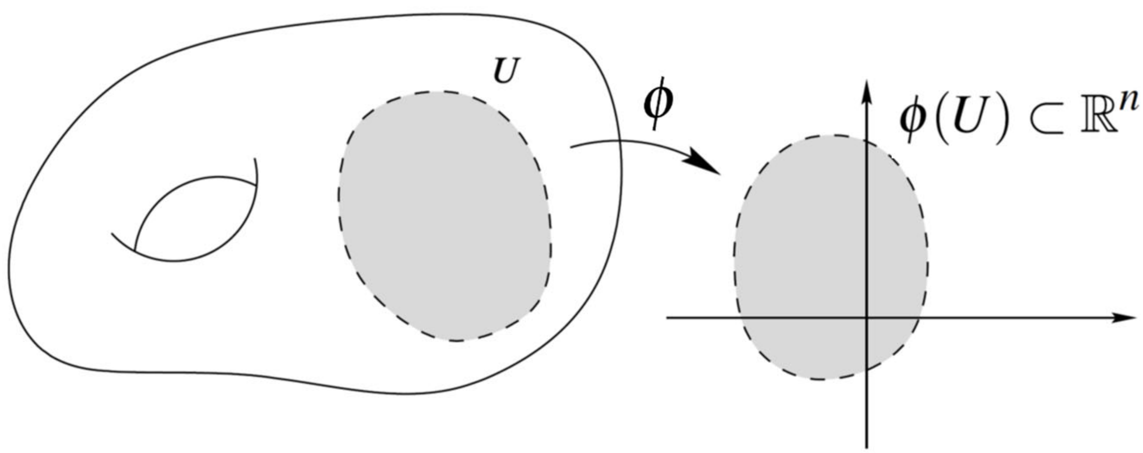

Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\begin{aligned} \mathbf{v} : U \subset \mathbb{R}^2 &\to \mathbb{R}^2, \\ (x, y) &\mapsto \begin{bmatrix} u \\ v \end{bmatrix}. \end{aligned}$$

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$$\mathbf{v} : U \subset \mathbb{R}^3 \to \mathbb{R}^3,$$
$$(x, y, z) \mapsto \begin{bmatrix} u \\ v \\ w \end{bmatrix}.$$

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Vector Fields vs. Vectors in Components

$$\mathbf{v} : U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

$$\mathbf{v} : U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{pmatrix} v^1(x^1, x^2, \ldots, x^n) \\ v^2(x^1, x^2, \ldots, x^n) \\ \vdots \\ v^n(x^1, x^2, \ldots, x^n) \end{pmatrix}.$$

Vector Fields vs. Vectors in Components

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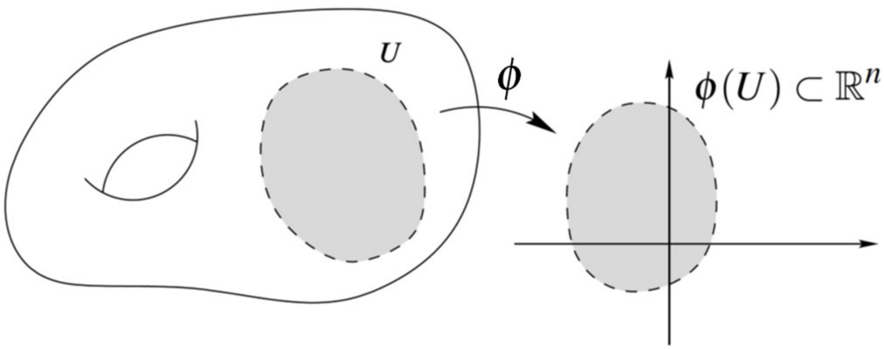

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$$\mathbf{v}\big|_U : \phi(U) \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\begin{aligned} \mathbf{e}_i : U \subset M \to TM, \\ x \mapsto \mathbf{e}_i(x) \end{aligned} \qquad \{\mathbf{e}_i(x)\}_{i=1}^{n} \quad \text{basis for } T_xM$$

Vector Fields vs. Vectors in Components

Need basis vector fields

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$$\begin{aligned} \mathbf{v} : U \subset M &\to TM, \\ x &\mapsto v^1 \mathbf{e}_1 + v^2 \mathbf{e}_2 + \ldots + v^n \mathbf{e}_n. \end{aligned}$$

$$\begin{aligned} \mathbf{v} : U \subset M &\to TM, \\ x &\mapsto v^1(x)\, \mathbf{e}_1(x) + v^2(x)\, \mathbf{e}_2(x) + \ldots + v^n(x)\, \mathbf{e}_n(x). \end{aligned}$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \quad x \mapsto \mathbf{e}_i(x)$$

$$\{\mathbf{e}_i(x)\}_{i=1}^n \quad \text{basis for } T_xM$$

Coordinate basis:

$$\mathbf{e}_i := \frac{\partial}{\partial x^i}$$

$$\mathbf{v} : U \subset M \to TM, \quad x \mapsto v^1\mathbf{e}_1 + v^2\mathbf{e}_2 + \ldots + v^n\mathbf{e}_n.$$

$$\mathbf{v} : U \subset M \to TM, \quad x \mapsto v^1(x)\,\mathbf{e}_1(x) + v^2(x)\,\mathbf{e}_2(x) + \ldots + v^n(x)\,\mathbf{e}_n(x).$$

Examples of Coordinate Curves and Bases

Coordinate functions, coordinate curves, bases

- Coordinate functions are real-valued ("scalar") functions on the domain
- On each coordinate curve, *one* coordinate changes, *all others stay constant*
- Basis: n linearly independent vectors at each point of domain

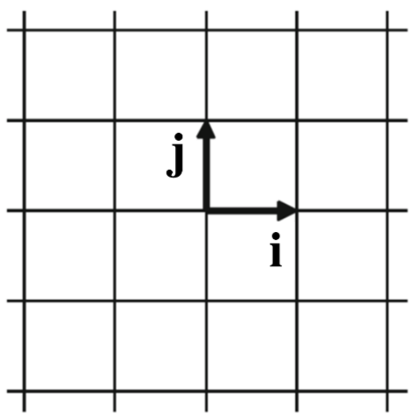
Cartesian coordinates

$$x^1 = x$$

$$x^2 = y$$

$$\mathbf{e}_1 = \frac{\partial}{\partial x} = \mathbf{i}$$

$$\mathbf{e}_2 = \frac{\partial}{\partial y} = \mathbf{j}$$



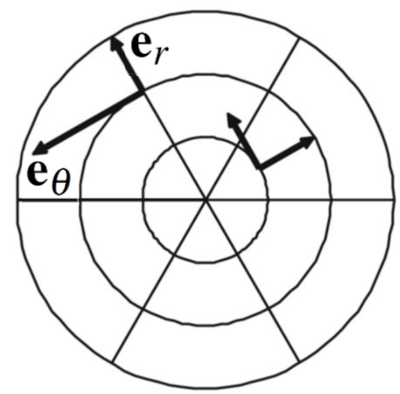
polar coordinates

$$x^1 = r$$

$$x^2 = \theta$$

$$\mathbf{e}_1 = \frac{\partial}{\partial r} = \mathbf{e}_r$$

$$\mathbf{e}_2 = \frac{\partial}{\partial \theta} = \mathbf{e}_\theta$$



Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama