CS 247 – Scientific Visualization
Lecture 15: Volume Rendering, Pt. 2

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Reading Assignment #8 (until Mar 21)

Read (required):

- Real-Time Volume Graphics, Chapter 1
  \textit{(Theoretical Background and Basic Approaches)},
  from beginning to 1.4.4 (inclusive)

- Real-Time Volume Graphics, Chapter 4 (Transfer Functions)
  until Sec. 4.4 (inclusive)

- Look at:
  \textit{Nelson Max, Optical Models for Direct Volume Rendering,}
  \textit{IEEE Transactions on Visualization and Computer Graphics, 1995}
  \url{http://dx.doi.org/10.1109/2945.468400}
Quiz #2: Mar 23

Organization

• First 30 min of lecture
• No material (book, notes, ...) allowed

Content of questions

• Lectures (both actual lectures and slides)
• Reading assignments (except optional ones)
• Programming assignments (algorithms, methods)
• Solve short practical examples
Volume Visualization
VolVis: Theory
Direct Volume Rendering
Direct Volume Rendering
**Image order approach:**

For each pixel {
  calculate color of the pixel
}
Object order approach:

For each slice {
  calculate contribution to the image
}
Physical Model of Radiative Transfer

Increase

true emission

Decrease

true absorption

in-scattering

out-scattering
Volume rendering integral for \textit{Emission Absorption} model

\[ I(s) = I(s_0) e^{-\tau(s_0, s)} + \int_{s_0}^{s} q(\tilde{s}) e^{-\tau(\tilde{s}, s)} \, d\tilde{s} \]

Numerical solutions:

\textbf{Back-to-front compositing}

\[ C'_i = C_i + (1 - A_i)C'_{i-1} \]

\textbf{Front-to-back compositing}

\[ C'_i = C'_{i+1} + (1 - A'_{i+1})C_i \]

\[ A'_i = A'_{i+1} + (1 - A'_{i+1})A_i \]

here, all colors are associated colors!
How do we determine the radiant energy along the ray?

*Physical model:* emission and absorption, no scattering

\[ I(s) = I(s_0) \]
How do we determine the radiant energy along the ray?

**Physical model:** emission and absorption, no scattering

Without absorption all the initial radiant energy would reach the point $s$. 

$$I(s) = I(s_0)$$

Initial intensity at $s_0$
How do we determine the radiant energy along the ray?

*Physical model:* emission and absorption, no scattering

\[
I(s) = I(s_0) e^{-\tau(s_0, s)}
\]

Absorption along the ray segment \( s_0 - s \)
Volume Rendering Integral

How do we determine the radiant energy along the ray?

*Physical model:* emission and absorption, no scattering

\[
I(s) = I(s_0) e^{-\tau(s_0, s)}
\]

**Optical depth** $\tau$

**Absorption** $\kappa$

\[
\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s) \, ds.
\]
How do we determine the radiant energy along the ray?

**Physical model:** emission and absorption, no scattering

One point \( \tilde{s} \) along the viewing ray emits additional radiant energy.

\[
I(s) = I(s_0) e^{-\tau(s_0, s)} + q(\tilde{s})
\]
How do we determine the radiant energy along the ray?

**Physical model:** emission and absorption, no scattering

Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

$$I(s) = I(s_0) e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s}) e^{-\tau(\tilde{s},s)} d\tilde{s}$$
Optical depth: $\tau(0, t) = \int_0^t \kappa(t) \, dt$
Volume Rendering Integral: Numerical Solution

**Optical depth:** $\tau(0, t) = \int_0^t \kappa(t) \, dt$

**Approximate Integral by Riemann sum:**

$$\tau(0, t) \approx \sum_{i=0}^{\left[\frac{t}{\Delta t}\right]} \kappa(i \cdot \Delta t) \Delta t$$
\[
\tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t
\]
Volume Rendering Integral: Numerical Solution

\[ \tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t \]

\[ e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t} \]
\[ \tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t \]

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} e^{-\kappa(i \cdot \Delta t) \Delta t} \]
Volume Rendering Integral: Numerical Solution

Now we introduce opacity:

\[
\tilde{\tau}(0, t) \approx \tau(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t
\]

\[
e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} e^{-\kappa(i \cdot \Delta t) \Delta t}
\]

Now we introduce opacity:

\[
A_i = 1 - e^{-\kappa(i \cdot \Delta t) \Delta t}
\]
Volume Rendering Integral: Numerical Solution

Now we introduce \( \text{opacity} \):

\[
\tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t
\]

\[
e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} e^{-\kappa(i \cdot \Delta t) \Delta t}
\]

Now we introduce \textit{opacity}:

\[
1 - A_i = e^{-\kappa(i \cdot \Delta t) \Delta t}
\]
Volume Rendering Integral: Numerical Solution

\[ \tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t \]

\[ e^{-\tilde{\tau}(0, t)} = \prod_{i=0}^{[t/\Delta t]} e^{-\kappa(i \cdot \Delta t) \Delta t} \]

Now we introduce opacity:

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Volume Rendering Integral: Numerical Solution

\[ \tau(0, t) \approx \tilde{\tau}(0, t) = \sum_{i=0}^{[t/\Delta t]} \kappa(i \cdot \Delta t) \Delta t \]

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

Now we introduce opacity:

\[ 1 - A_i = e^{-\kappa(i \cdot \Delta t) \Delta t} \]
Volume Rendering Integral: Numerical Solution

$q(t)$
Volume Rendering Integral: Numerical Solution

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

\[ q(t) \approx C_i = c(i \cdot \Delta t) \Delta t \]
Volume Rendering Integral: Numerical Solution

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

\[ q(t) \approx C_i = c(i \cdot \Delta t) \Delta t \]

\[ \tilde{C} = \sum_{i=0}^{[T/\Delta t]} C_i e^{-\tilde{\tau}(0,t)} \]
Volume Rendering Integral: Numerical Solution

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

\[ q(t) \approx C_i = c(i \cdot \Delta t) \Delta t \]

\[ \tilde{C} = \sum_{i=0}^{[T/\Delta t]} C_i e^{-\tilde{\tau}(0,t)} \]
Volume Rendering Integral: Numerical Solution

\[ e^{-\tau(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

\[ q(t) \approx C_i = c(i \cdot \Delta t) \Delta t \]

\[ \tilde{C} = \sum_{i=0}^{[T/\Delta t]} C_i \prod_{j=0}^{i-1} (1 - A_j) \]
Volume Rendering Integral: Numerical Solution

\[ e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{[t/\Delta t]} (1 - A_i) \]

\[ q(t) \approx C_i = c(i \cdot \Delta t) \Delta t \]

\[ \tilde{C} = \sum_{i=0}^{[T/\Delta t]} C_i \prod_{j=0}^{i-1} (1 - A_j) \]

can be computed recursively/iteratively!
Volume Rendering Integral: Numerical Solution

\[ q(t) \]

\[ i=0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots \]

\[ \Delta t \]

Note: we just changed the convention from \( i=0 \) is at the front of the volume (previous slides) to \( i=0 \) is at the back of the volume!

can be computed recursively/iteratively:

\[
C'_{i} = C_{i} + (1 - A_{i}) C'_{i-1}
\]

- Radiant energy observed at position \( i \)
- Radiant energy emitted at position \( i \)
- Absorption at position \( i \)
- Radiant energy observed at position \( i-1 \)
Volume Rendering Integral: Numerical Solution

Back-to-front compositing

\[ C'_i = C_i + (1 - A_i)C'_{i-1} \]

Iterate from \( i=0 \) (back) to \( i=\text{max} \) (front): \( i \) increases

Front-to-back compositing

\[ C'_i = C''_{i+1} + (1 - A'_{i+1})C_i \]
\[ A'_i = A'_{i+1} + (1 - A'_{i+1})A_i \]

Iterate from \( i=\text{max} \) (front) to \( i=0 \) (back): \( i \) decreases
Volume Rendering Integral: Numerical Solution

**Back-to-front compositing**

\[ C'_i = C_i + (1 - A'_i) C_i \]

Iterate from \( i = 0 \) (back) to \( i = \text{max} \) (front): \( i \) decreases

**Front-to-back compositing**

\[ C'_i = C''_{i+1} + (1 - A'_i) C_i \]

\[ A'_i = A'_{i+1} + (1 - A'_i) A_i \]

Iterate from \( i = \text{max} \) (front) to \( i = 0 \) (back): \( i \) decreases

**Early Ray Termination:**

Stop the calculation when

\[ A'_i \approx 1 \]
Volume rendering integral for *Emission Absorption* model

\[ I(s) = I(s_0) e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s}) e^{-\tau(\tilde{s},s)} \, d\tilde{s} \]

Numerical solutions:

**Back-to-front compositing**

\[ C'_i = C_i + (1 - A_i)C'_{i-1} \]

**Front-to-back compositing**

\[ C'_i = C'_{i+1} + (1 - A'_{i+1})C_i \]

\[ A'_i = A'_{i+1} + (1 - A'_{i+1})A_i \]

here, all colors are associated colors!
VolVis: Opacity Correction

[preview]
Simple compositing only works as far as the opacity values are correct... and they depend on the sample distance!

\[ T_i = e^{-\int_{s_i}^{s_i+\Delta t} \kappa(t) \, dt} \approx e^{-\kappa(s_i) \Delta t} = e^{-\kappa_i \Delta t} \]

\[ A_i = 1 - e^{-\kappa_i \Delta t} \]

\[ \tilde{T}_i = T_i \left( \frac{\Delta \tilde{t}}{\Delta t} \right) \]

\[ \tilde{A}_i = 1 - (1 - A_i) \left( \frac{\Delta \tilde{t}}{\Delta t} \right) \]

opacity correction formula

Beware that usually this is done for each different scalar value (every transfer function entry), not actually at spatial positions/intervals \( i \)
Associated Colors

Associated (or “opacity-weighted” colors) are often used in compositing equations.

Every color is *pre-multiplied* by its corresponding opacity.

\[
\begin{pmatrix}
R \\
G \\
B \\
A
\end{pmatrix} 
\rightarrow 
\begin{pmatrix}
R*A \\
G*A \\
B*A \\
A
\end{pmatrix}
\]

Our compositing equations assume associated colors!

**Important:** After opacity-correction, all associated colors must be updated! (or combined/multiplied correctly on-the-fly!)
Associated Colors in Volume Rendering

Standard emission-absorption optical model

- Only one kind of particle: the same particles that absorb light, emit light
- Aha! Therefore lower absorption means lower emission as well

Light observed from (in front of) segment $i$ (without any light behind it):

$$C_i = \frac{q_i}{\kappa_i} \left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i A_i$$

$$q_i := \hat{C}_i \kappa_i$$

$$A_i := 1 - e^{-\kappa_i \Delta t}$$

$$\lim_{\kappa_i \to 0} q_i \frac{1 - e^{-\kappa_i \Delta t}}{\kappa_i} = \lim_{\kappa_i \to 0} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = 0$$

$$\lim_{\kappa_i \to \infty} q_i \frac{1 - e^{-\kappa_i \Delta t}}{\kappa_i} = \lim_{\kappa_i \to \infty} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i$$
Associated Colors in Volume Rendering

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$$A_i := 1 - e^{-\kappa_i \Delta t}$$

$$\lim_{\kappa_i \to 0} q_i \left( 1 - e^{-\kappa_i \Delta t} \right) = \lim_{\kappa_i \to 0} \hat{C}_i \left( 1 - e^{-\kappa_i \Delta t} \right) = 0$$

$$\lim_{\kappa_i \to \infty} q_i \left( 1 - e^{-\kappa_i \Delta t} \right) = \lim_{\kappa_i \to \infty} \hat{C}_i \left( 1 - e^{-\kappa_i \Delta t} \right) = \hat{C}_i$$
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Associated Colors in Volume Rendering

Standard emission-absorption optical model

- Only one kind of particle: the same particles that absorb light, emit light
- Aha! Therefore lower absorption means lower emission as well

Light observed from (in front of) segment $i$ (without any light behind it):

$$C_i = \frac{q_i}{k_i} \left( 1 - e^{-k_i \Delta t} \right) = \hat{C}_i A_i$$

$q_i := \hat{C}_i k_i$

$A_i := 1 - e^{-k_i \Delta t}$

$$\lim_{k_i \to 0} \frac{q_i (1 - e^{-k_i \Delta t})}{k_i} = \lim_{k_i \to 0} \hat{C}_i \left( 1 - e^{-k_i \Delta t} \right) = 0$$

$$\lim_{k_i \to \infty} \frac{q_i (1 - e^{-k_i \Delta t})}{k_i} = \lim_{k_i \to \infty} \hat{C}_i \left( 1 - e^{-k_i \Delta t} \right) = C_i$$
Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama