CS 247 – Scientific Visualization
Lecture 9: Scalar Fields, Pt. 5

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Reading Assignment #5 (until Feb 28)

Read (required):

- Gradients of scalar-valued functions

  https://en.wikipedia.org/wiki/Gradient

- Critical points

  https://en.wikipedia.org/wiki/Critical_point_(mathematics)

- Multivariable derivatives and differentials

  https://en.wikipedia.org/wiki/Total_derivative

  https://en.wikipedia.org/wiki/Differential_of_a_function#Differentials_in_several_variables

  https://en.wikipedia.org/wiki/Hessian_matrix

- Dot product, inner product (more general)

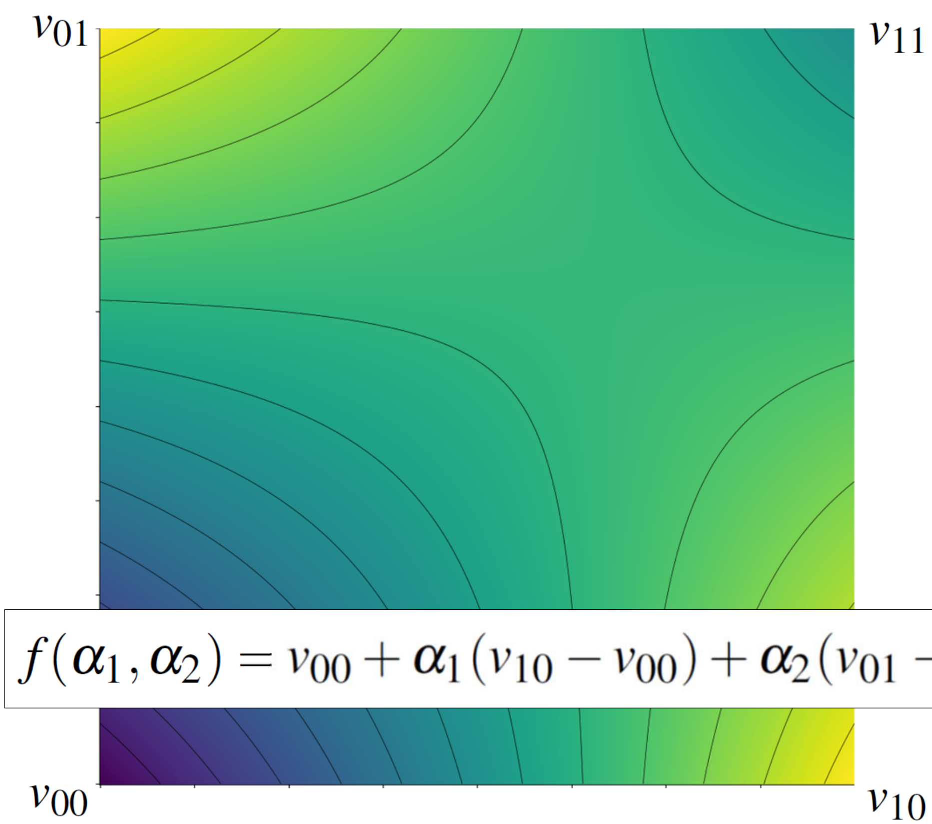
  https://en.wikipedia.org/wiki/Dot_product

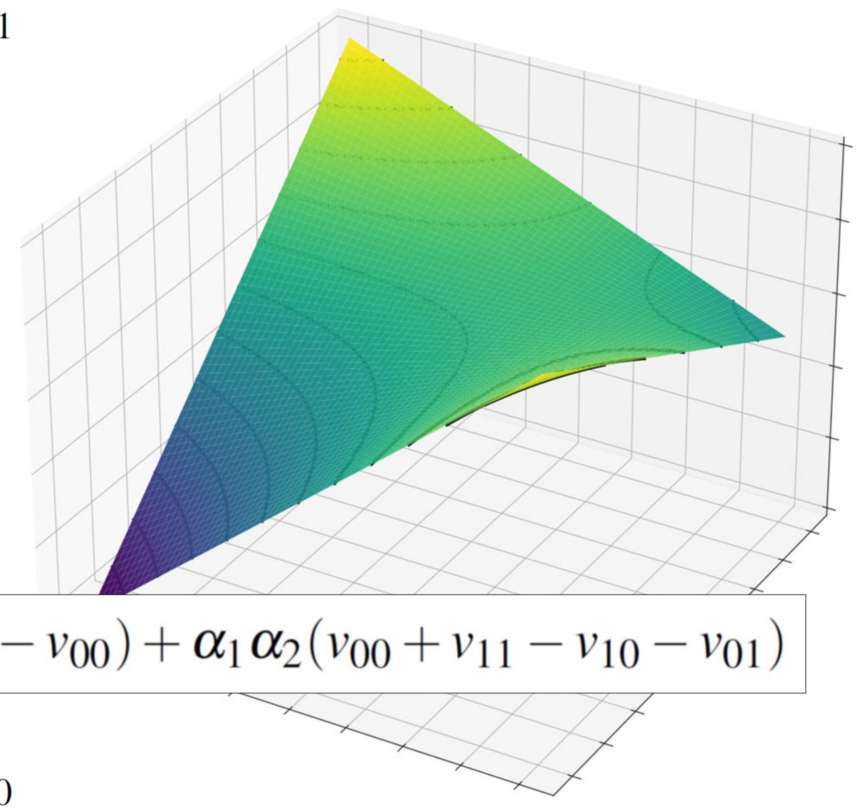
  https://en.wikipedia.org/wiki/Inner_product_space

Bi-Linear Interpolation

Consider area between 2x2 adjacent samples

Example: 1.0 at top-left and bottom-right, 0.0 at bottom-left, 0.5 at top-right

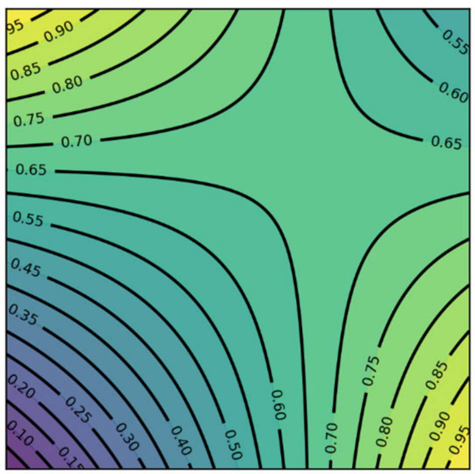


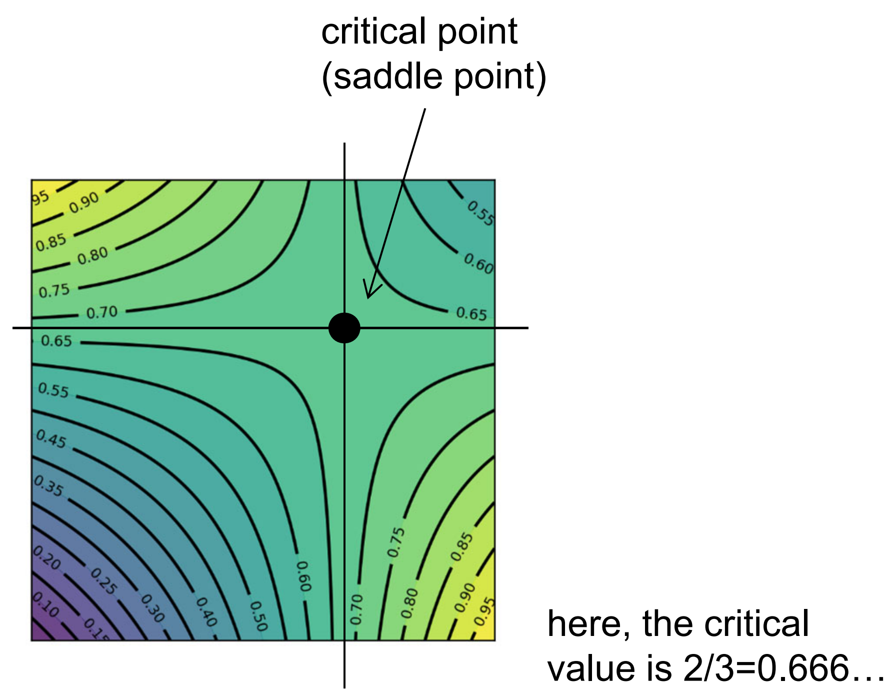


$$f(\alpha_1, \alpha_2) = v_{00} + \alpha_1 (v_{10} - v_{00}) + \alpha_2 (v_{01} - v_{00}) + \alpha_1 \alpha_2 (v_{00} + v_{11} - v_{10} - v_{01})$$

Bi-Linear Interpolation: Critical Points

Critical points are where the gradient vanishes (i.e., is the zero vector)





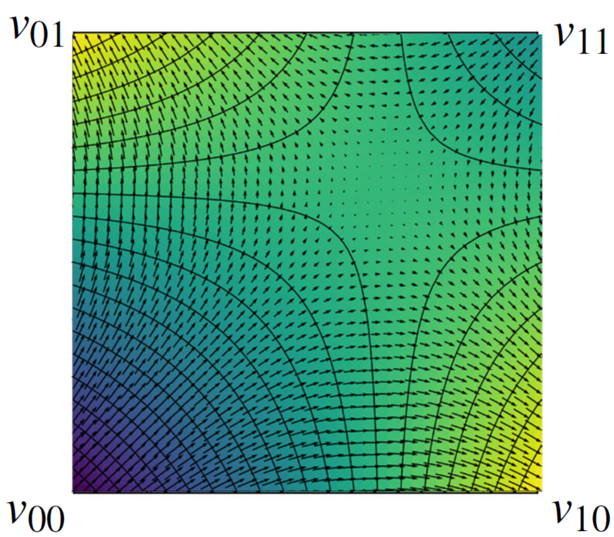
"Asymptotic decider": resolve ambiguous configurations (6 and 9) by comparing specific iso-value with critical value (scalar value at critical point)

Preview: Critical Point and Value (Details Later)

Compute gradient (critical points are where gradient is zero vector):

$$\frac{\partial f(\alpha_1,\alpha_2)}{\partial \alpha_1} = (v_{10} - v_{00}) + \alpha_2 (v_{00} + v_{11} - v_{10} - v_{01})$$

$$\frac{\partial f(\alpha_1,\alpha_2)}{\partial \alpha_2} = (v_{01} - v_{00}) + \alpha_1 (v_{00} + v_{11} - v_{10} - v_{01})$$

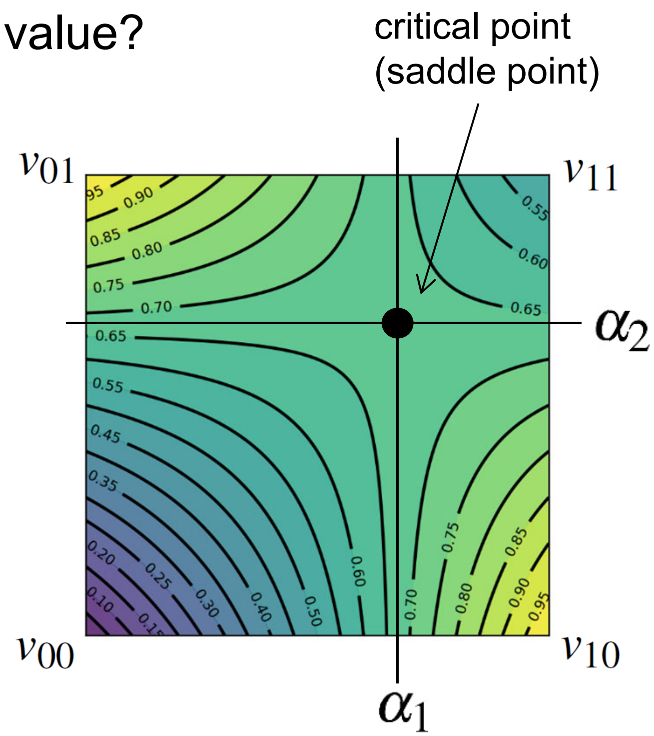


Preview: Critical Point and Value (Details Later)

Where is the critical point, and what is the critical value?

$$\frac{\partial f(\alpha_1,\alpha_2)}{\partial \alpha_1} = 0: \qquad \alpha_2 = \frac{v_{00} - v_{10}}{v_{00} + v_{11} - v_{10} - v_{01}}$$

$$\frac{\partial f(\alpha_1,\alpha_2)}{\partial \alpha_2} = 0: \qquad \alpha_1 = \frac{v_{00} - v_{01}}{v_{00} + v_{11} - v_{10} - v_{01}}$$

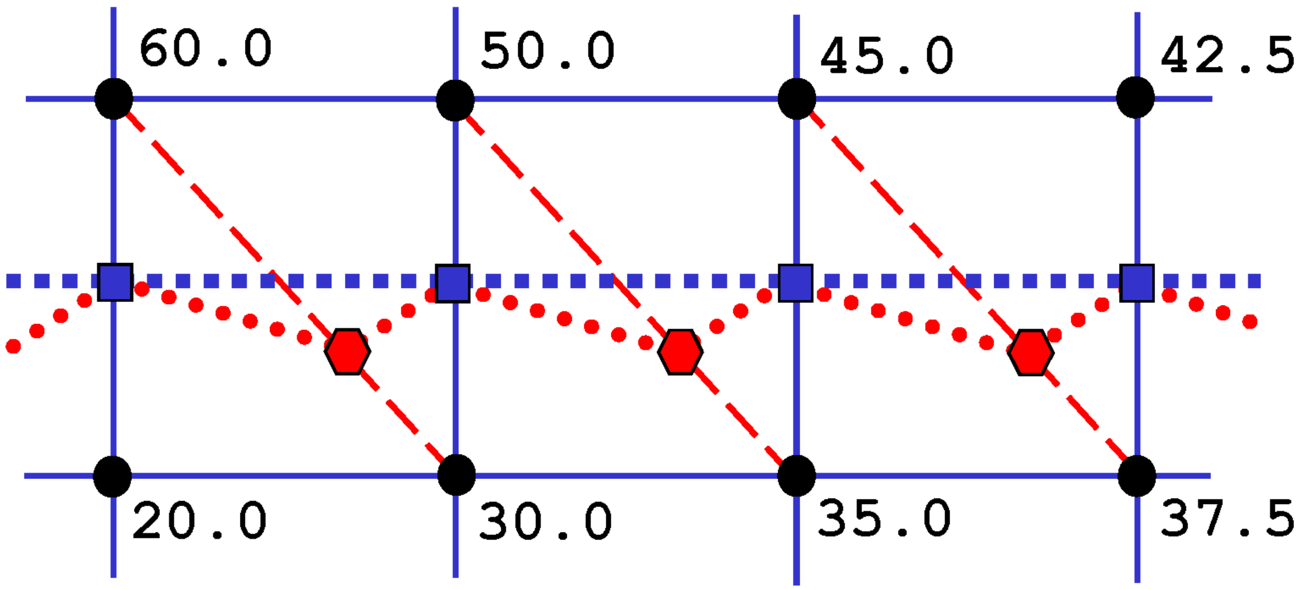


$$f(\alpha_1,\alpha_2) = v_{00} + \alpha_1(v_{10} - v_{00}) + \alpha_2(v_{01} - v_{00}) + \alpha_1\alpha_2(v_{00} + v_{11} - v_{10} - v_{01})$$

Contours in triangle/tetrahedral cells

Illustrative example: Find contour at level c=40.0 !

(note that in each cell the average value of the four vertices is also 40.0; in bi-linear interpolation this is the value of the cell's center point)



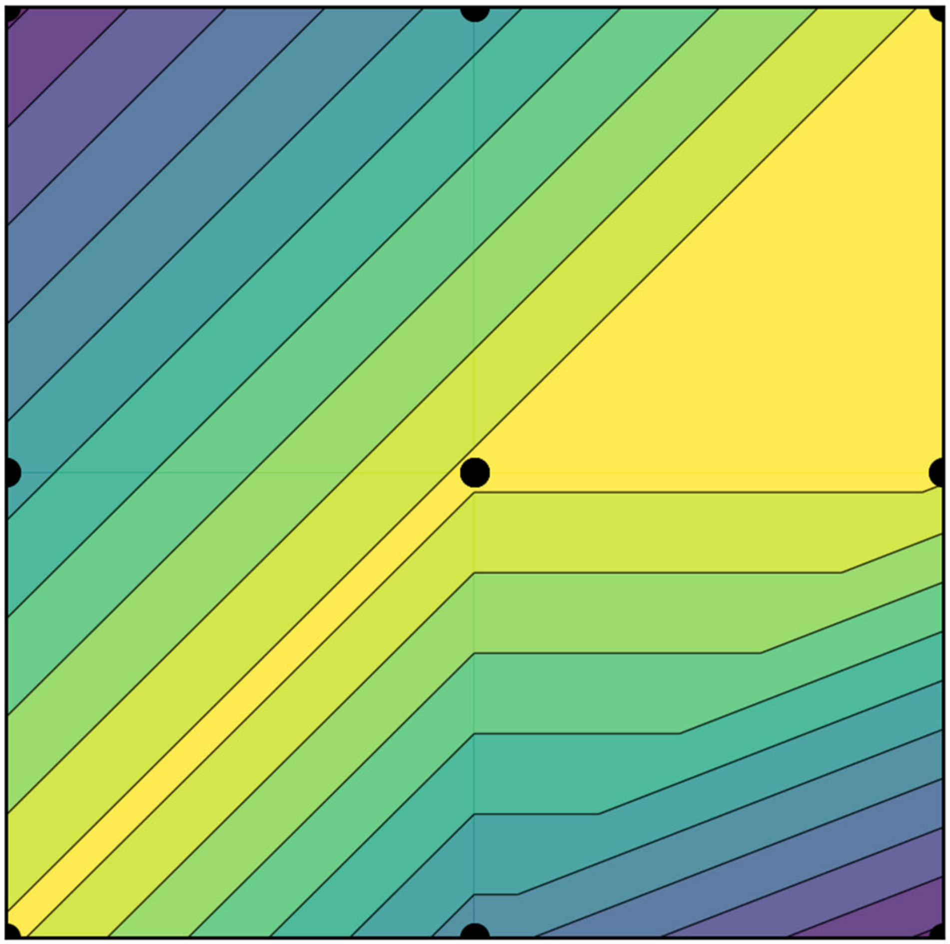
original quad grid, yielding vertices ■ and contour ▪▪▪▪▪▪

triangulated grid, yielding vertices ⬣ and contour ••••••

Bi-Linear Interpolation: Comparisons

linear

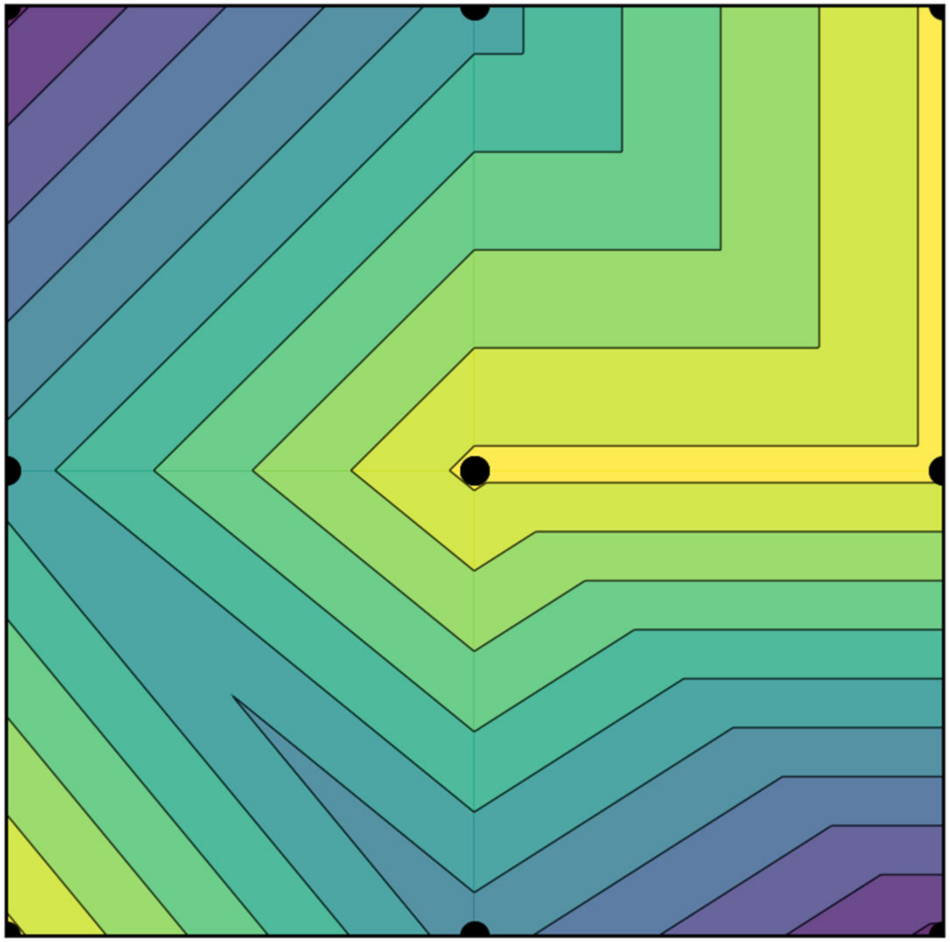
(2 triangles per quad;
diagonal:
bottom-left,
top-right)



Bi-Linear Interpolation: Comparisons

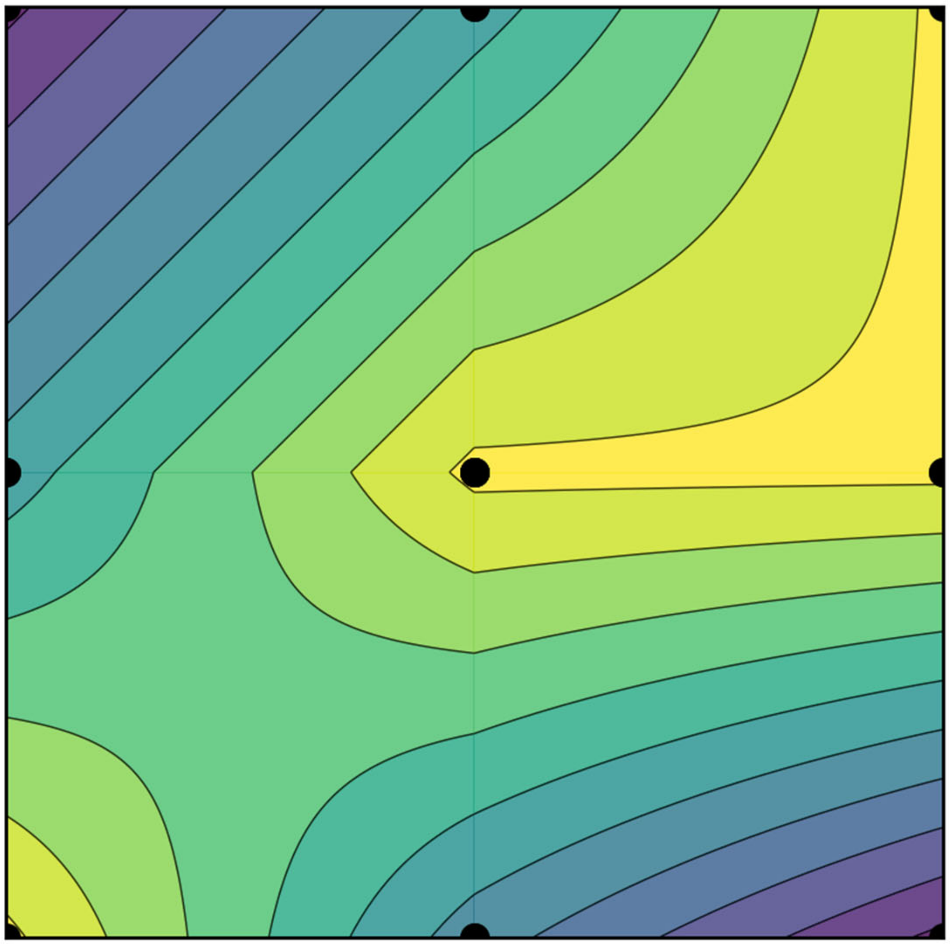
linear

(2 triangles per quad;
  diagonal:
  top-left,
  bottom-right)



Bi-Linear Interpolation: Comparisons

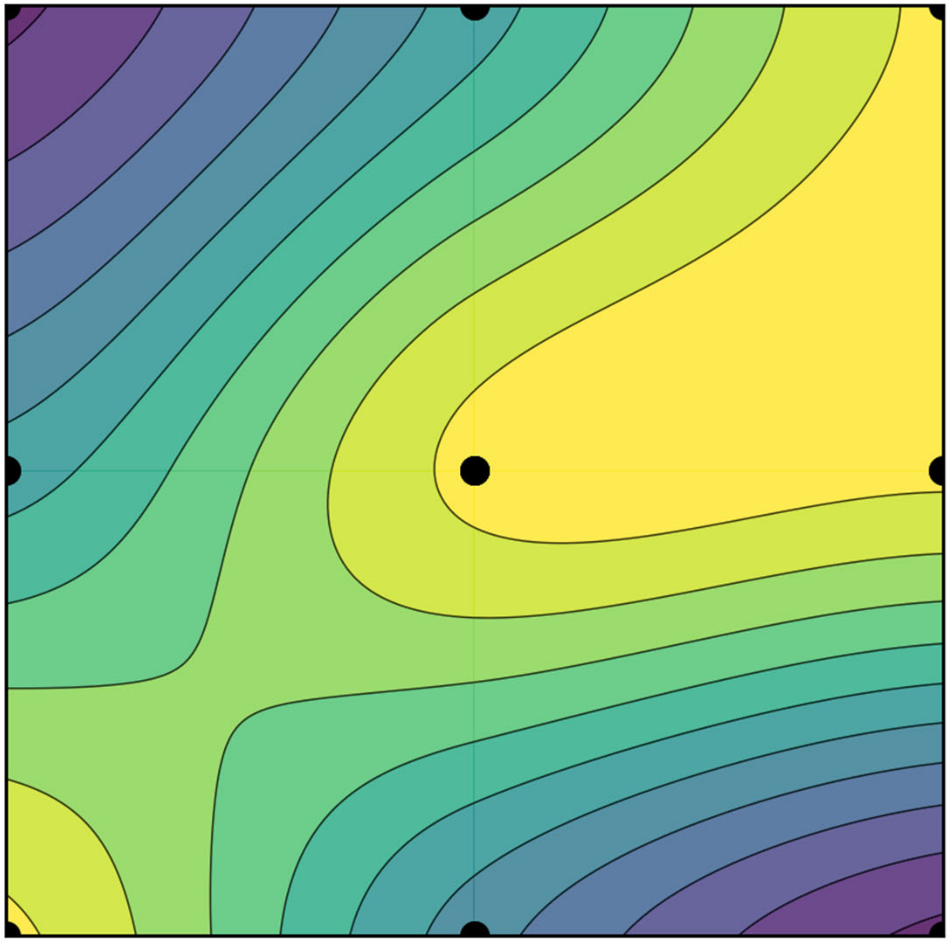
bi-linear



Bi-Linear Interpolation: Comparisons

bi-cubic

(Catmull-Rom spline)



Thank you.

Thanks for material

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- Christof Rezk-Salama