CS 247 – Scientific Visualization
Lecture 23: Vector / Flow Visualization, Pt. 5

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Reading Assignment #13 (until Apr 26)

Read (required):

- Data Visualization book, Chapter 6.7
- B. Cabral, C. Leedom:
  Imaging Vector Fields Using Line Integral Convolution, SIGGRAPH 1993

  http://dx.doi.org/10.1145/166117.166151
- Learn how convolution (the convolution of two functions) works:

  https://en.wikipedia.org/wiki/Convolution
- Refresh your memory on eigenvectors and eigenvalues:

  https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors

Read (optional):

- Paper: Streak Lines as Tangent Curves of a Derived Vector Field,
  Tino Weinkauf and Holger Theisel, IEEE Vis 2010

  http://dx.doi.org/10.1109/TVCG.2010.198

The Flow / Flow Map of a Vector Field (1)

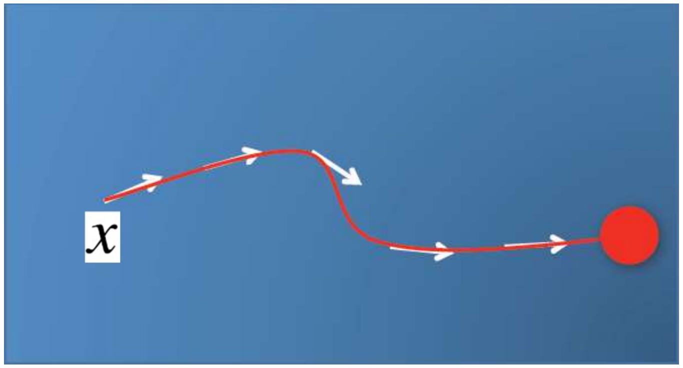
Flow of a *steady (time-independent)* vector field

- Map source position *x* "forward" ($t>0$) or "backward" ($t<0$) by time *t*

$$\boxed{\phi(x,t)} \qquad \boxed{\phi_t(x)} \qquad \text{with} \qquad \phi_0(x) = x$$

$$\phi_s(\phi_t(x)) = \phi_{s+t}(x)$$

$$\phi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n, \quad (x,t) \mapsto \phi(x,t). \qquad \phi_t : \mathbb{R}^n \to \mathbb{R}^n, \quad x \mapsto \phi_t(x).$$



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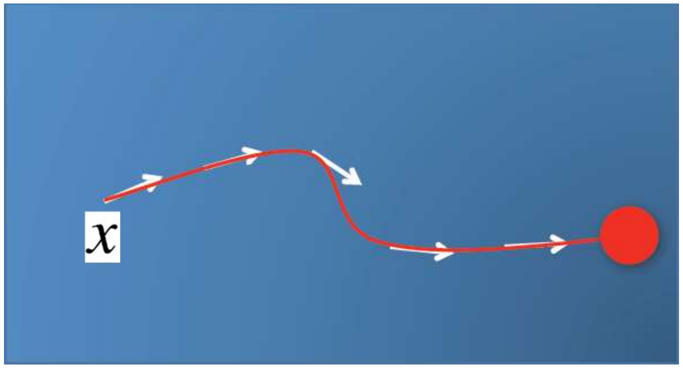
$$\phi : M \times \mathbb{R} \to M, \quad (x,t) \mapsto \phi(x,t).$$

$$\phi_t : M \to M, \quad x \mapsto \phi_t(x).$$

$$\phi(x,t) = x + \int_0^t \mathbf{v}(\phi(x,\tau))\, \mathrm{d}\tau$$



(on a general manifold *M*, integration is performed in coordinate charts)

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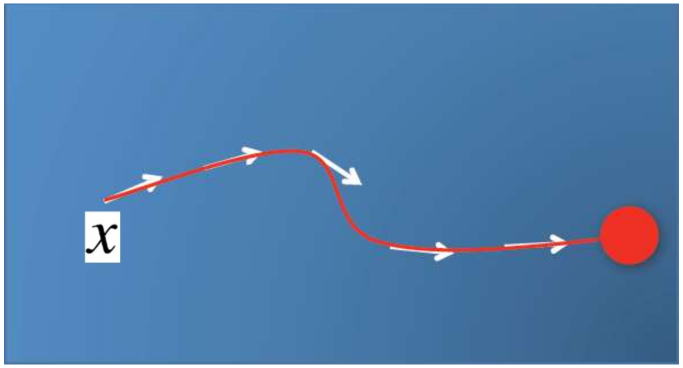
- Unsteady flow? Just fix arbitrary time T

$$\phi(x,t) = x + \int_0^t \mathbf{v}(\phi(x,\tau),\mathrm{T})\,\mathrm{d}\tau$$



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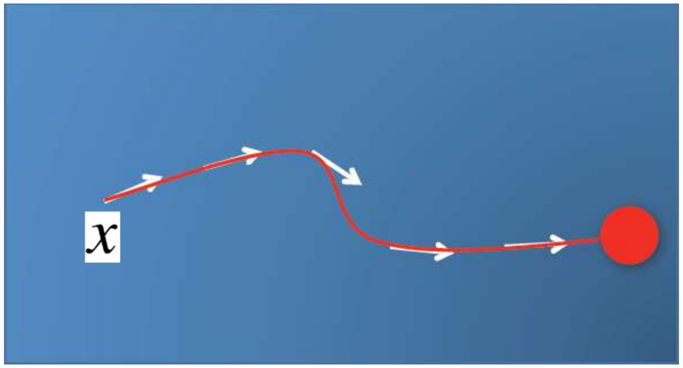
$$\phi : M \times \mathbb{R} \to M, \quad (x,t) \mapsto \phi(x,t). \qquad \phi_t : M \to M, \quad x \mapsto \phi_t(x).$$

Can write explicitly as function of independent variable *t*, with *position x fixed*

$$t \mapsto \phi(x,t) \qquad t \mapsto \phi_t(x)$$

= stream line going through point *x*



The Flow / Flow Map of a Vector Field (2)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t*
  (t < s is allowed: map forward or backward in time)

$$\boxed{\psi_{t,s}(x)}$$

$$\psi : M \times \mathbb{R} \times \mathbb{R} \to M, \quad (x,s,t) \mapsto \psi_{t,s}(x).$$

with

$$\psi_{s,s}(x) = x$$

$$\psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\,\mathrm{d}\tau$$

$$\psi_{t,r}\big(\psi_{r,s}(x)\big) = \psi_{t,s}(x)$$

The Flow / Flow Map of a Vector Field (2)

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$$\psi_{t,r}\big(\psi_{r,s}(x)\big) = \psi_{t,s}(x)$$

The Flow / Flow Map of a Vector Field (3)

Flow of an *unsteady (time-dependent)* vector field

- Map source position *x* from time *s* to destination position at time *t* (t < s is allowed: map forward or backward in time)

$$\boxed{\psi_{t,s}(x)} \qquad \psi_{t,s}(x) = x + \int_s^t \mathbf{v}\big(\psi_{\tau,s}(x), \tau\big)\, \mathrm{d}\tau$$

Can write explicitly as function of t, *with s and x fixed*

$t \mapsto \psi_{t,s}(x)$ → path line

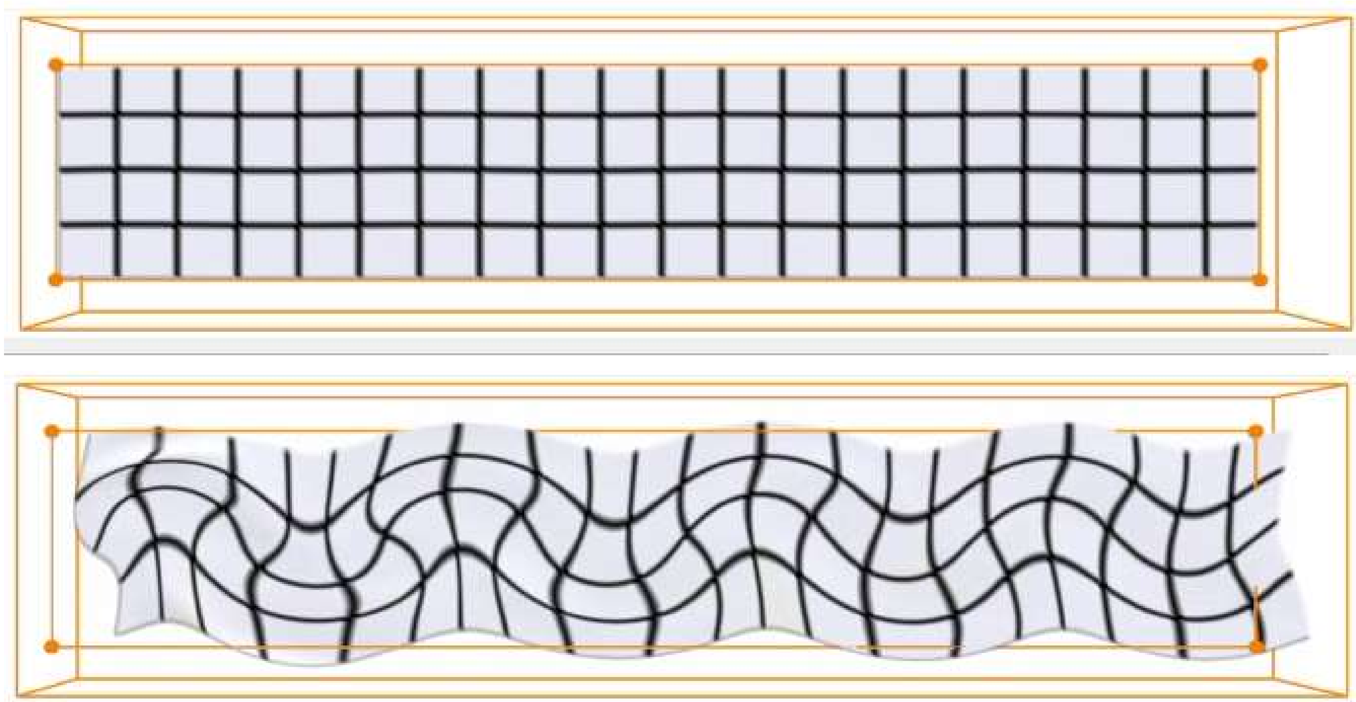
Can write explicitly as function of s, *with t and x fixed*

$s \mapsto \psi_{t,s}(x)$ → streak line

$\psi_{t,s}(x)$ is also often written as ***flow map*** $\phi_t^{\tau}(x)$ (with t:=s and either τ:=t or τ:=t-s)

The Flow / Flow Map of a Vector Field (4)

Can map a whole set of points (or the entire domain) through the flow map (this map is a *diffeomorphism*): $t \mapsto \psi_{t,s}(U)$



U

$= \psi_{s,s}(U)$

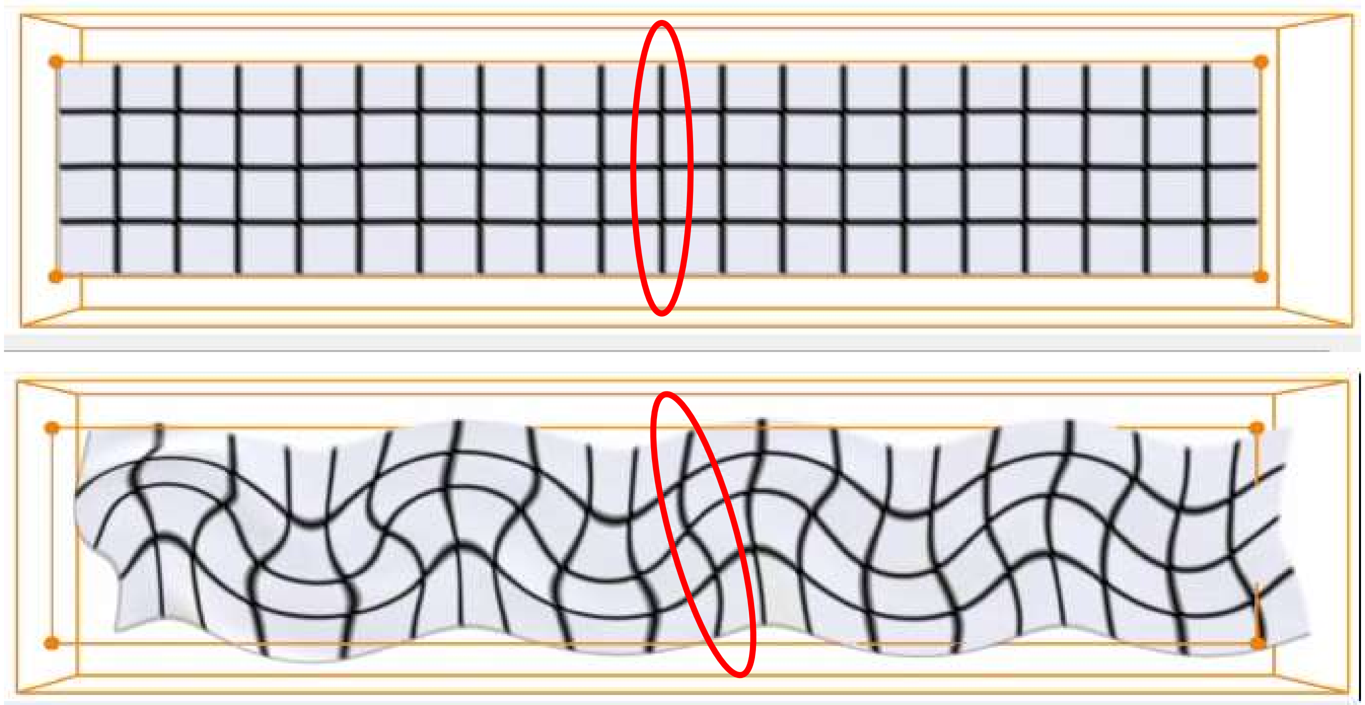
$\psi_{t,s}(U)$

(this is a *time surface!*)

The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (*s*) to another time (*t*)

$$t \mapsto \psi_{t,s}(c(\lambda))$$



$c(\lambda)$

$= \psi_{s,s}(c(\lambda))$

$\psi_{t,s}(c(\lambda))$

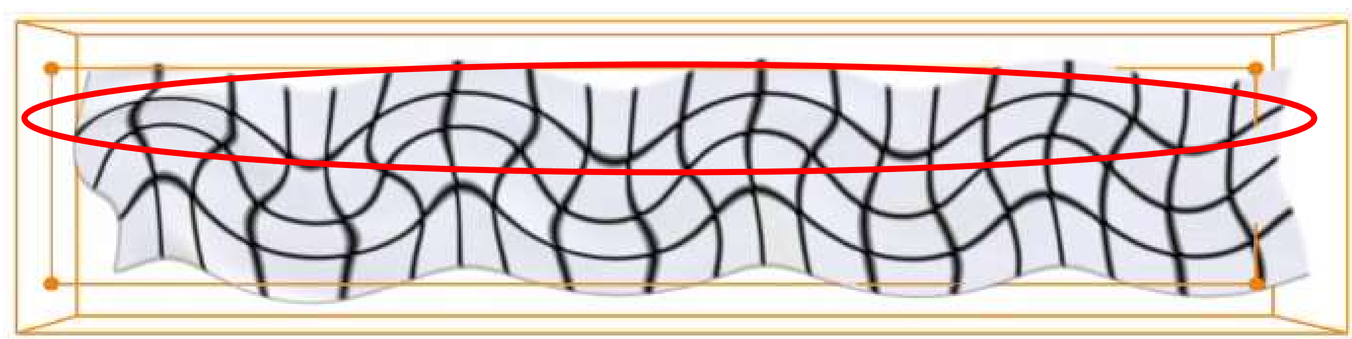
The Flow / Flow Map of a Vector Field (5)

Time line: Map a whole curve from one fixed time (s) to another time (t)

$$t \mapsto \psi_{t,s}(c(\lambda))$$

$c(\lambda)$

$= \psi_{s,s}(c(\lambda))$



$\psi_{t,s}(c(\lambda))$

Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

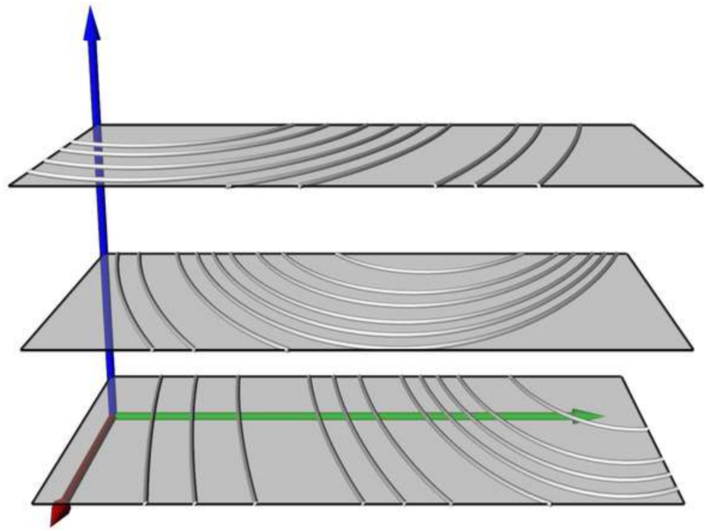
- Describes motion of a massless particle over time

Streakline

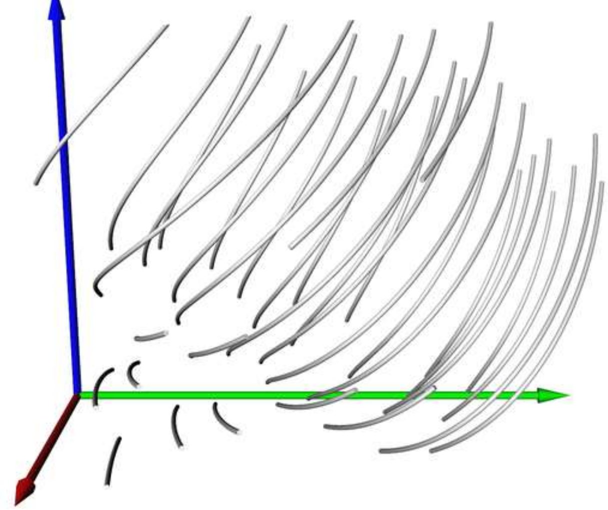
- Location of all particles released at a *fixed position* over time

Timeline

- Location of all particles released along a line at a *fixed time*

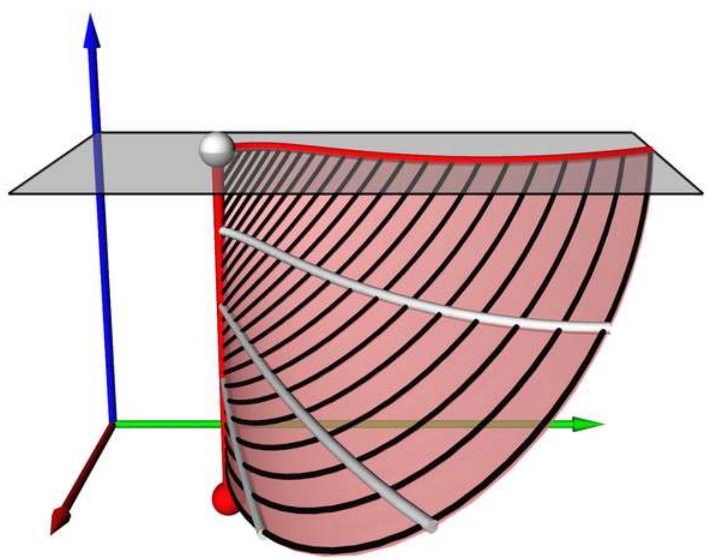


stream lines

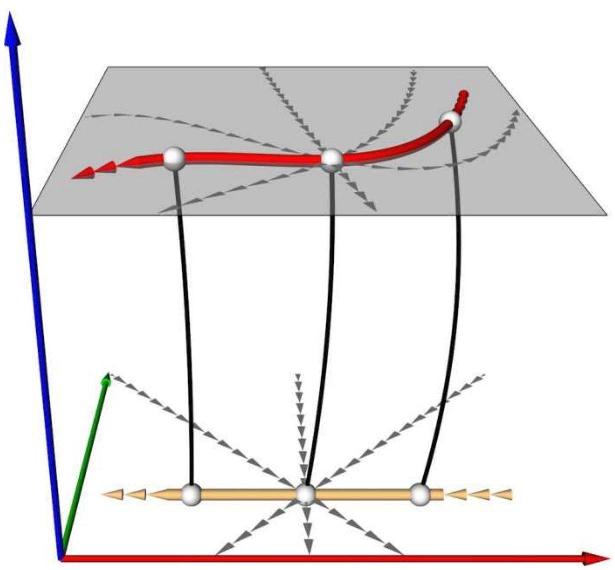


path lines

streak lines



time lines



Comparison of techniques:

(1) Pathlines:
- are physically meaningful
- allow comparison with experiment (observe marked particles)
- are well suited for dynamic visualization (of particles)

(2) Streamlines:
- are only geometrically, not physically meaningful
- are easiest to compute (no temporal interpolation, single IVP)
- are better suited for static visualization (prints)
- don't intersect (under reasonable assumptions)

(3) Streaklines:

- are physically meaningful
- allow comparison with experiment (dye injection)
- are well suited for static and dynamic visualization
- good choice for fast moving vortices
- can be approximated by set of disconnected particles

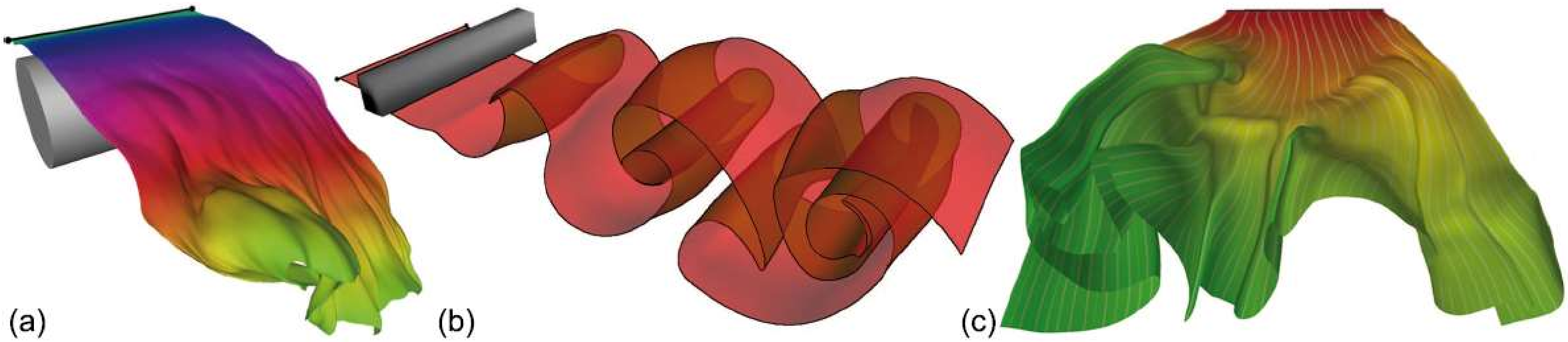
(4) Timelines:

- are physically meaningful
- are well suited for static and dynamic visualization
- can be approximated by set of disconnected particles

Surfaces Instead of Lines

Seeding from a line instead of from a point

Example: streak surfaces



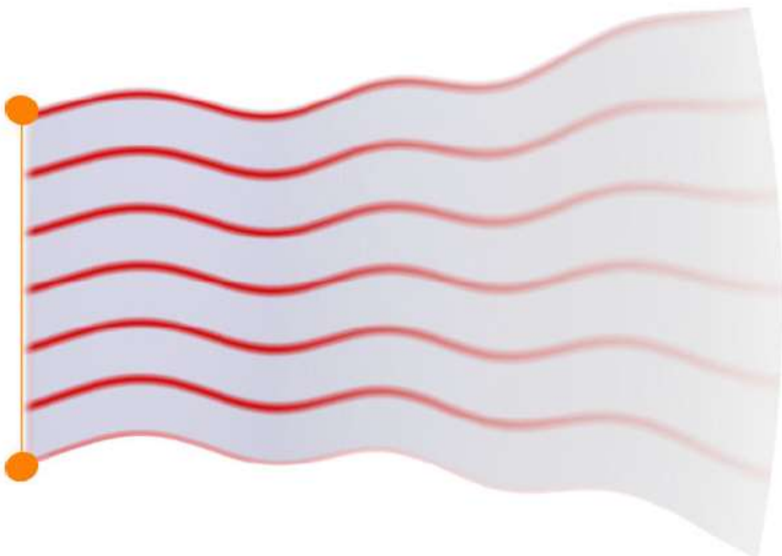
Volumes: seeding from a surface instead of a line

Real "Streak Surfaces"

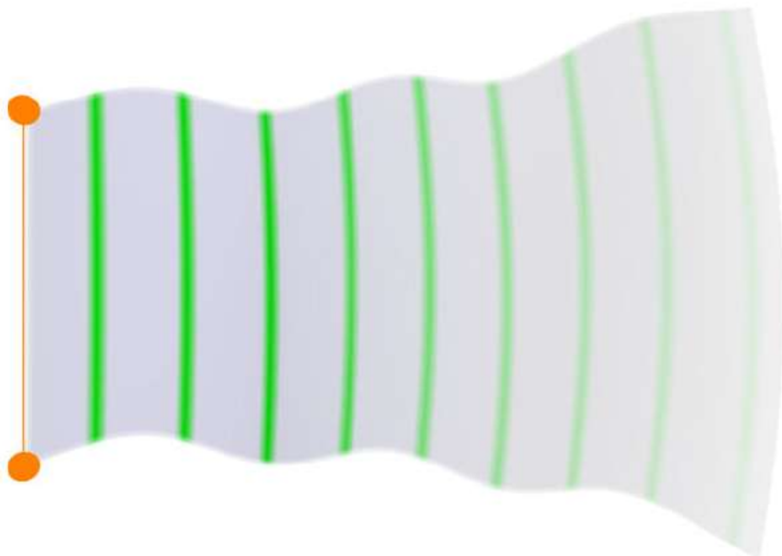
Artistic photographs of smoke

Streak Lines vs. Time Lines

(on a streak surface)

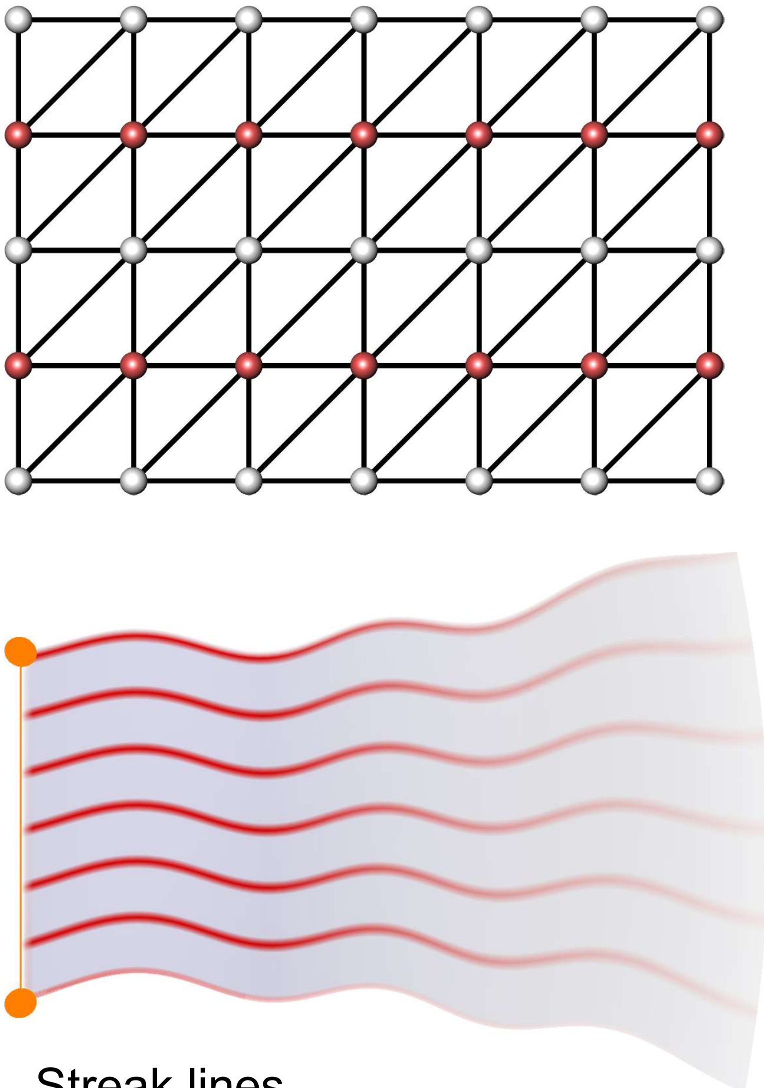


Streak Lines

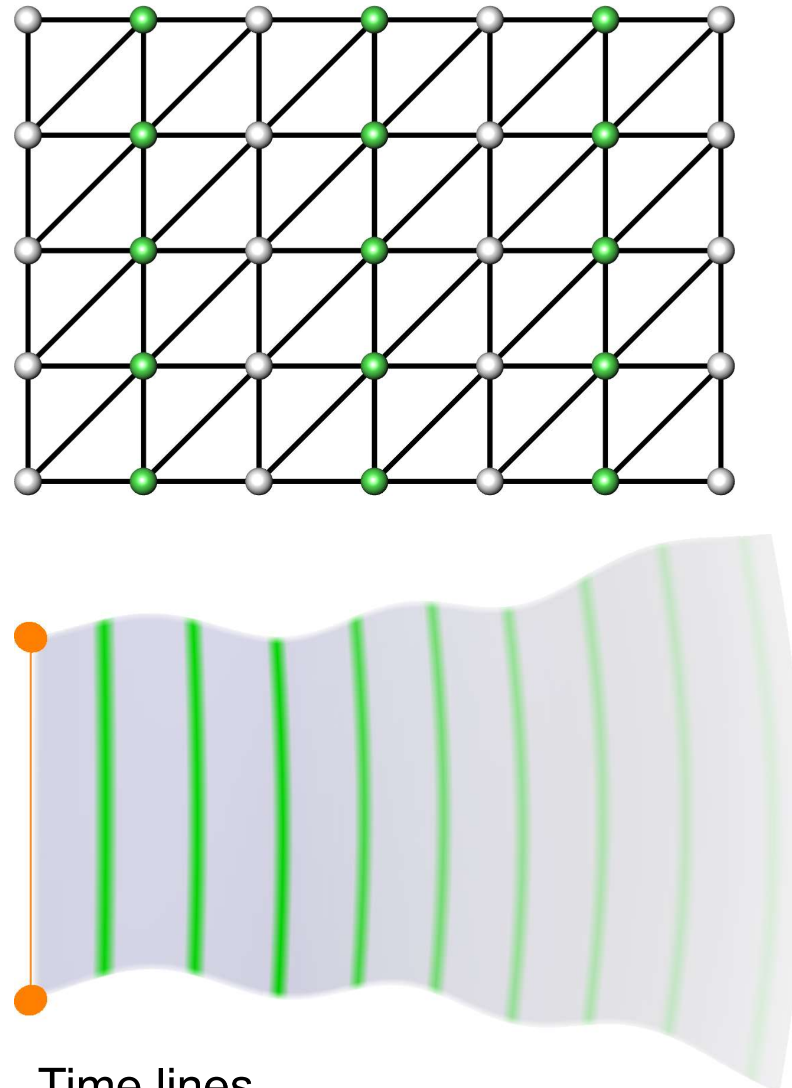


Time Lines

Streak and Time Lines



Streak lines

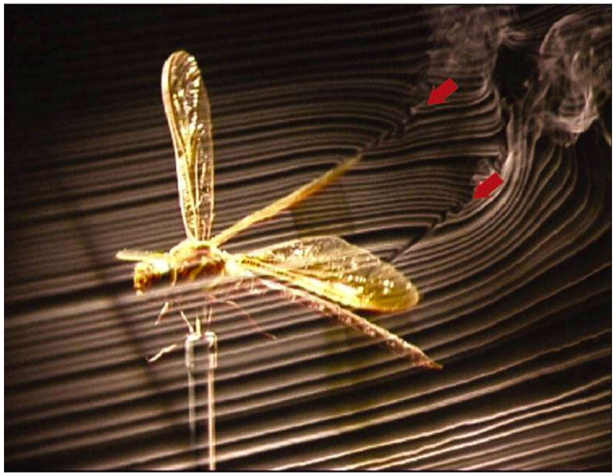


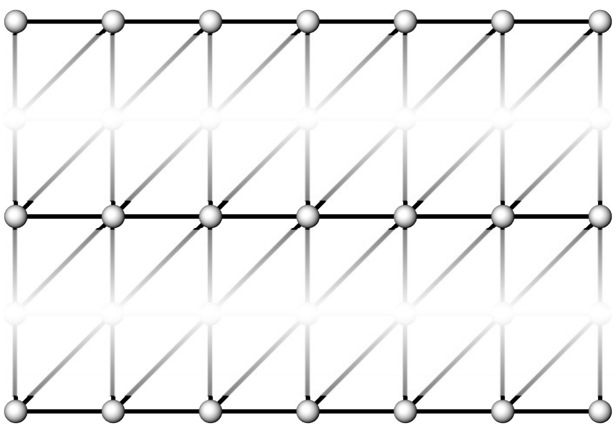
Time lines

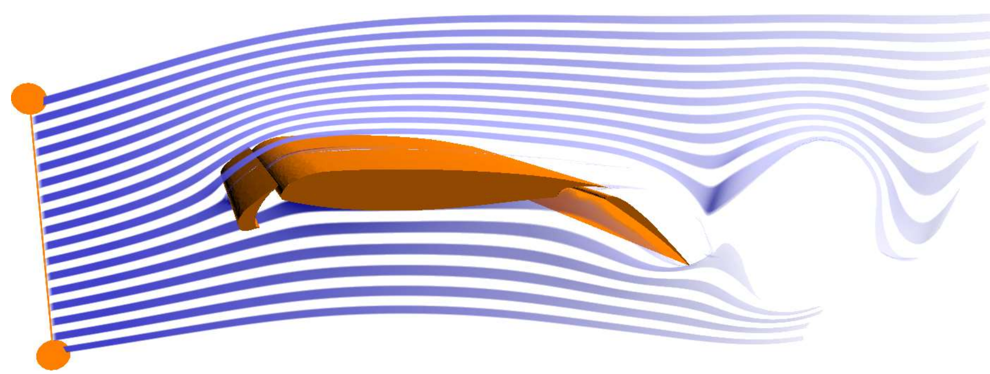
Time

streak line

streak surface







[Data courtesy of Günther (TU Berlin)]

fixed zero opacity rows

break connectivity

More Fun with Flow Maps (1)

Can compute when/where different curves intersect

- Two path lines intersect (at same position, but at different times)

$$\psi_{t,s}(x) = \psi_{t',\tau}(\tilde{x})$$

- One path line intersects itself (at same position, but at different times)

$$\psi_{t,s}(x) = \psi_{t',s}(x)$$

- Special case when the “two” path lines are in fact the same path line

$$\psi_{t,s}(x) = \psi_{t,\tau}(\tilde{x}) \qquad \tilde{x} = \psi_{\tau,s}(x)$$

More Fun with Flow Maps (2)

Can compute when/where different curves intersect

- Two streak lines (with different seeding positions) only intersect in the special case when some point on the first/second streak line is at some time at the seeding position of the second/first streak line

$$\psi_{t,s}(x) = \psi_{t,\tilde{s}}(\tilde{x})$$

- Then, the particles (x, s) and $(\tilde{x}, \tilde{s})$ are *the same particle*

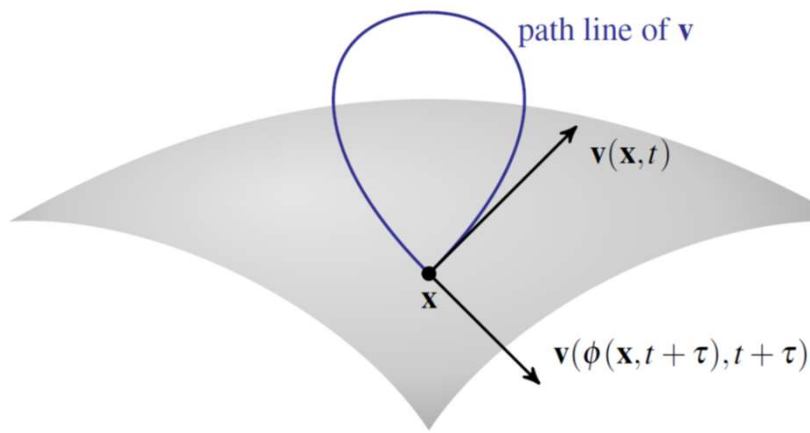
$$\tilde{x} = \psi_{\tilde{s},s}(x) \qquad \psi_{t,\tilde{s}}(\psi_{\tilde{s},s}(x)) = \psi_{t,s}(x)$$

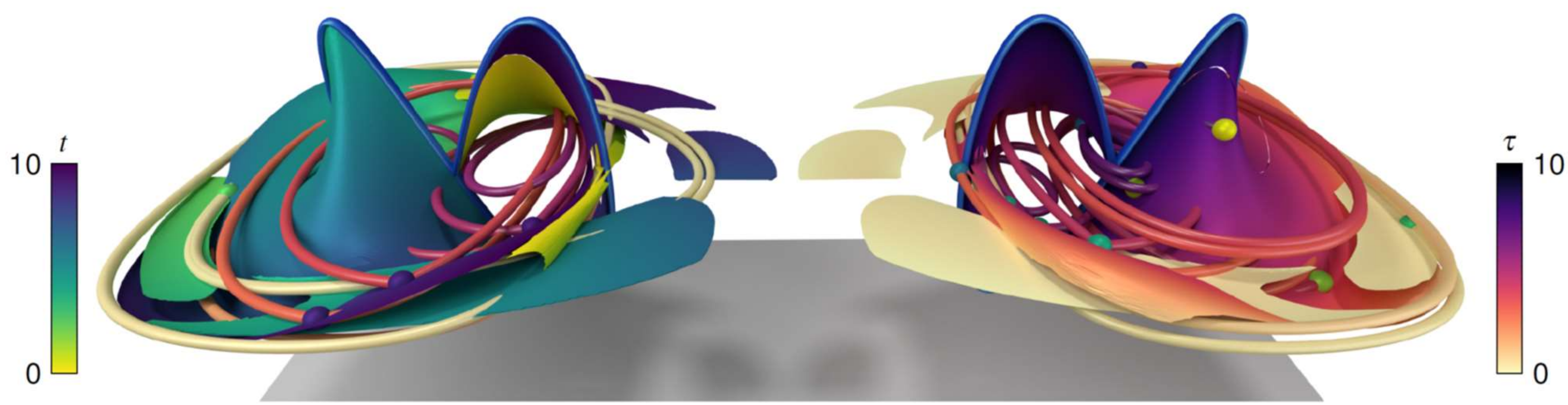
- Even more special case:
  Streak line "intersecting" itself = looping back on itself (**recirculation**)

$$\psi_{t,s}(x) = \psi_{t,\tilde{s}}(x)$$

Recirculation (Surfaces)

Wilde, Roessl, Theisel; Recirculation Surfaces for Flow Visualization





Thank you.

Thanks for material

- Helwig Hauser
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- Philipp Muigg
- Christof Rezk-Salama