CS 247 – Scientific Visualization
Lecture 20: Vector / Flow Visualization, Pt. 2

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Reading Assignment #11 (until Apr 12)

Read (required):

- Data Visualization book
  - Chapter 6 (Vector Visualization)
    - Beginning (before 6.1)
    - Chapters 6.2, 6.3, 6.5
- More general vector field basics (the book is not very precise on the basics)

  `https://en.wikipedia.org/wiki/Vector_field`

Read (optional):

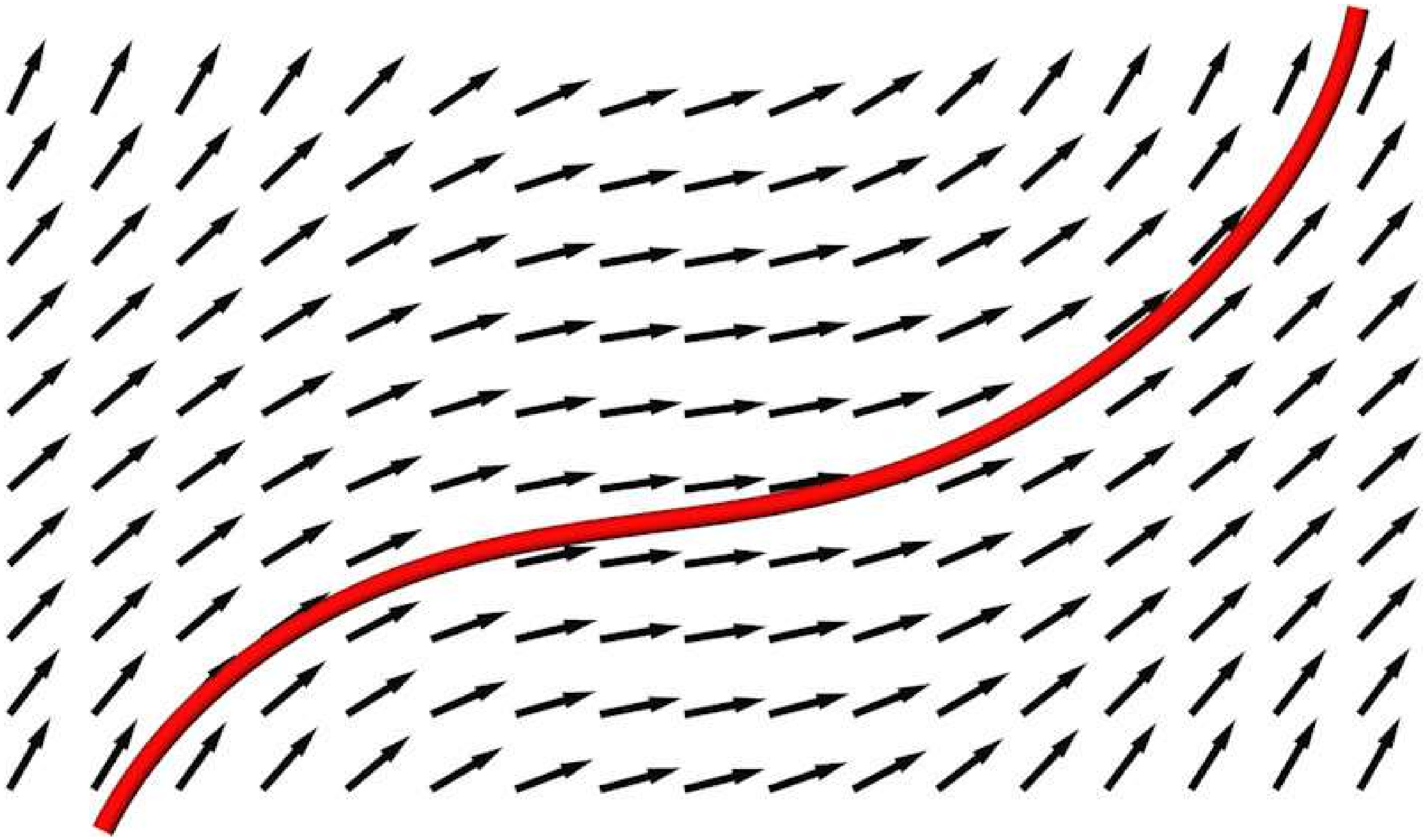
- Paper:
  Bruno Jobard and Wilfrid Lefer
  Creating Evenly-Spaced Streamlines of Arbitrary Density,

  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.9498`

- **Vortex/ Vortex core lines**
  - There is no exact definition of vortices
  - capturing some swirling behavior

Integral Curves / Stream Objects

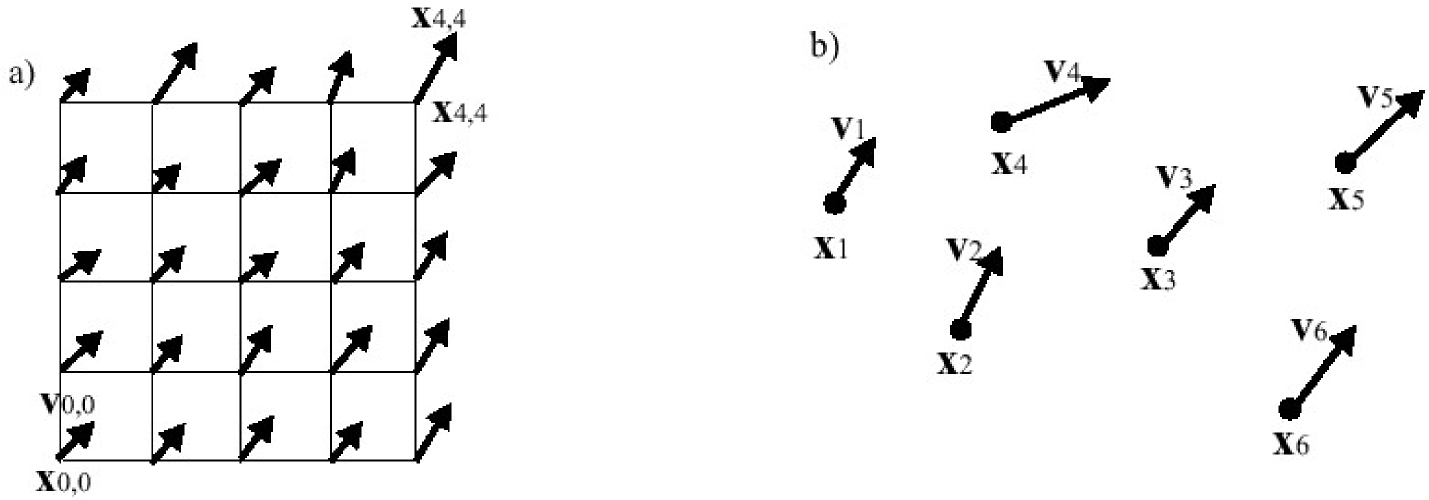
Integrating velocity over time yields spatial motion



Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)



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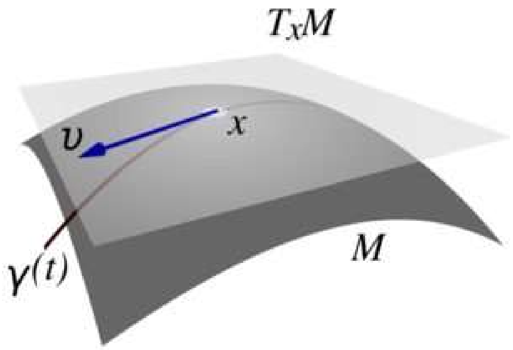
Each vector in a vector field
lives in the **tangent space**
of the manifold at that point:

Each vector is a **tangent vector**



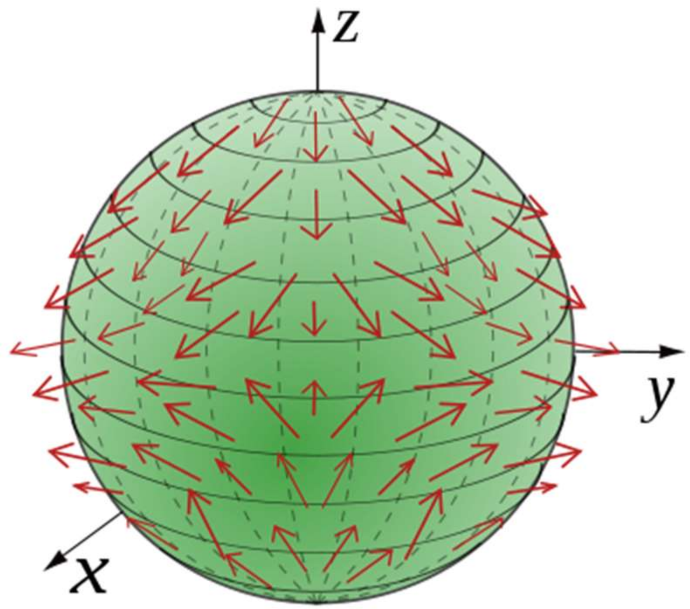


image from wikipedia

Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\mathbf{v}: U \subset \mathbb{R}^2 \to \mathbb{R}^2, \quad (x,y) \mapsto \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^3 \to \mathbb{R}^3, \quad (x,y,z) \mapsto \begin{bmatrix} u \\ v \\ w \end{bmatrix}.$$

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Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \quad x \mapsto \mathbf{e}_i(x) \qquad \{\mathbf{e}_i(x)\}_{i=1}^n \text{ basis for } T_xM$$

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$$\mathbf{v} : U \subset M \to TM, \quad x \mapsto v^1 \mathbf{e}_1 + v^2 \mathbf{e}_2 + \ldots + v^n \mathbf{e}_n.$$

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Coordinate basis:

$$\mathbf{e}_i := \frac{\partial}{\partial x^i}$$

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Examples of Coordinate Curves and Bases

Coordinate functions, coordinate curves, bases

- Coordinate functions are real-valued ("scalar") functions on the domain
- On each coordinate curve, *one* coordinate changes, *all others stay constant*
- Basis: n linearly independent vectors at each point of domain

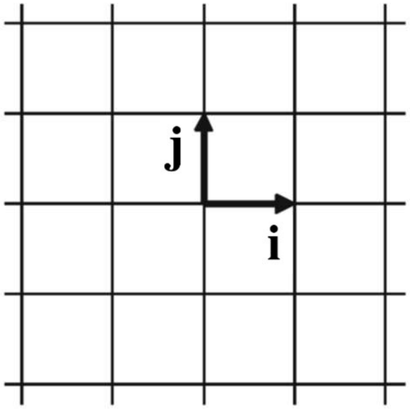
Cartesian coordinates

$$x^1 = x$$

$$x^2 = y$$

$$\mathbf{e}_1 = \frac{\partial}{\partial x} = \mathbf{i}$$

$$\mathbf{e}_2 = \frac{\partial}{\partial y} = \mathbf{j}$$



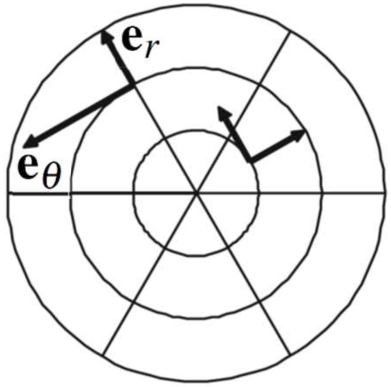
polar coordinates

$$x^1 = r$$

$$x^2 = \theta$$

$$\mathbf{e}_1 = \frac{\partial}{\partial r} = \mathbf{e}_r$$

$$\mathbf{e}_2 = \frac{\partial}{\partial \theta} = \mathbf{e}_\theta$$



Vectors as Derivative Operators

A vector applied to a (real) function on the manifold gives the *directional derivative* in that direction

- From this viewpoint, the vector is a derivative operator (actually, a *derivation*)
- Can be used as *definition* of a vector (must fulfill props. of a derivation; esp. Leibniz rule)

$$f : M \to \mathbb{R}, \qquad \mathbf{v} f$$
$$x \mapsto f(x).$$

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Kronecker delta
("identity matrix")

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Kronecker delta ("identity matrix")

For vector field: obtain directional derivative at each point

$$\mathbf{v} f : M \to \mathbb{R}, \quad x \mapsto \mathbf{v}(x) f = df(\mathbf{v}(x)).$$

(remember that this just *looks* scary (maybe) ...)

Integral Curves / Stream Objects

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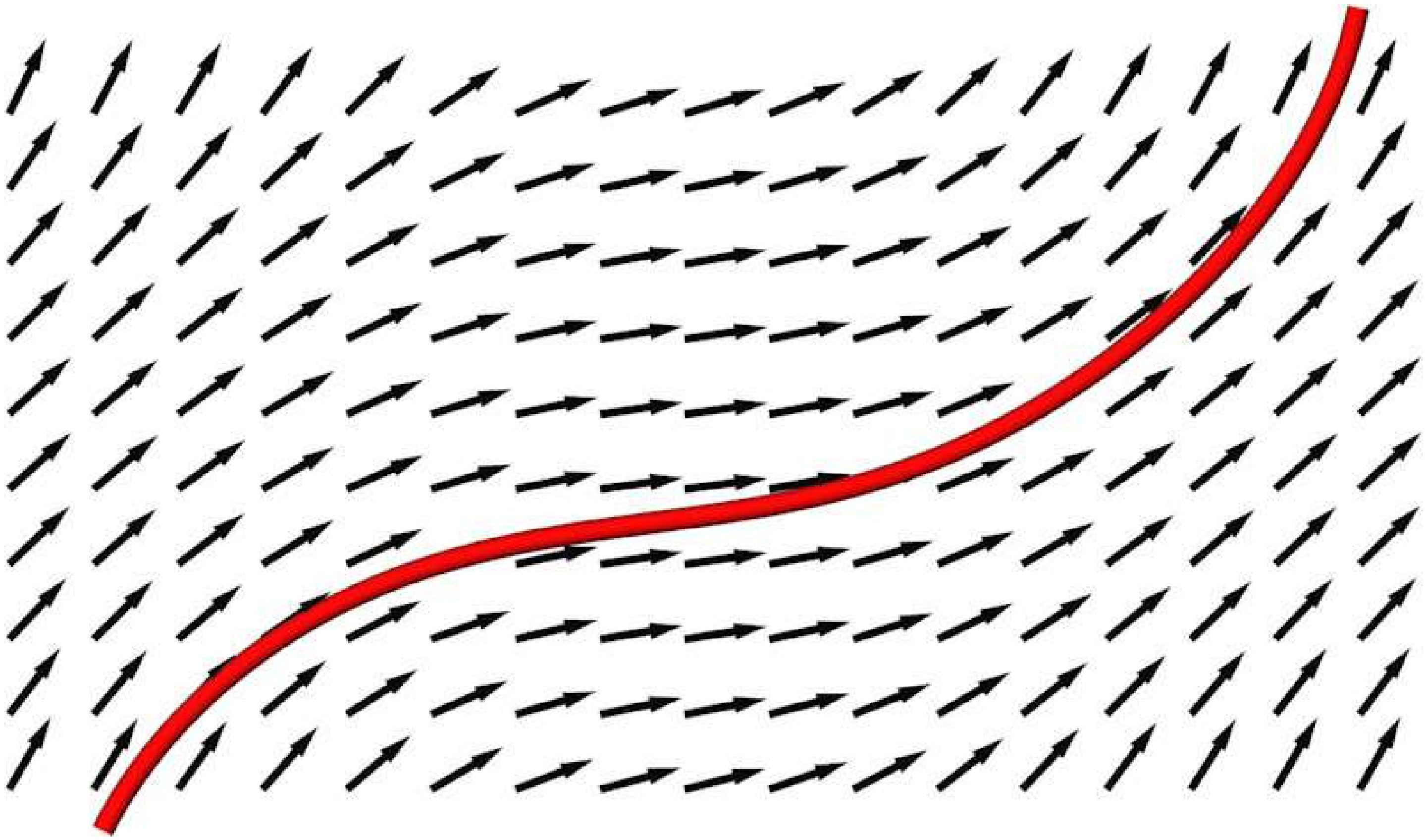

Particle Trajectories

Courtesy Jens Krüger

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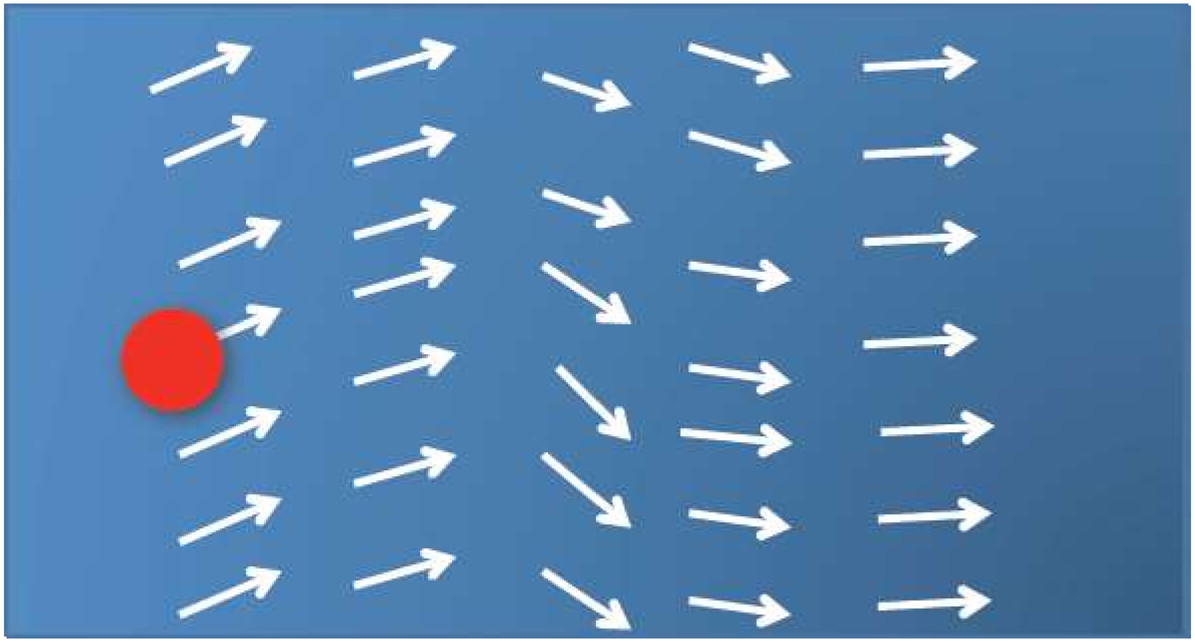

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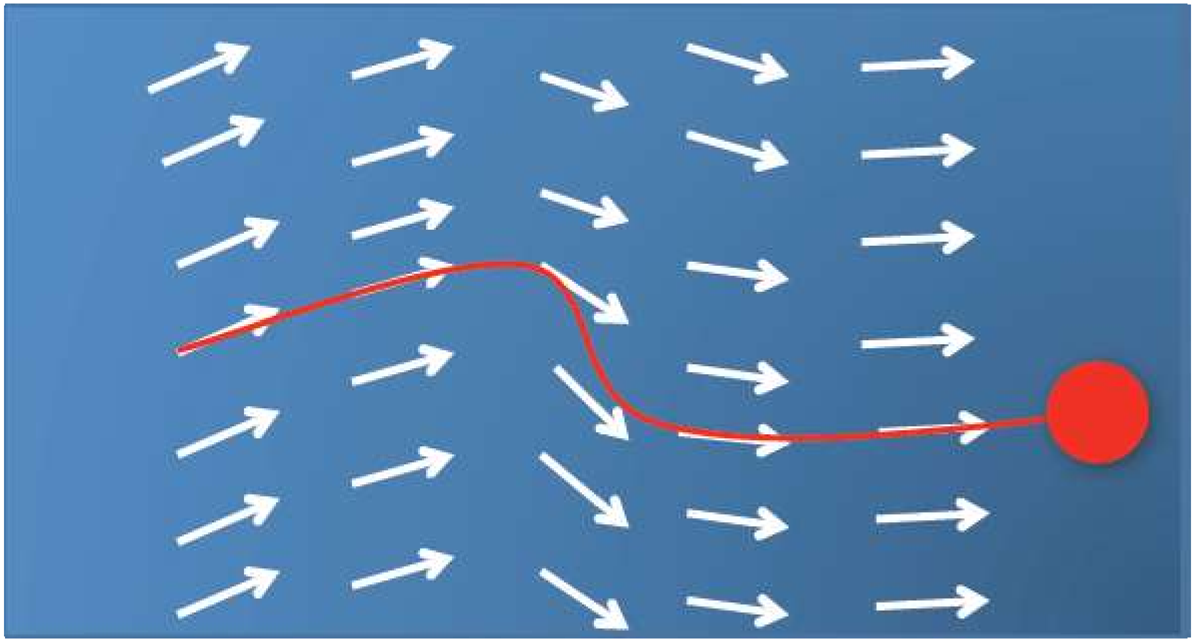

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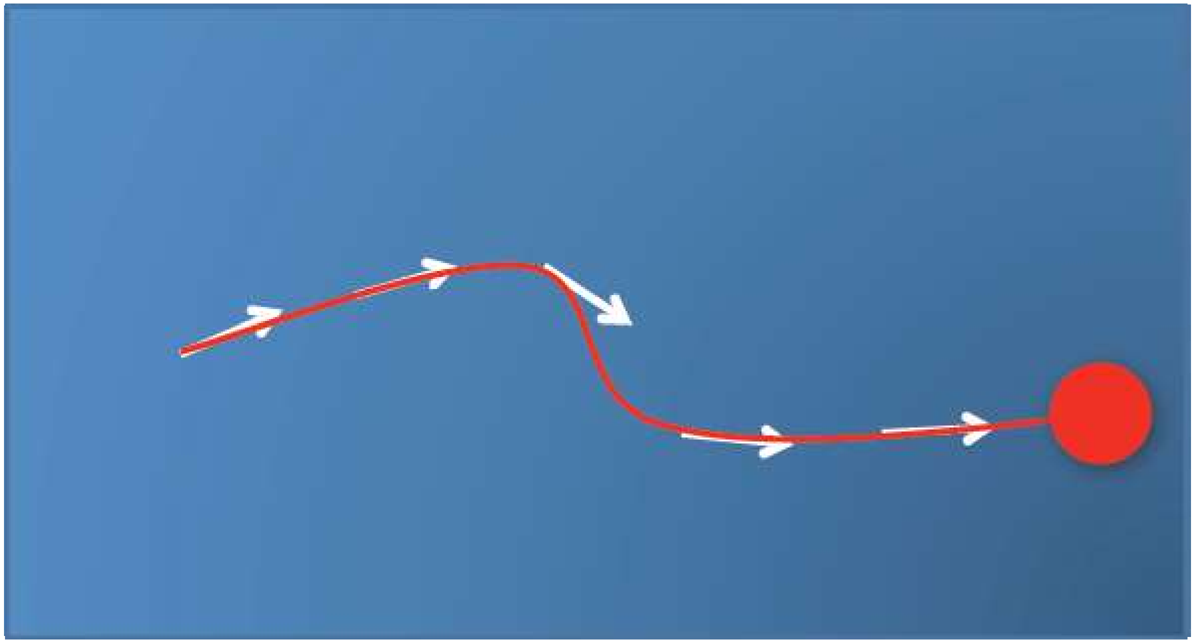

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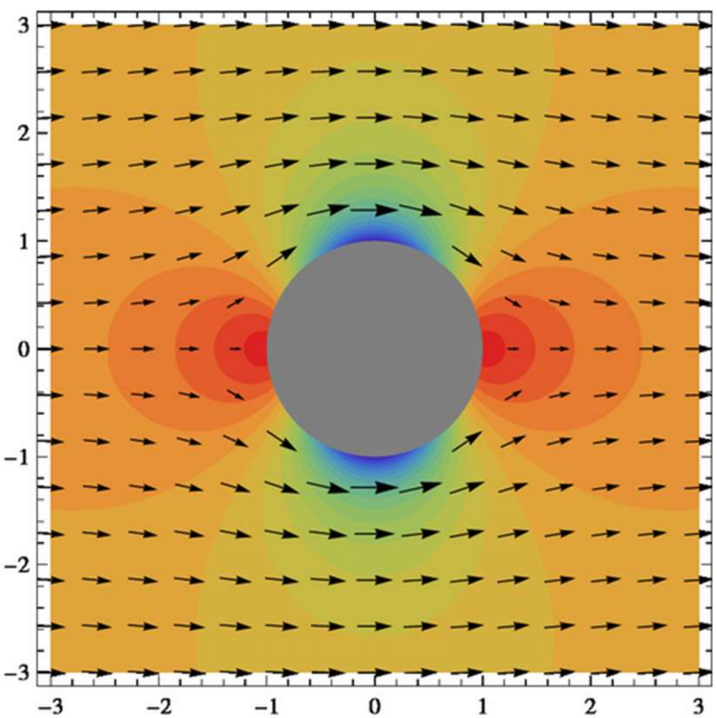
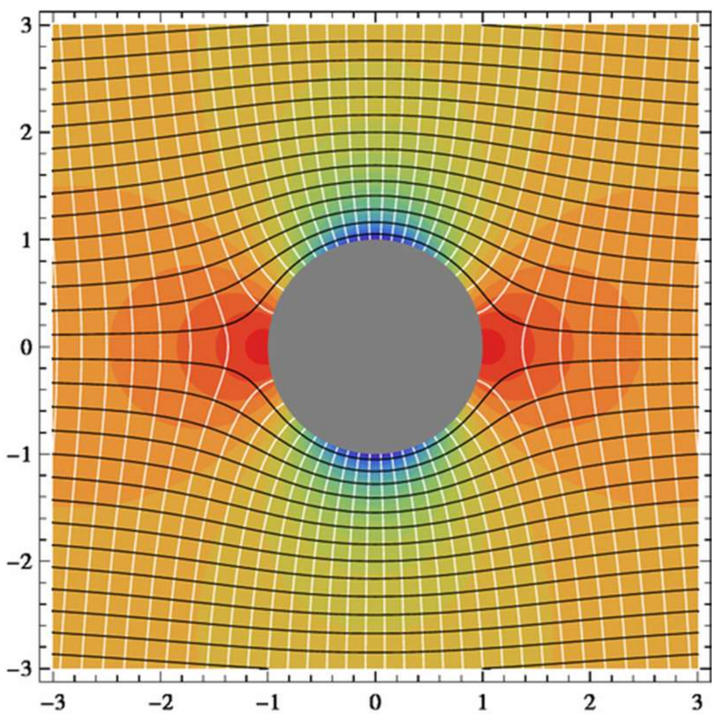
Flow Field Example (1)

Potential flow around a circular cylinder

https://en.wikipedia.org/wiki/Potential_flow_around_a_circular_cylinder

Inviscid, incompressible flow that is irrotational (curl-free) and can be modeled as the gradient of a scalar function called the (scalar) velocity potential





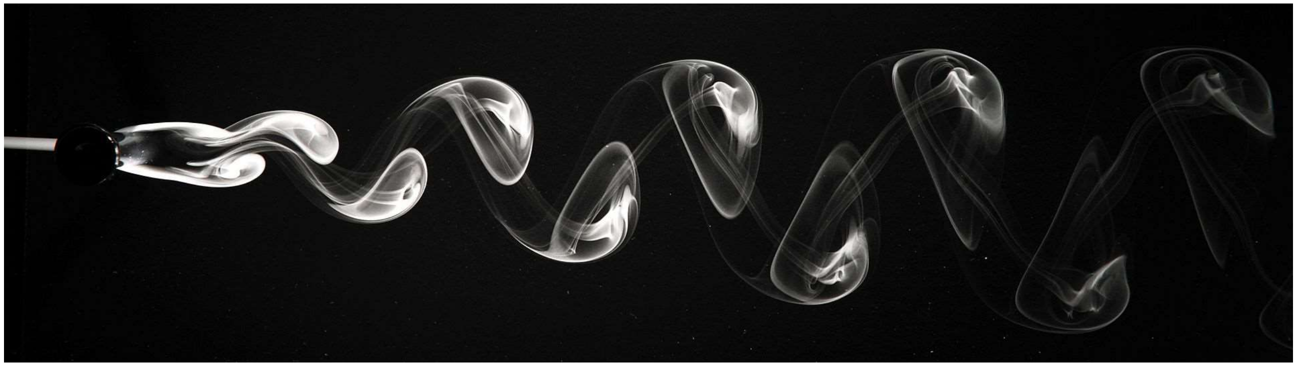
images from wikipedia

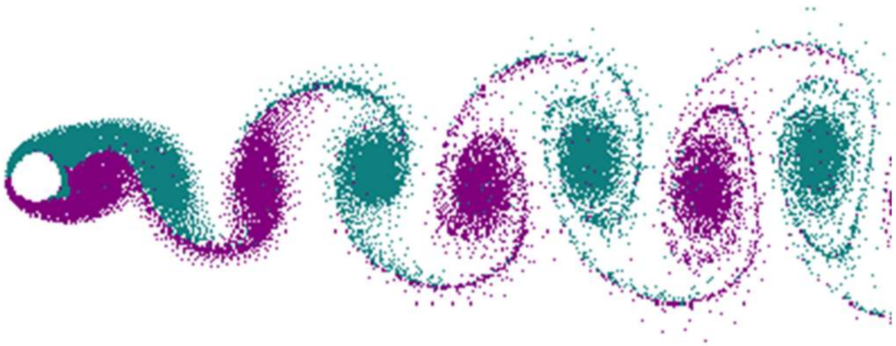
Flow Field Example (2)

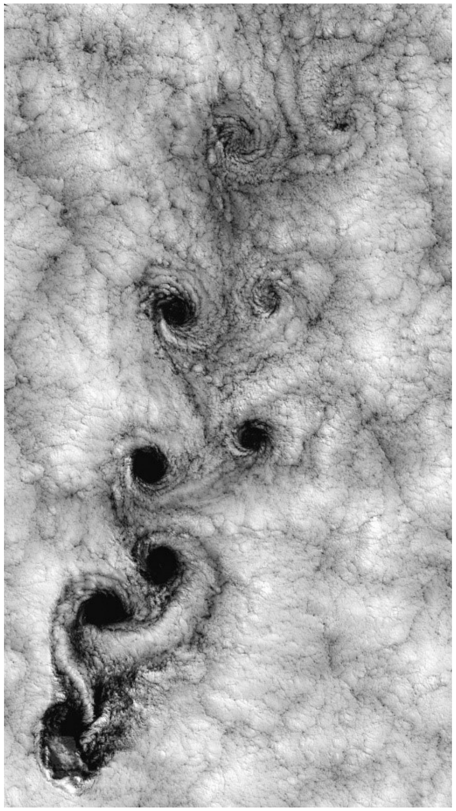
Depending on Reynolds number, turbulence will develop

Example: von Kármán vortex street: vortex shedding

`https://en.wikipedia.org/wiki/Karman_vortex_street`







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Steady vs. Unsteady Flow

- Steady flow: time-independent
  - Flow itself is static over time: $\mathbf{v}(\mathbf{x})$
  - Example: laminar flows

$$\begin{aligned} \mathbf{v} : \mathbb{R}^n &\to \mathbb{R}^n, \\ x &\mapsto \mathbf{v}(x). \end{aligned}$$

- Unsteady flow: time-dependent
  - Flow itself changes over time: $\mathbf{v}(\mathbf{x}, t)$
  - Example: turbulent flows

$$\begin{aligned} \mathbf{v} : \mathbb{R}^n \times \mathbb{R} &\to \mathbb{R}^n, \\ x &\mapsto \mathbf{v}(x, t). \end{aligned}$$

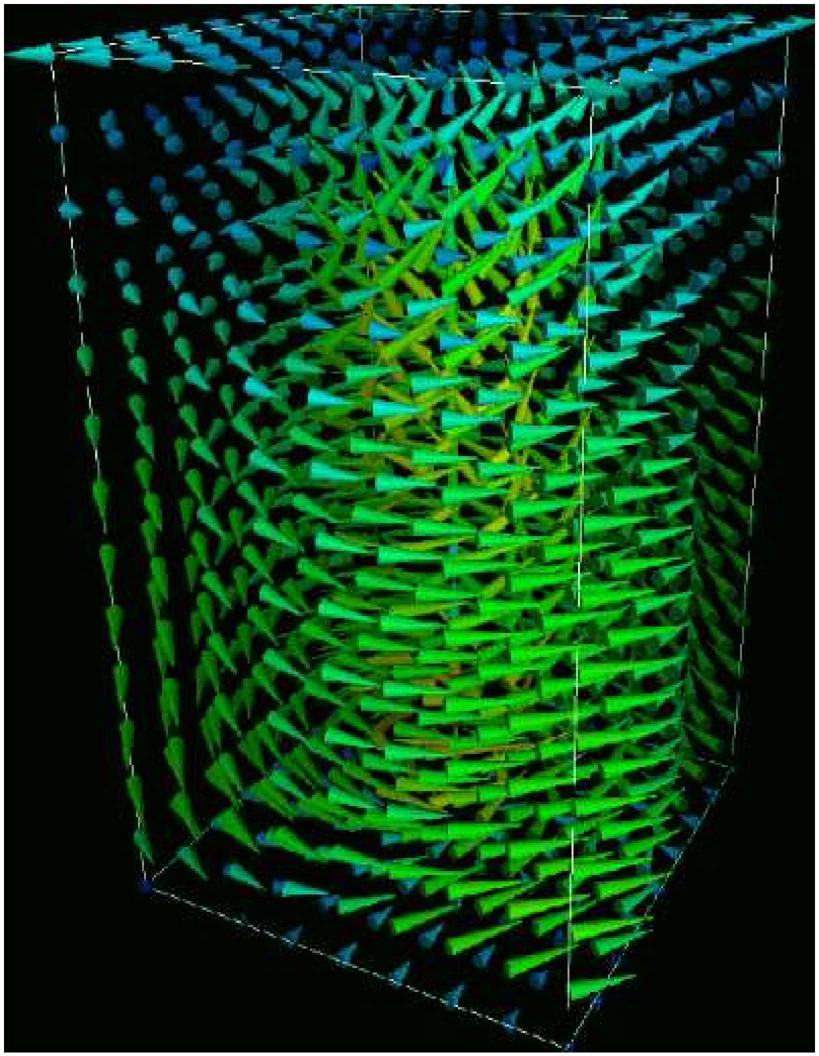
(here just for Euclidean domain; analogous on general manifolds)

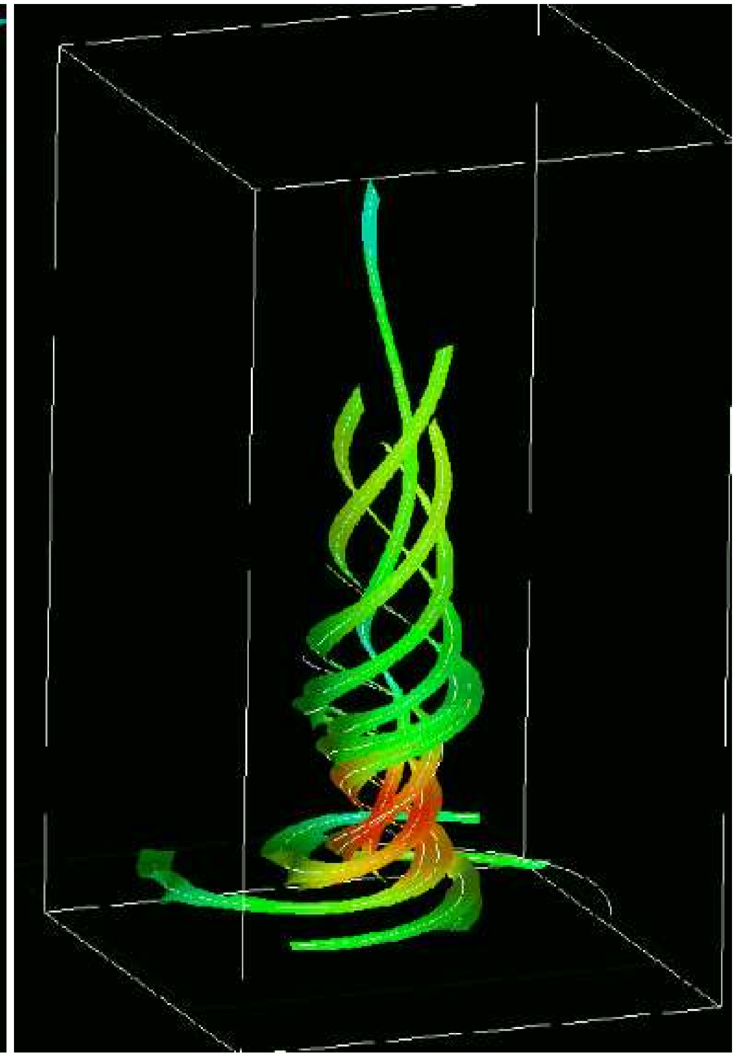
Direct vs. Indirect Flow Visualization

- Direct flow visualization
  - Overview of current flow state
  - Visualization of vectors: arrow plots (“hedgehog” plots)

- Indirect flow visualization
  - Use intermediate representation: vector field integration over time
  - Visualization of temporal evolution
  - Integral curves: streamlines, pathlines, streaklines, timelines
  - Integral surfaces: streamsurfaces, pathsurfaces, streaksurfaces

Direct vs. Indirect Flow Visualization

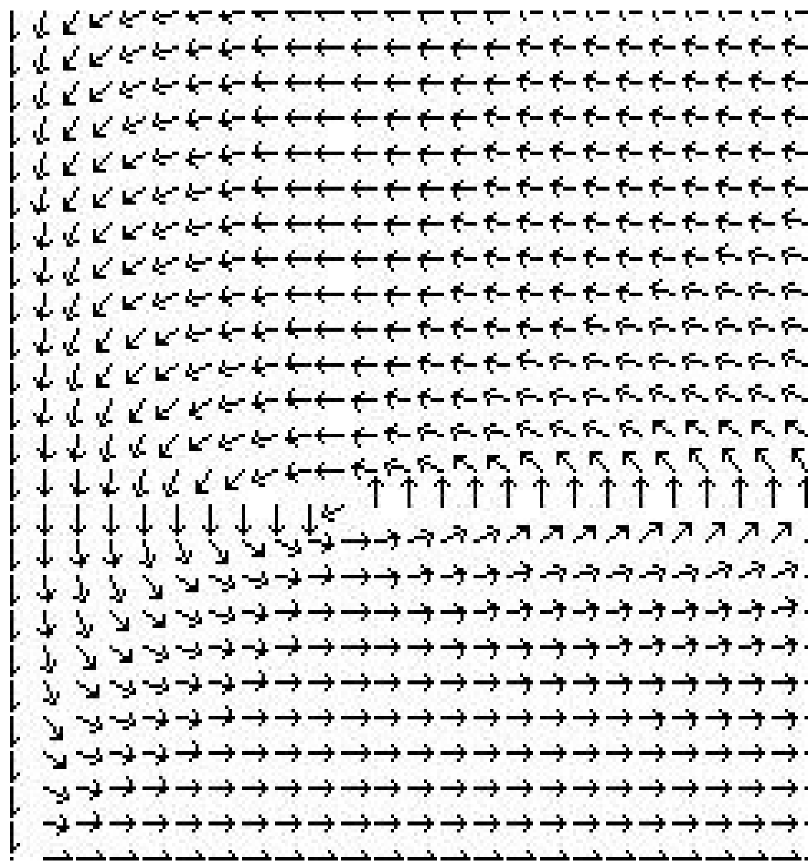




Direct Flow Visualization

Flow Visualization with Arrows

- Hedgehog plots
  - Direct flow visualization
  - Normalized arrows vs. scaling with velocity
    - 2D: quite usable
    - 3D: often problematic
  - Limited expressivity (temporal component missing)
  - But often used as basic technique!



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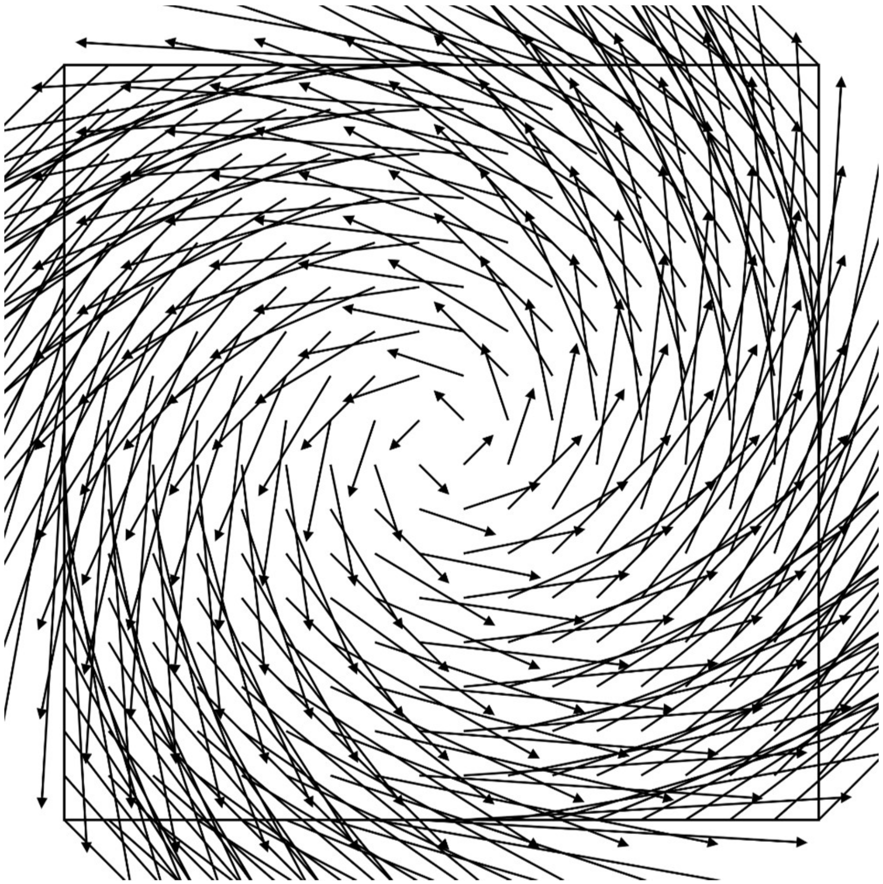


An elementary visualization is to draw arrows

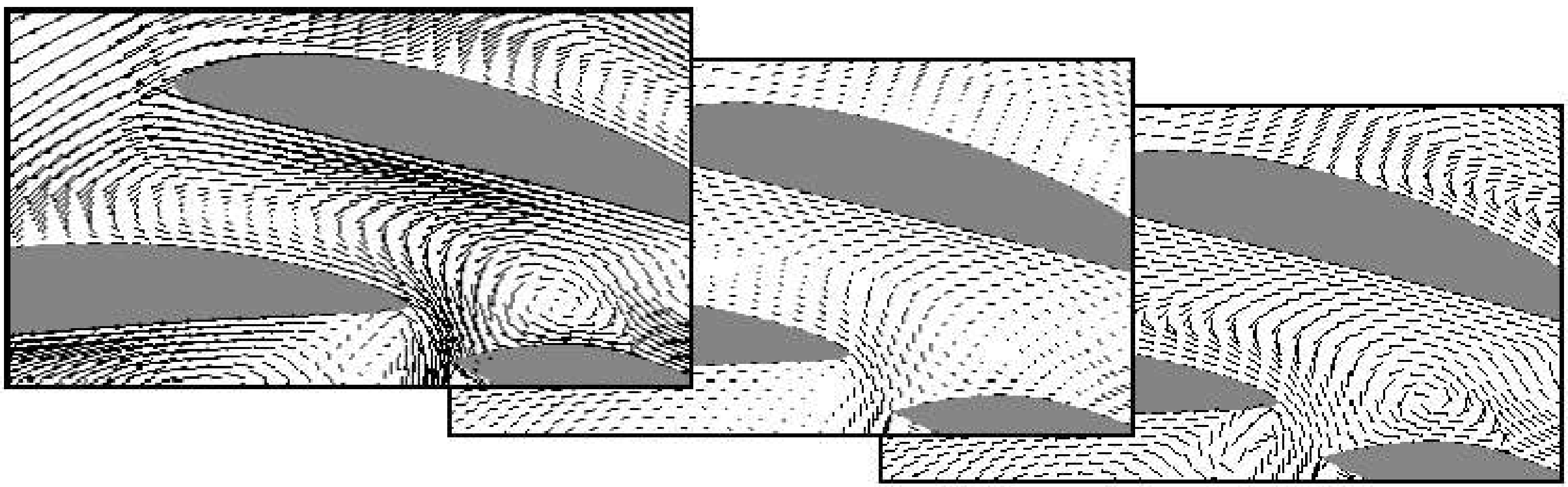
- at the data points (grid nodes or cell centers), or
- at a new (uniform) grid, for 3D fields often a 2D slice

Arrows can visualize:

- direction
- relative magnitude (when approproiately scaled)
- time dependency (when animated)

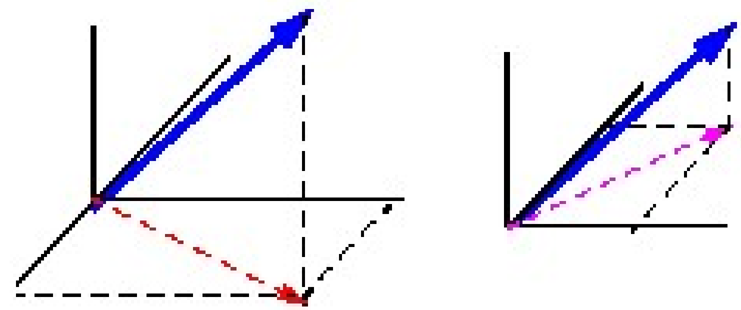
Problems of visualization with arrows:

- It is not clear whether arrows represent vector values at the start point or at the midpoint of the arrow
- Often no satisfactory scaling factor exists:
  - large scaling: Arrows occlude each other
  - small scaling: Direction is not recognizable in some regions
  - fixed length: Magnitude information is lost

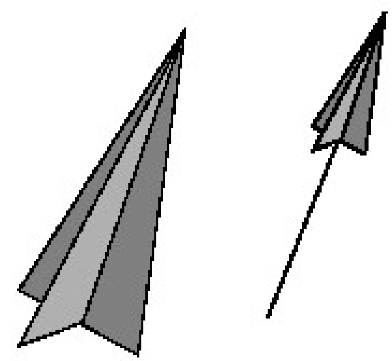


Arrows in 3D

- Problems:
  - Ambiguity
  - Perspective
  - 1D objects in 3D:
    difficult spatial perception
  - Visual clutter

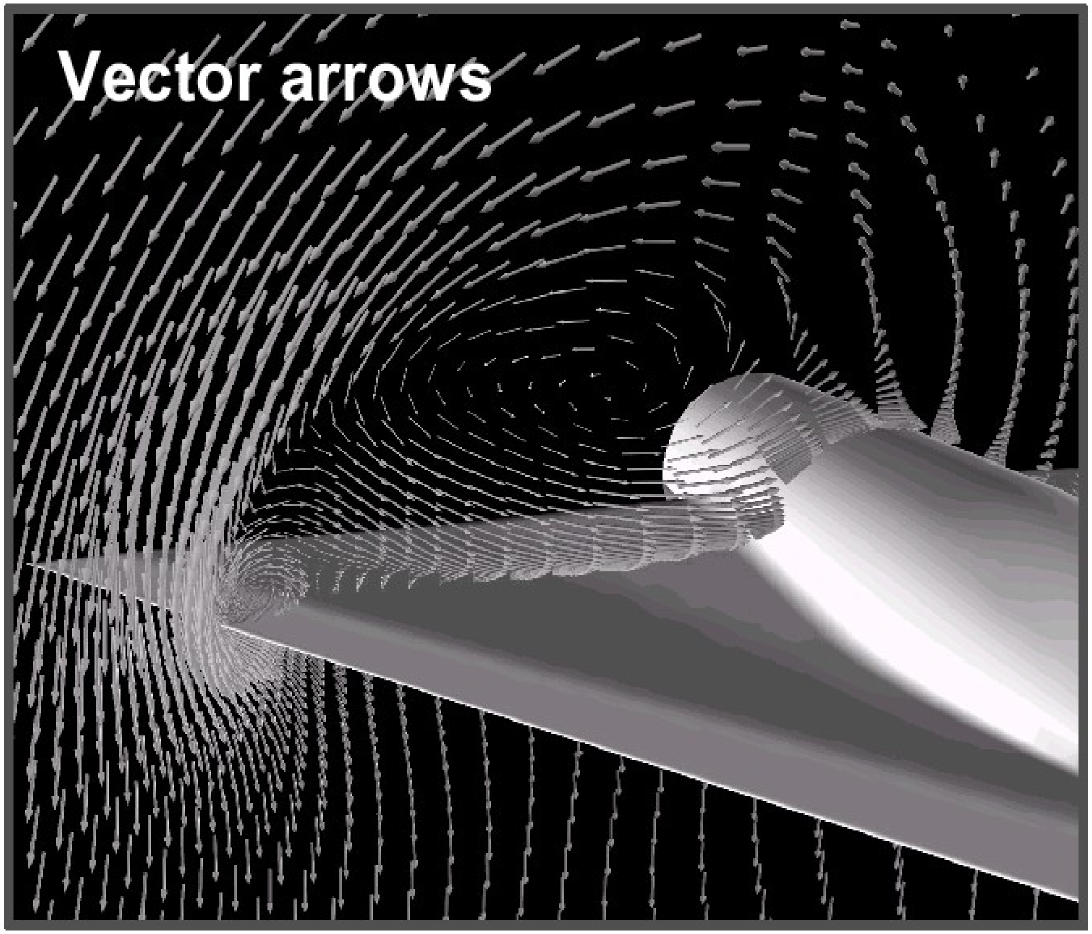


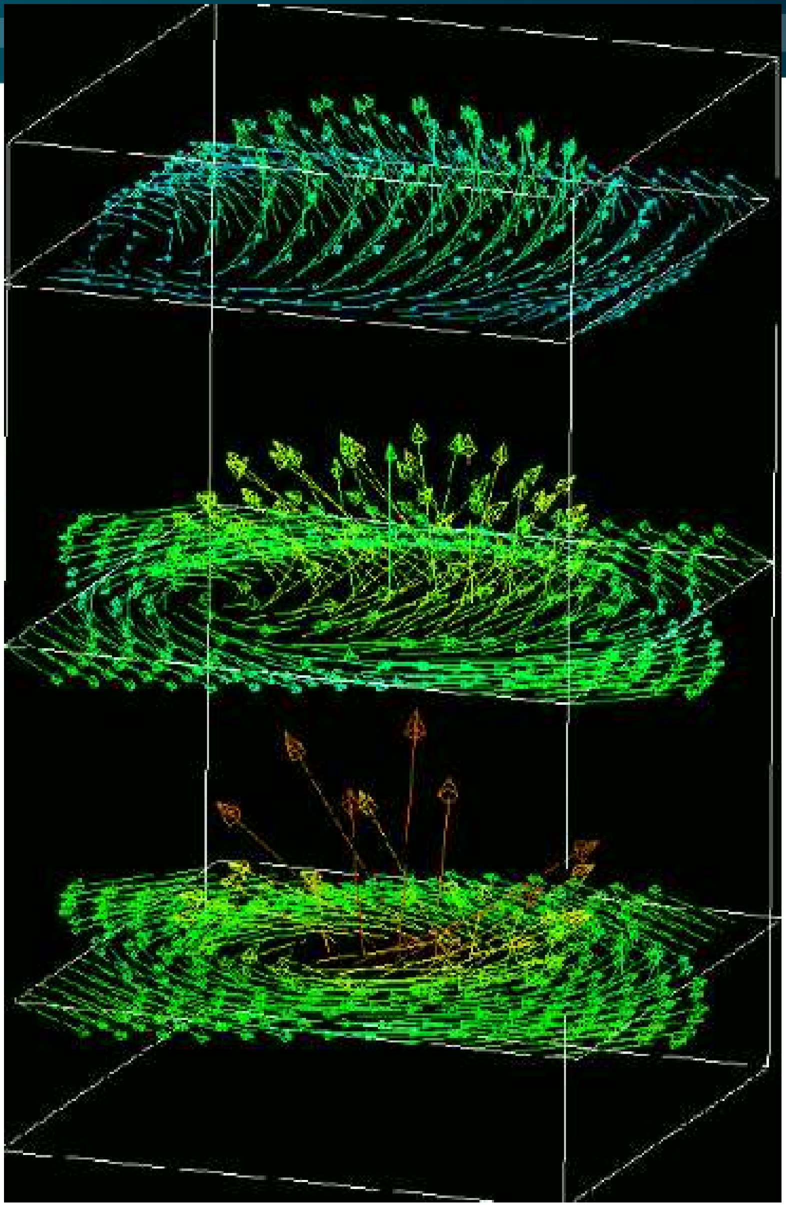
- Improvement:
  - 3D arrows (help to a certain extent)



Arrows in 3D

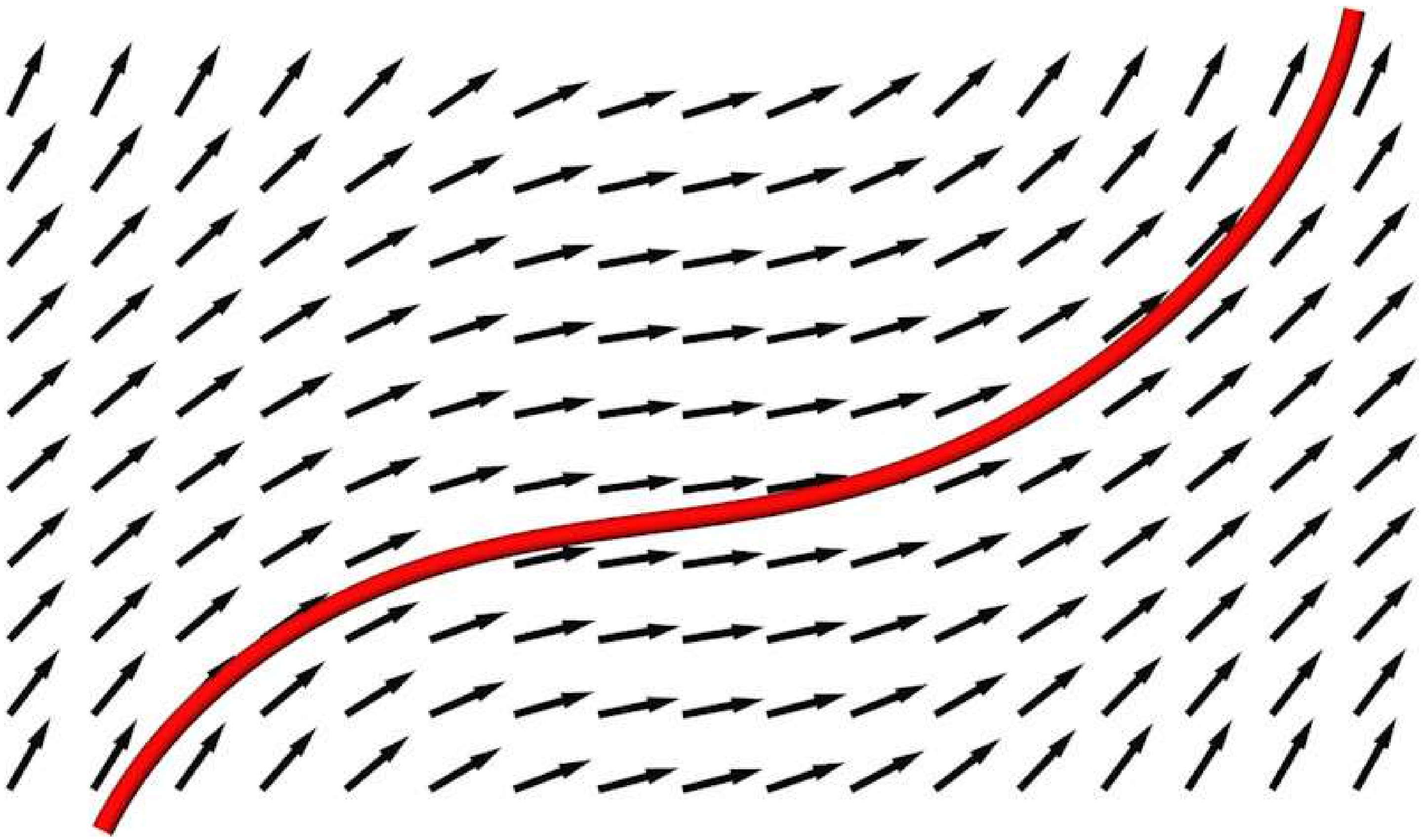
- Compromise:
  Arrows only in slices



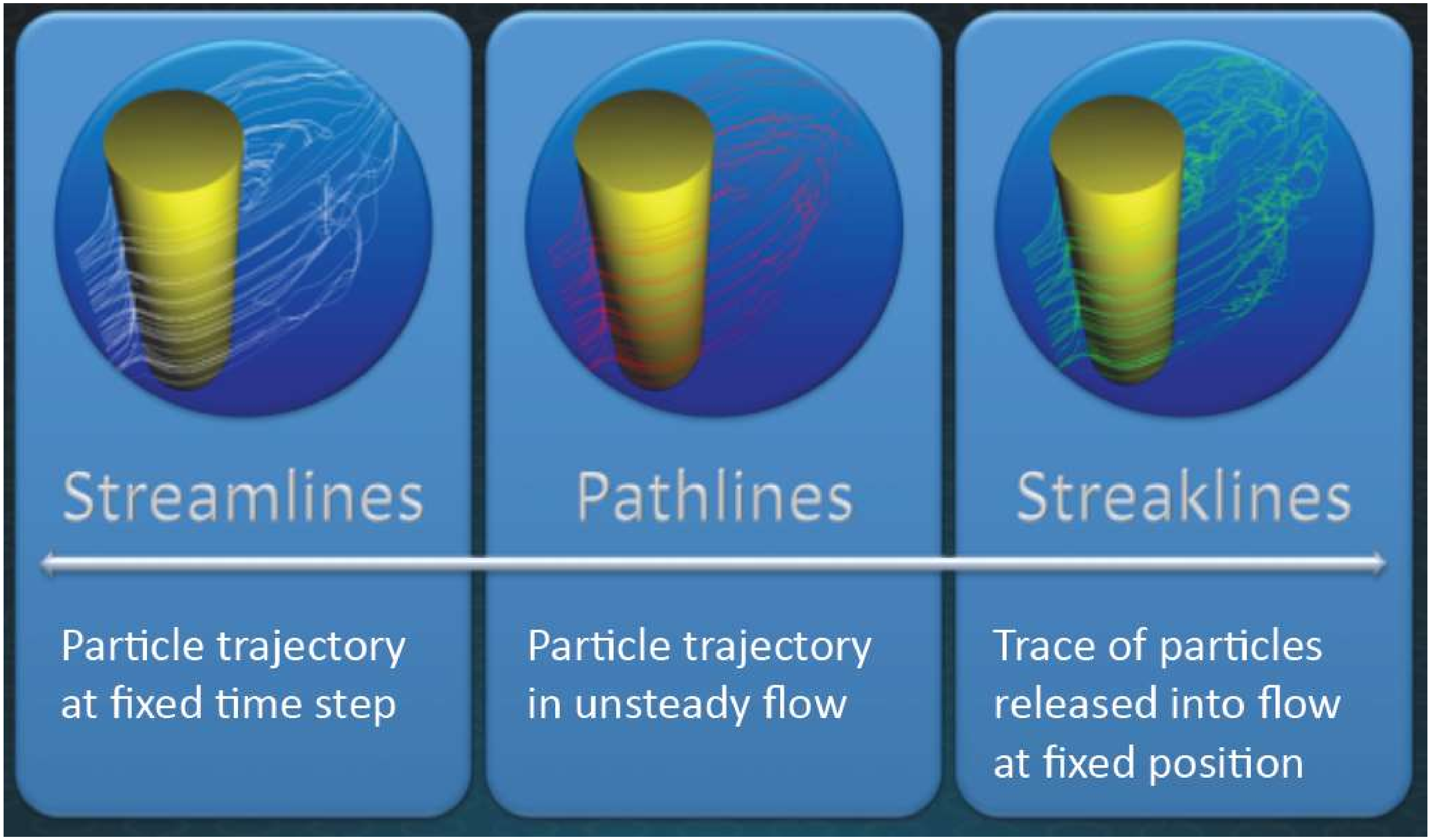


Indirect Flow Visualization

Integral Curves / Stream Objects



Integral Curves



Streamline

- Curve parallel to the vector field in each point for a fixed time

Pathline

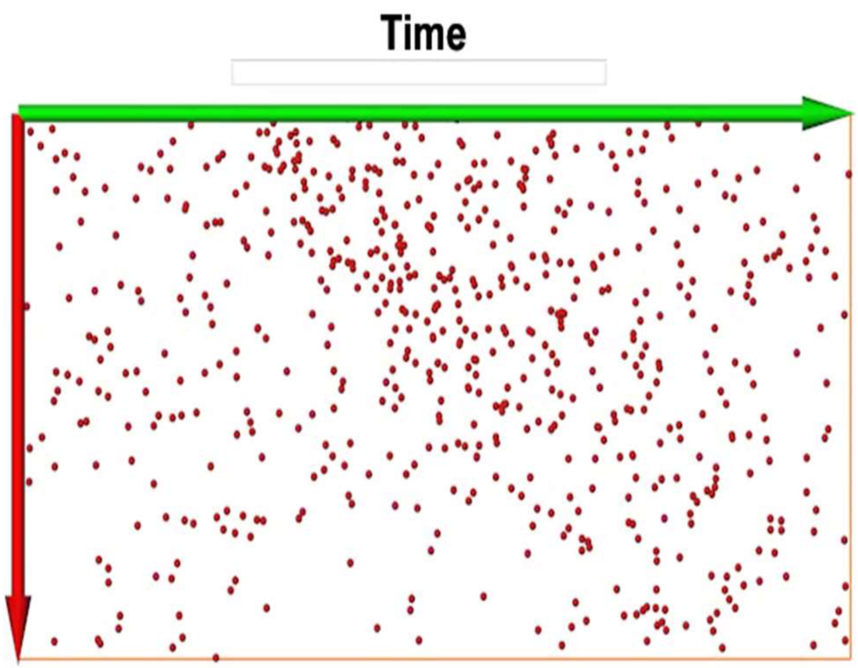
- Describes motion of a massless particle over time

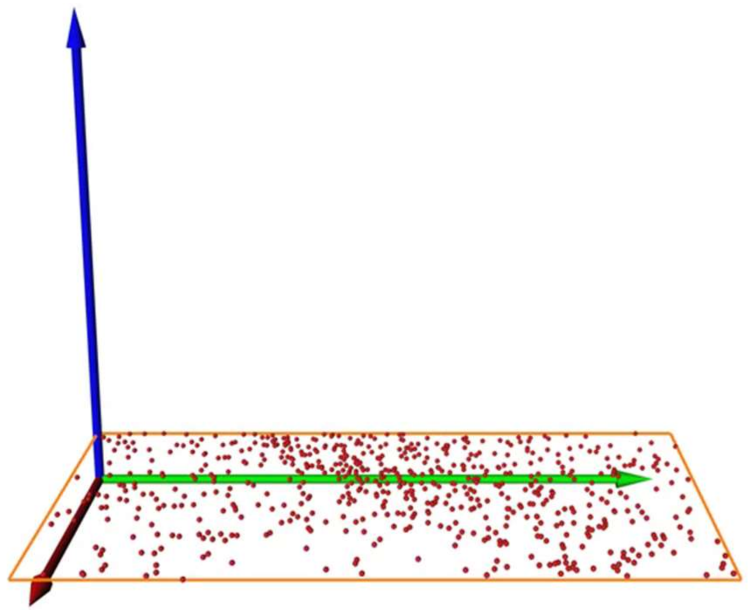
Streakline

- Location of all particles released at a *fixed position* over time

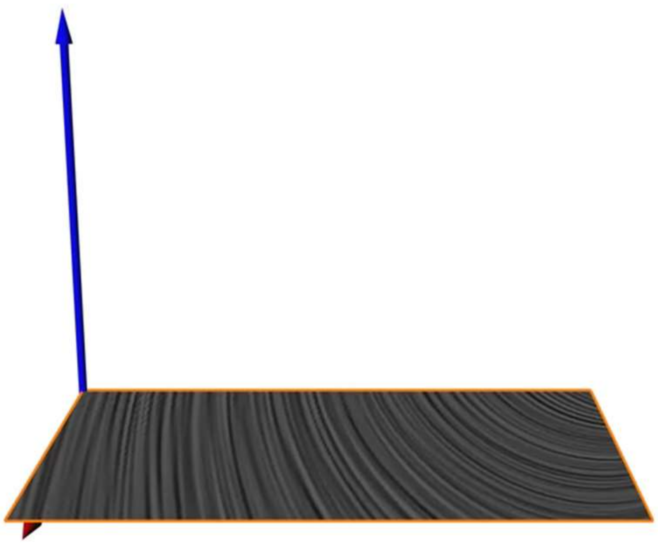
Timeline

- Location of all particles released along a line at a *fixed time*

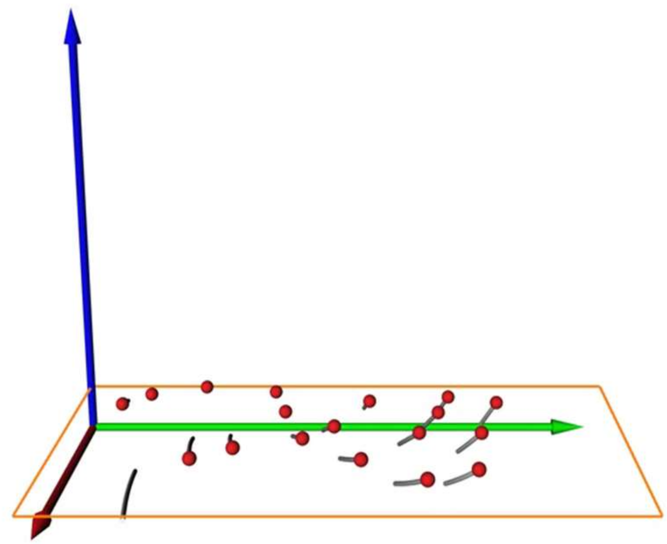




2D time-dependent vector field
particle visualization



stream lines



path lines

curve parallel to the vector field in each point for a **fixed time**

describes motion of a massless particle in an **steady** flow field

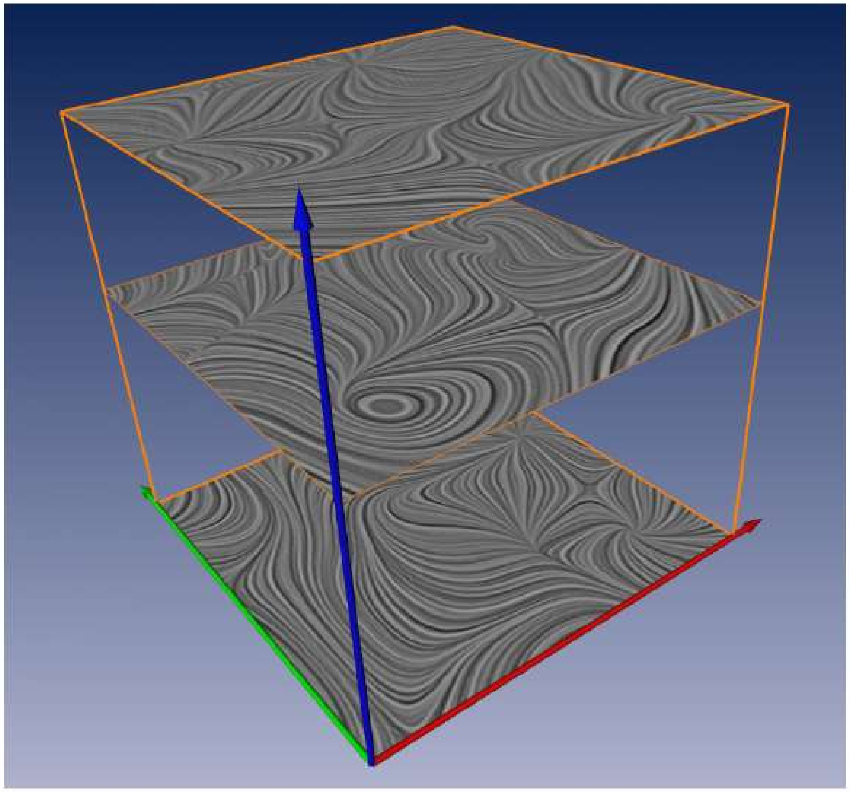
curve parallel to the vector field in each point **over time**

describes motion of a massless particle in an **unsteady** flow field

Streamlines Over Time

Defined only for steady flow or for a fixed time step (of unsteady flow)

Different tangent curves in every time step for time-dependent vector fields (unsteady flow)

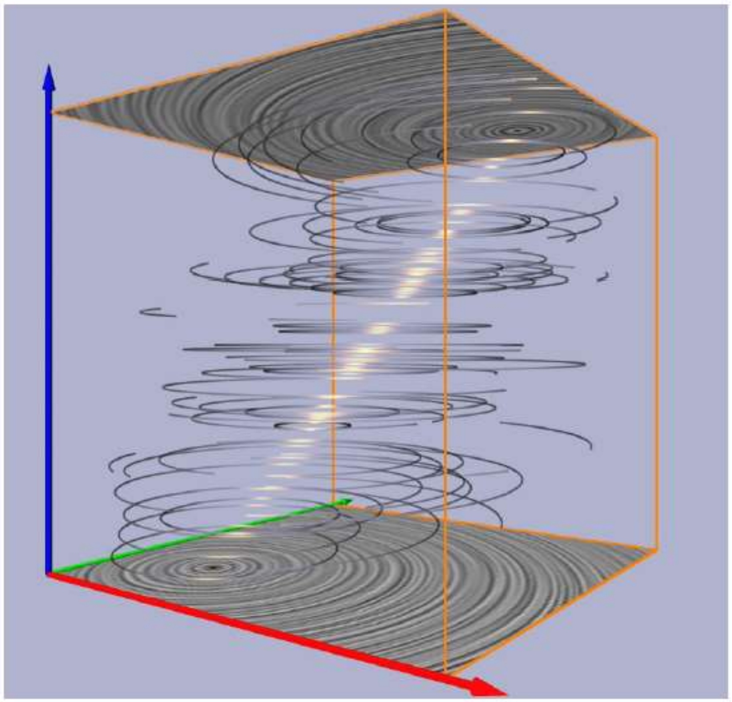


Tino Weinkauf

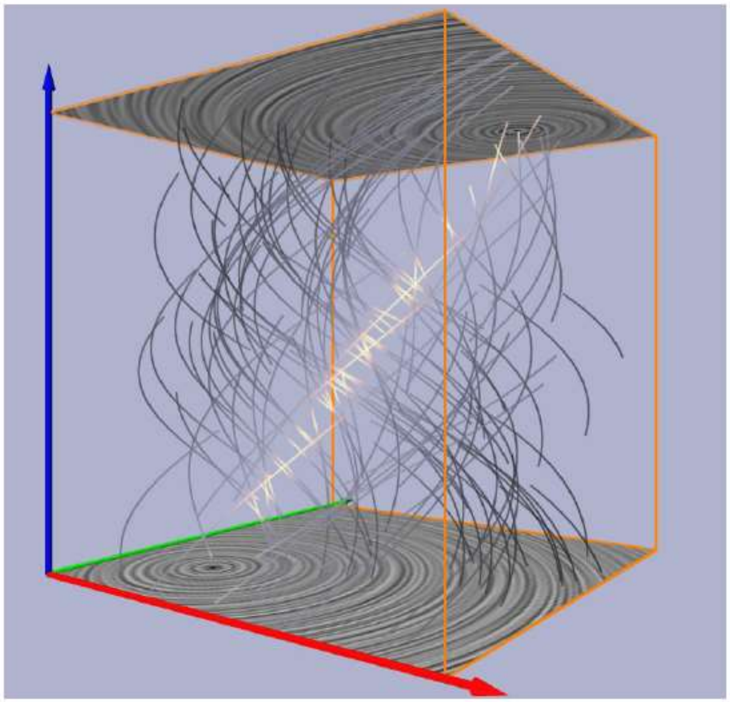
Stream Lines vs. Path Lines Viewed Over Time

Plotted with time as third dimension

- Tangent curves to a (n + 1)-dimensional vector field



Stream Lines



Path Lines

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama