King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 19: Vector / Flow Visualization, Pt. 1

Markus Hadwiger, KAUST

Reading Assignment #11 (until Apr 12)

Read (required):

- Data Visualization book
  - Chapter 6 (Vector Visualization)
    - Beginning (before 6.1)
    - Chapters 6.2, 6.3, 6.5
- More general vector field basics (the book is not very precise on the basics)

  `https://en.wikipedia.org/wiki/Vector_field`

Read (optional):

- Paper:
  Bruno Jobard and Wilfrid Lefer
  Creating Evenly-Spaced Streamlines of Arbitrary Density,

  `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.9498`

Online Demos and Info

Numerical ODE integration methods (Euler vs. Runge Kutta, etc.)

https://demonstrations.wolfram.com/NumericalMethodsForDifferentialEquations/

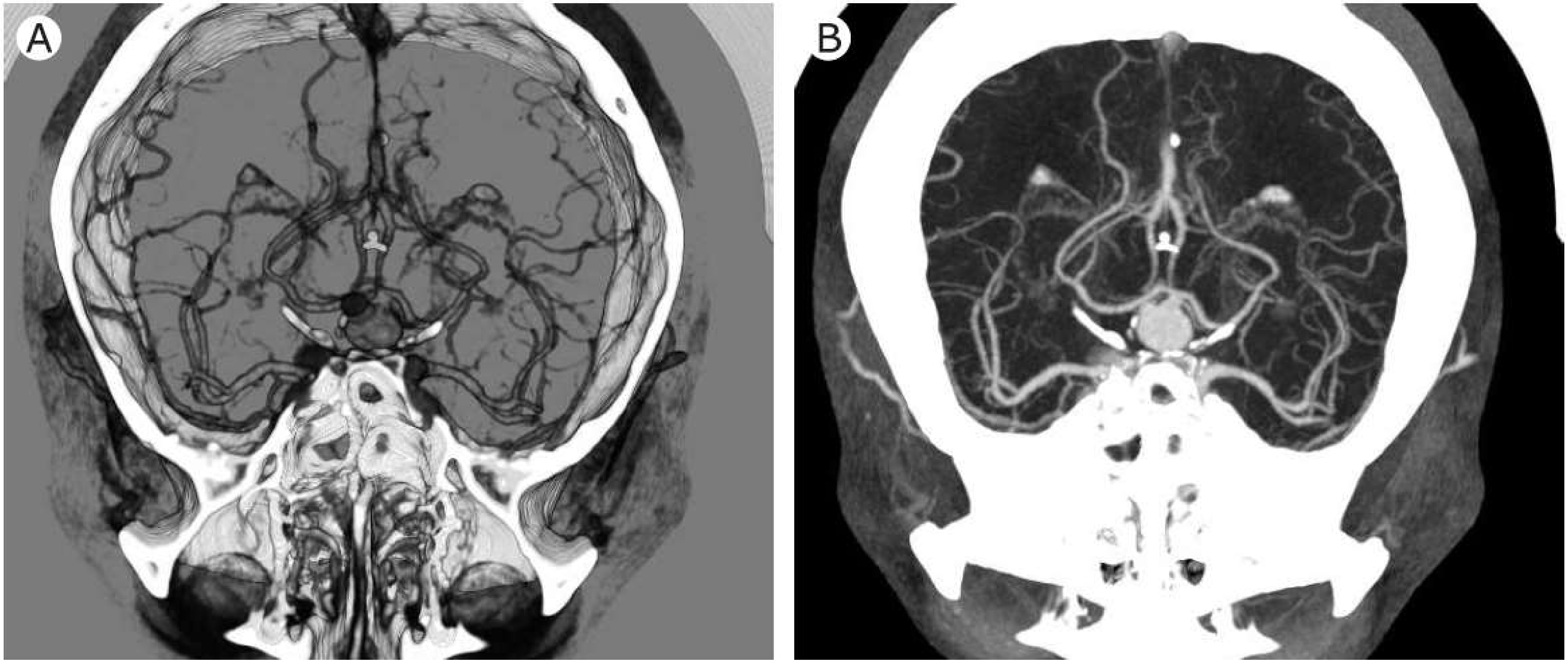
Flow visualization concepts

https://www3.nd.edu/~cwang11/flowvis.html

Maximum Intensity Projection

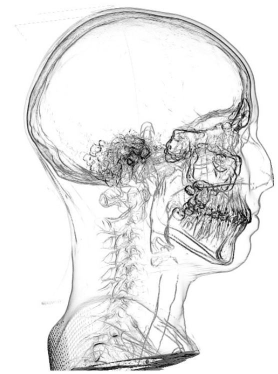
Alternative compositing mode (no alpha blending)

Keeps structure of maximum intensity visible



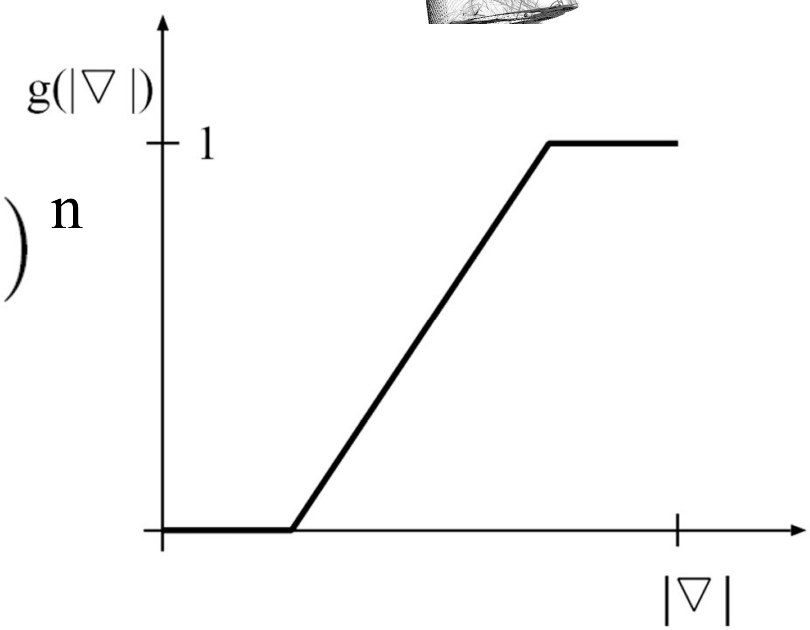
Volumetric Boundary Contours (1)

Based on view direction and gradient magnitude



Global boundary detection
instead of isosurface

Gradient magnitude window g(.)



$$\mathbf{I} = g\big(|\nabla f|\big) \cdot \big(1 - |\mathbf{v} \cdot \mathbf{n}|\big)^{n}$$

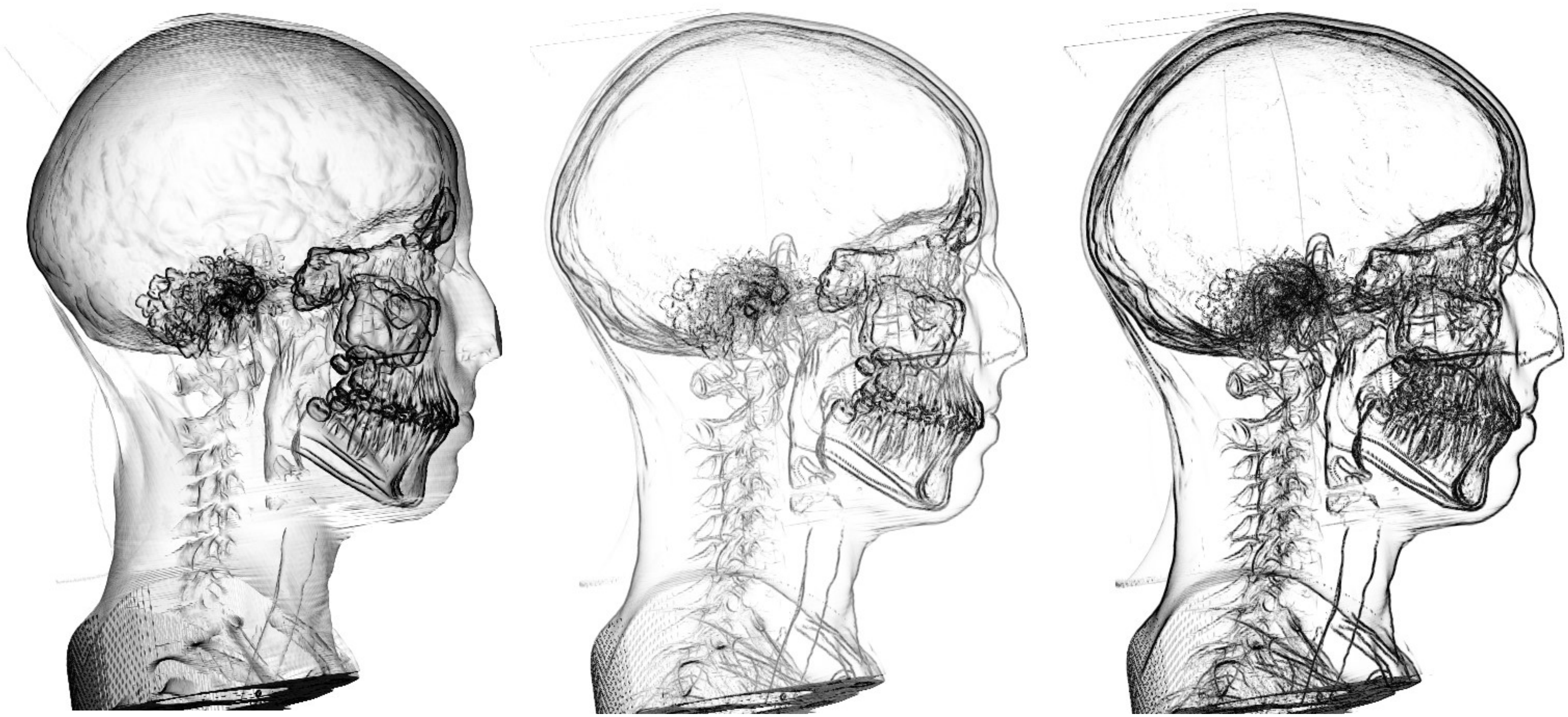
Exponent determines silhouette range

Does not work for distance fields!

Volumetric Boundary Contours (2)

Gradient magnitude window is main parameter

Exponent between 4 and 16 is good choice

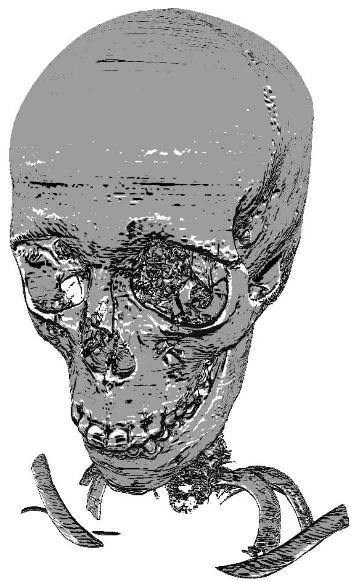


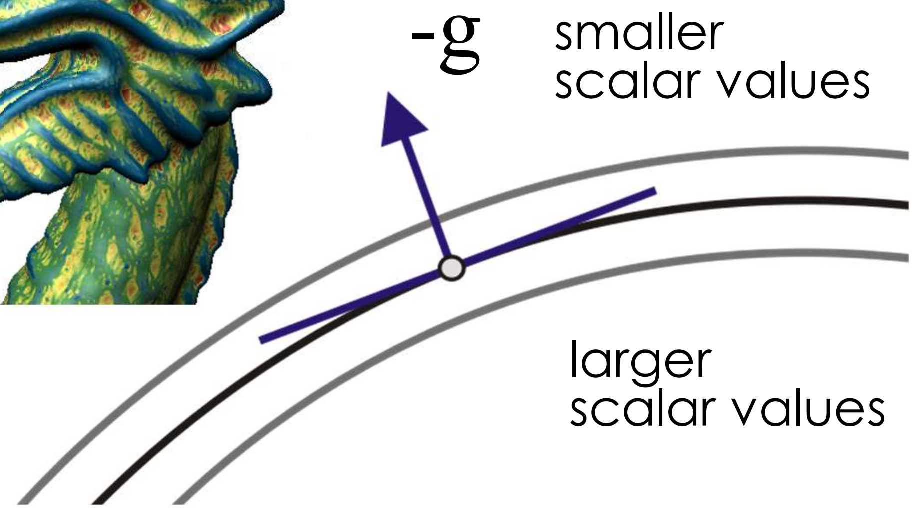
Curvature-Based Transfer Functions

Curvature-Based Isosurface Illustration

Curvature measure color mapping

Curvature directions; ridges and valleys



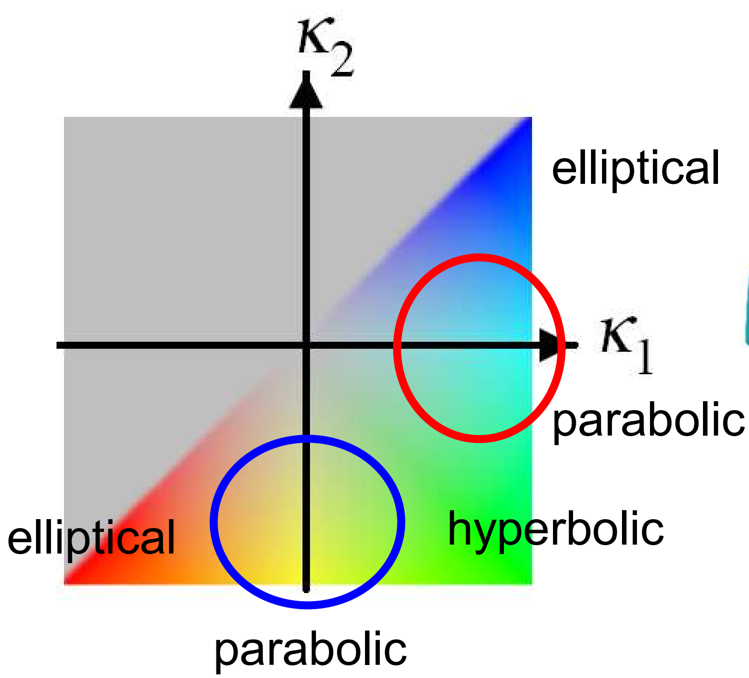


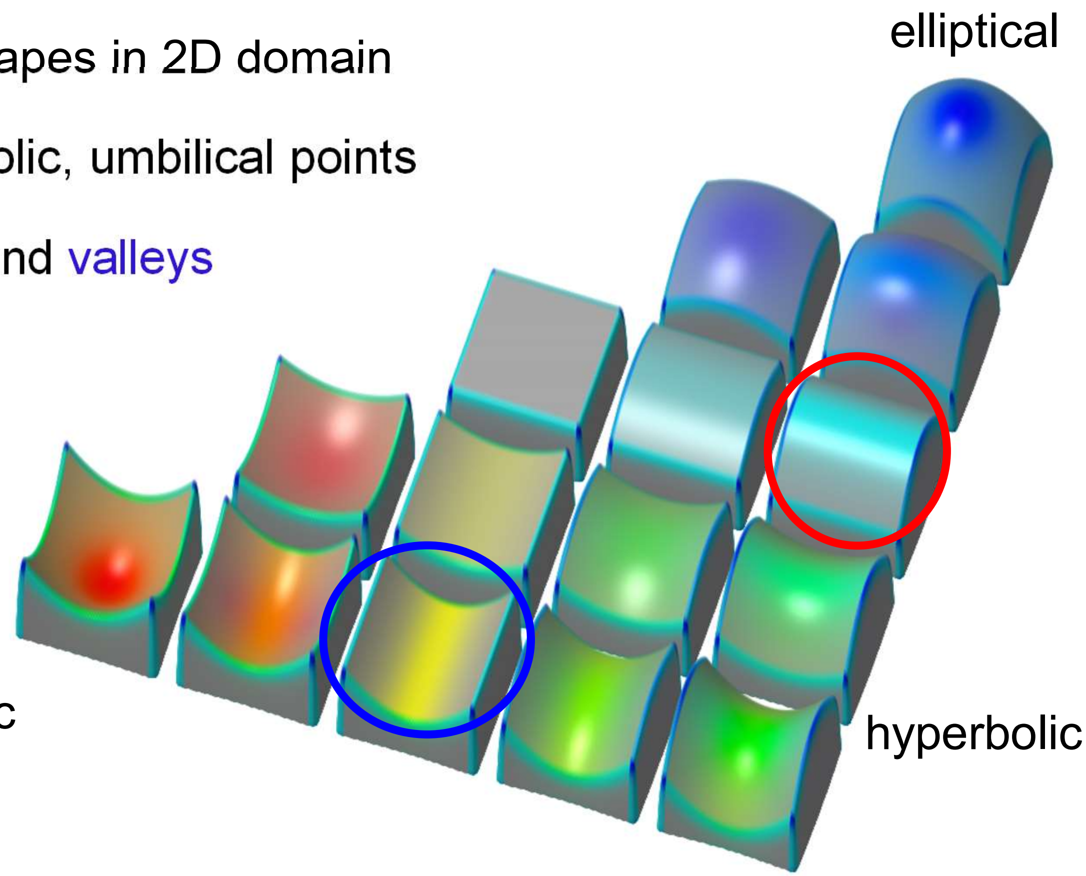
- Implicit surface curvature
- Isosurface through a point

The Principal Curvature Domain

Maximum/minimum principal curvature magnitude

- Identification of different shapes in 2D domain
- Elliptical, parabolic, hyberbolic, umbilical points
- Feature lines: e.g., ridges and valleys

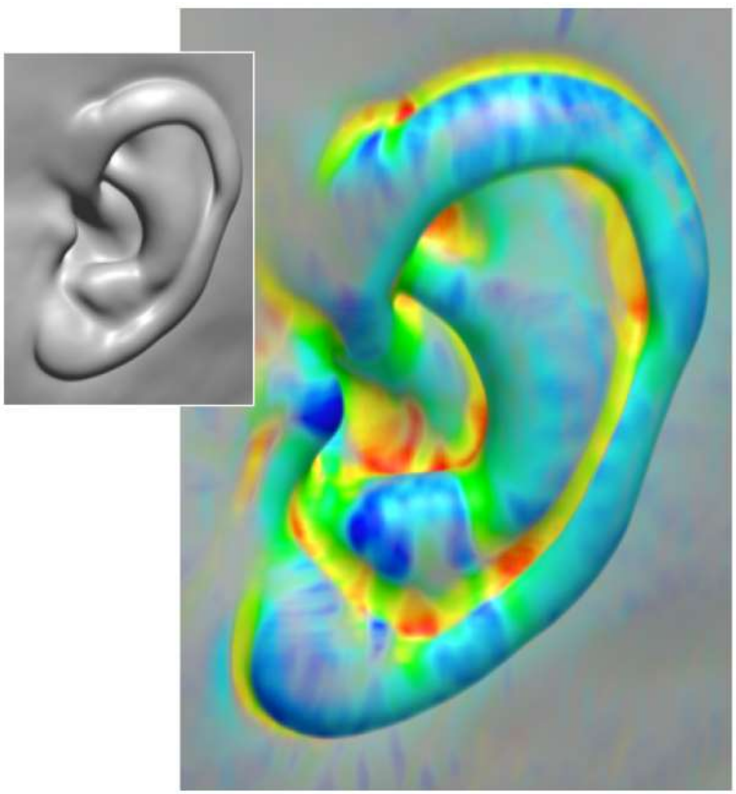


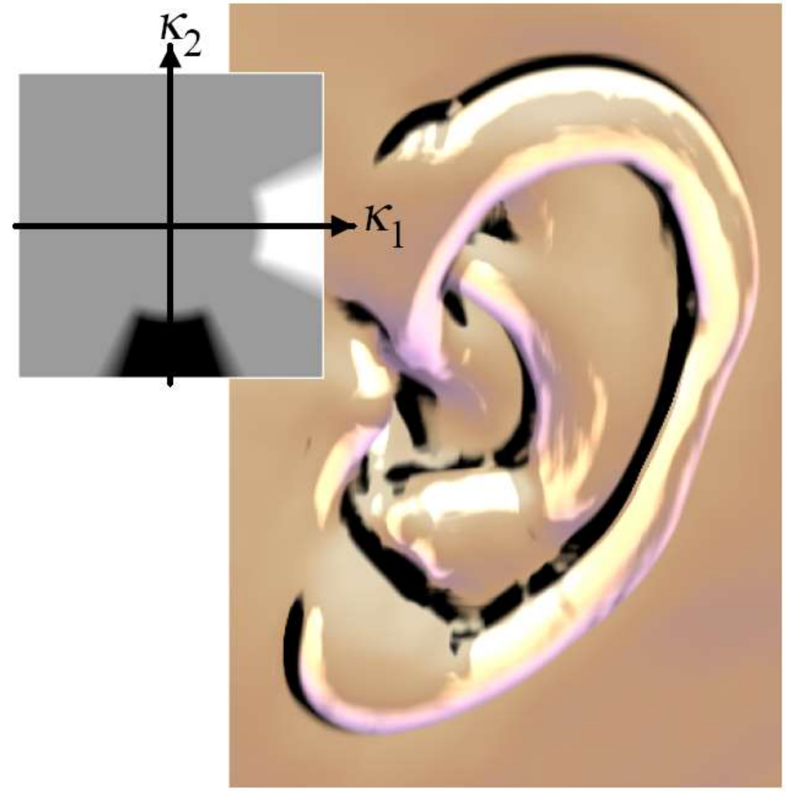


courtesy of Gordon Kindlmann

Curvature Transfer Functions

- Color coding of curvature domain
- Paint features: ridge and valley lines



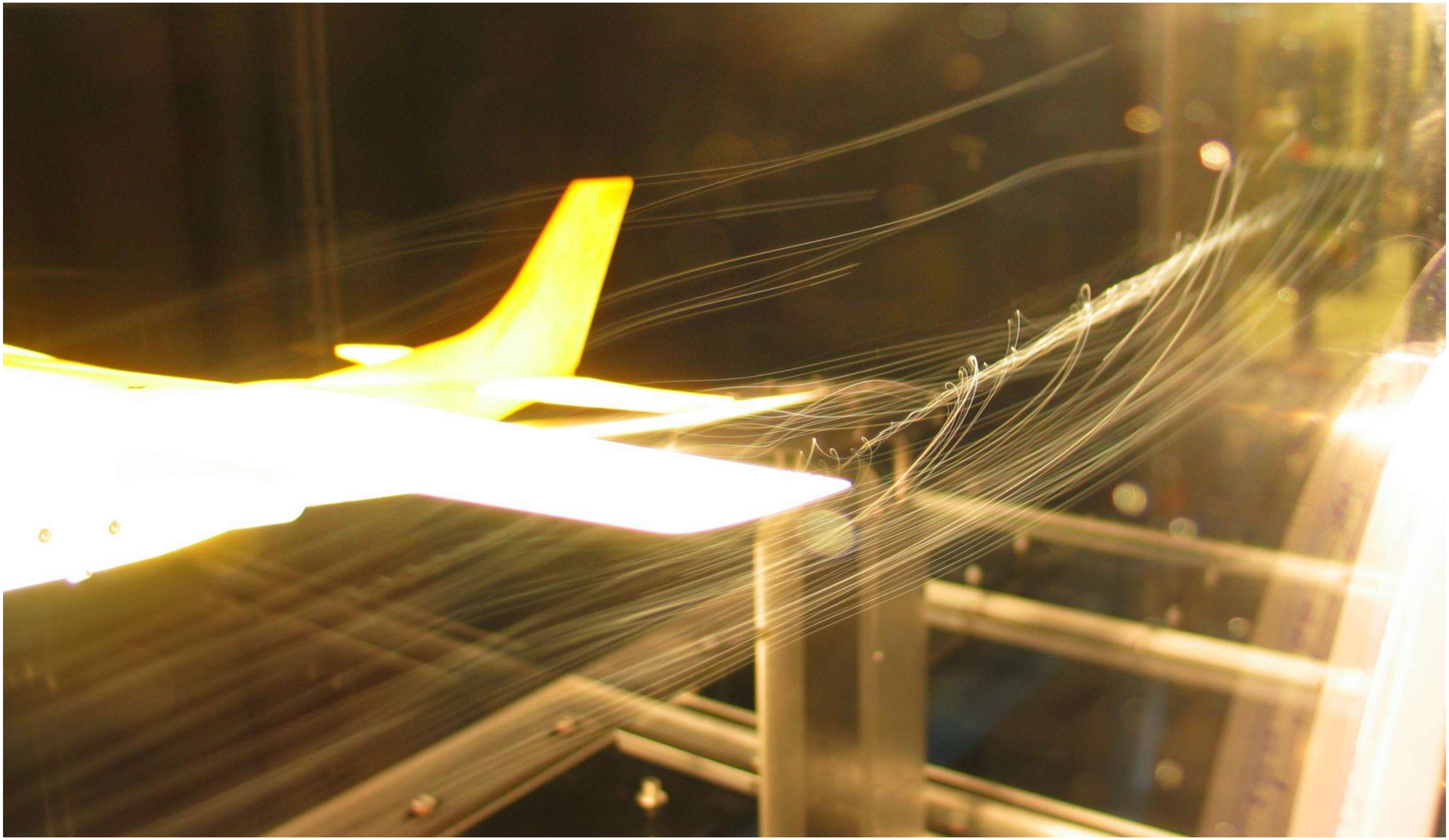


courtesy of Gordon Kindlmann

Example

Vector Field / Flow Visualization

Smoke angel
A C-17 Globemaster III from the 14th Airlift Squadron, Charleston Air Force Base, S.C. flies off after releasing flares over the Atlantic Ocean near Charleston, S.C., during a training mission on Tuesday, May 16, 2006. The "smoke angel" is caused by the vortex from the engines.
(U.S. Air Force photo/Tech. Sgt. Russell E. Cooley IV)



A wind tunnel model of a Cessna 182 showing a wingtip vortex.
Tested in the RPI (Rensselaer Polytechnic Institute) Subsonic Wind Tunnel.
By Ben FrantzDale (2007).

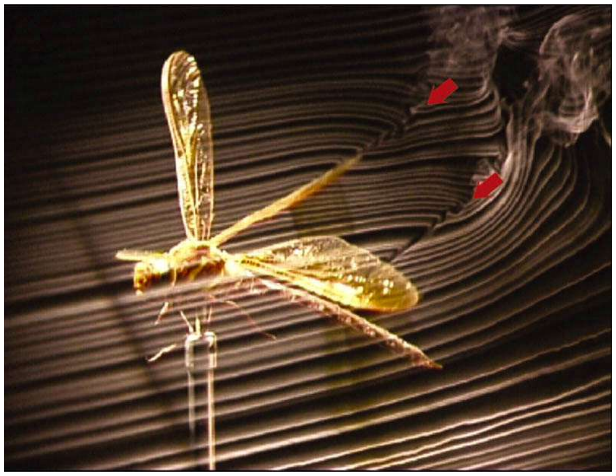

http://autospeed.com/cms/A_108677/article.html

wool tufts


http://autospeed.com/cms/A_108677/article.html

smoke injection

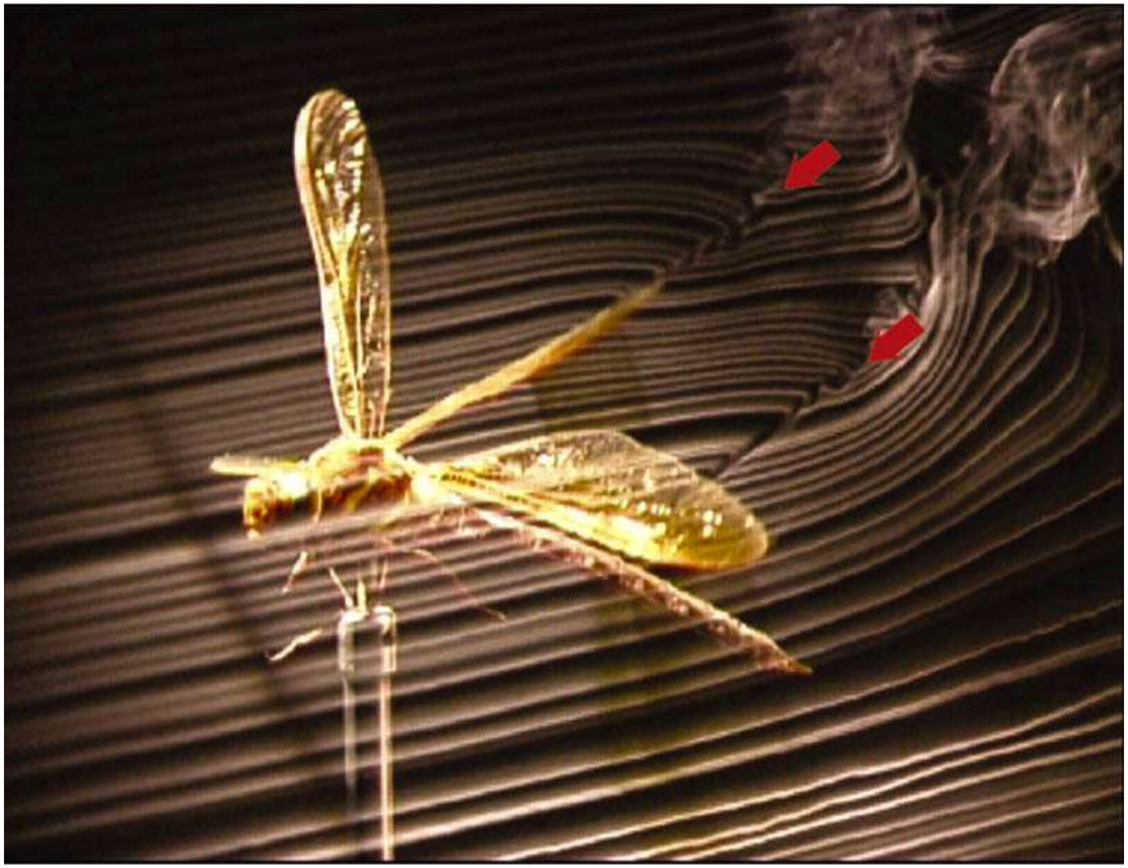
http://autospeed.com/cms/A_108677/article.html smoke nozzles



[NASA, J. Exp. Biol.]

http://autospeed.com/cms/A_108677/article.html smoke nozzles

Smoke injection

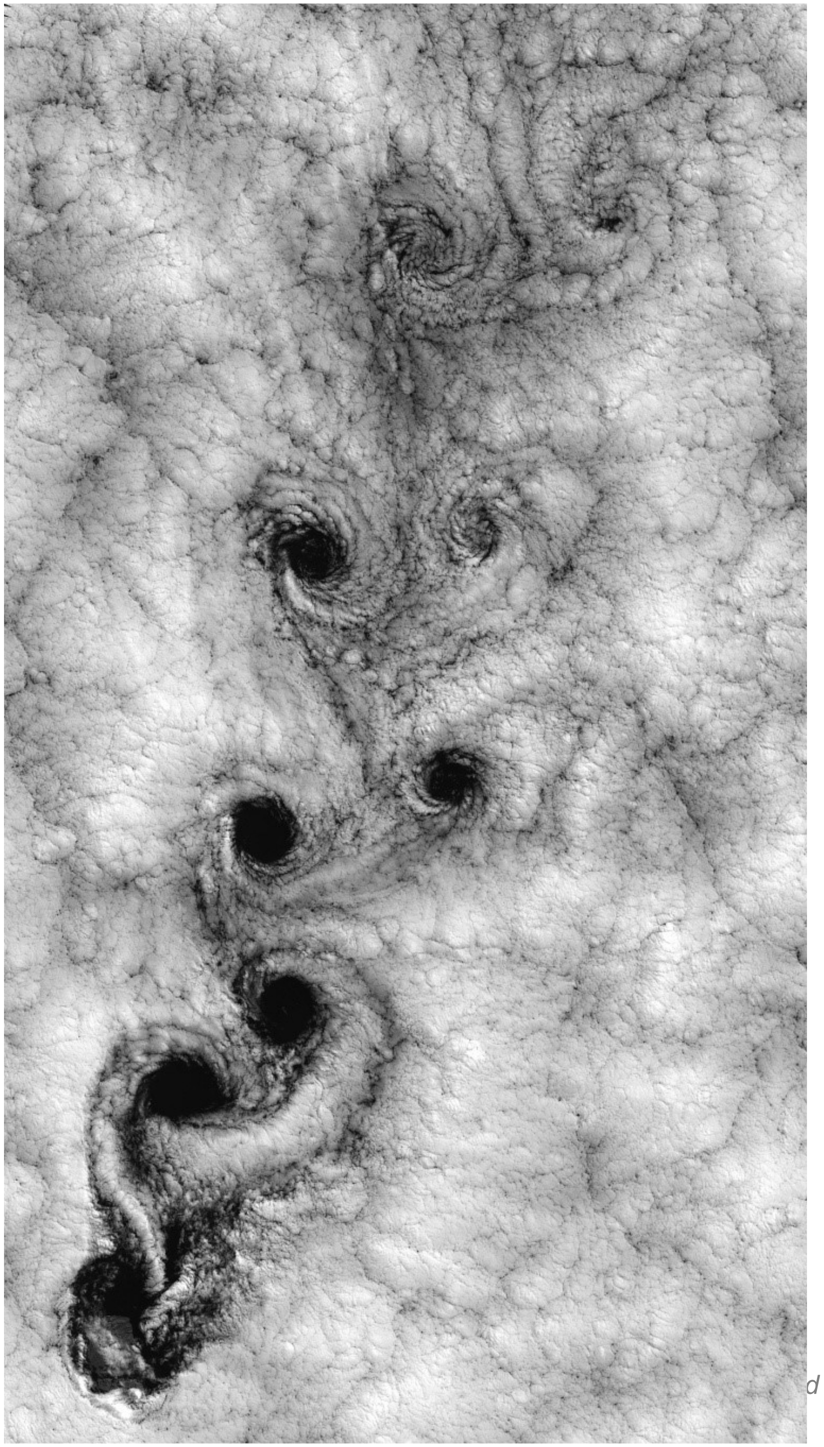


A. L. R. Thomas, G. K. Taylor, R. B. Srygley, R. L. Nudds, and R. J. Bomphrey. Dragonfly flight: free-flight and tethered flow visualizations reveal a diverse array of unsteady lift-generating mechanisms, controlled primarily via angle of attack. J Exp Biol, 207(24):4299–4323, 2004.

http://de.wikipedia.org/wiki/Bild:Airplane_vortex_edit.jpg

Smoke injection

http://www-me.ccny.cuny.edu/research/aerolab/facilities/images/wt2.jpg



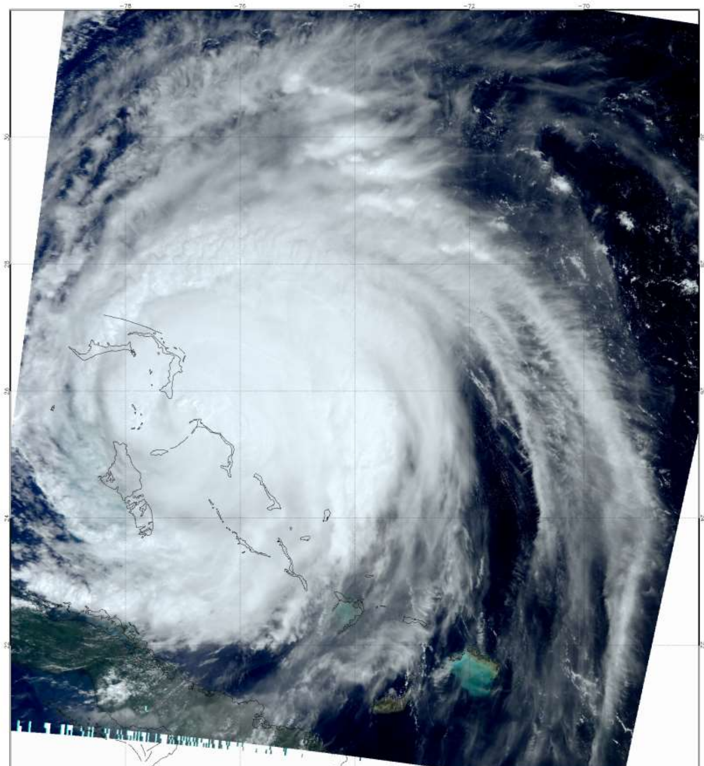
Clouds (satellite image)

Juan Fernandez Islands

http://de.wikipedia.org/wiki/Bild:Vortex-street-1.jpg

Clouds (satellite image)

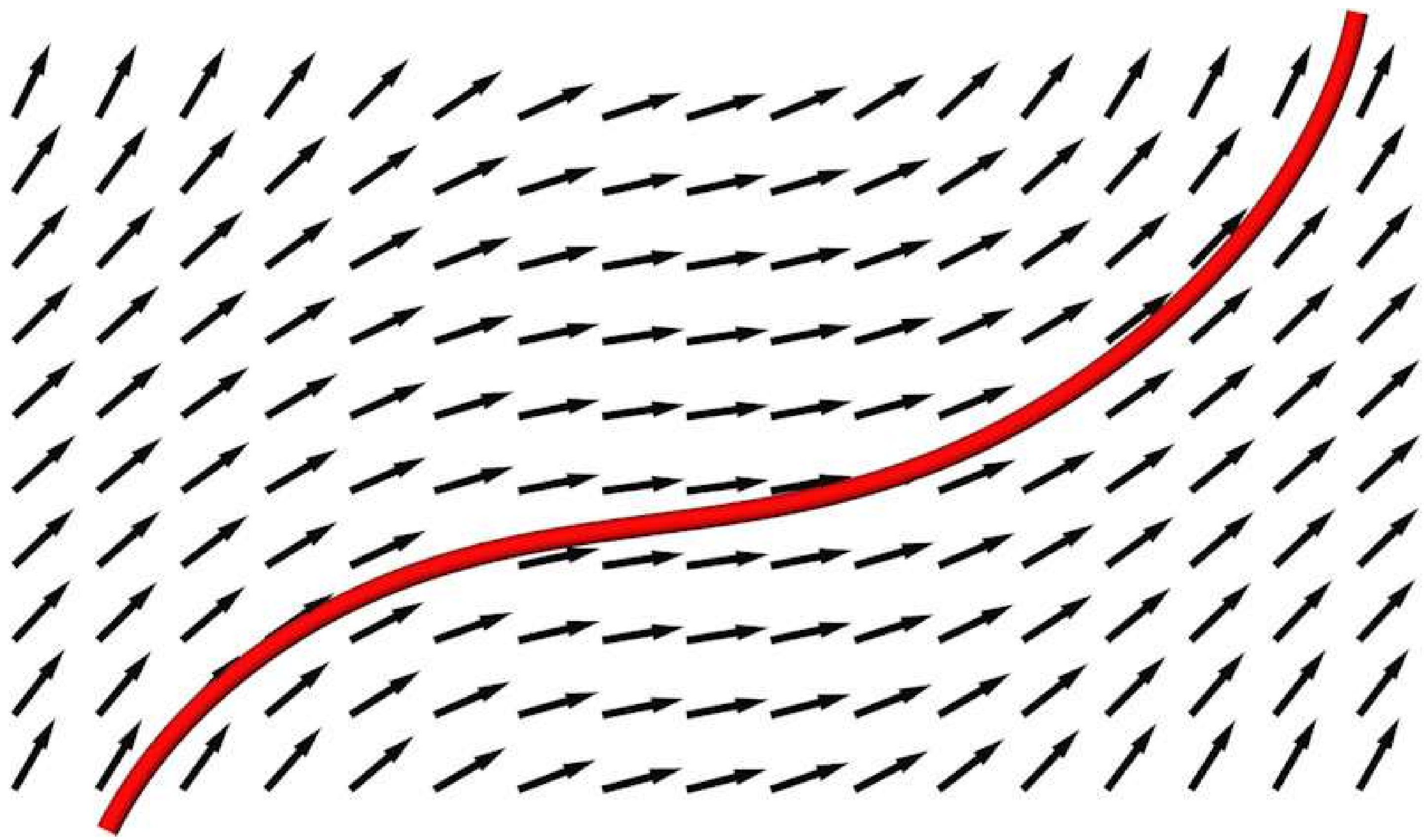
http://daac.gsfc.nasa.gov/gallery/frances/



- **Vortex/ Vortex core lines**
  - There is no exact definition of vortices
  - capturing some swirling behavior

Integral Curves / Stream Objects

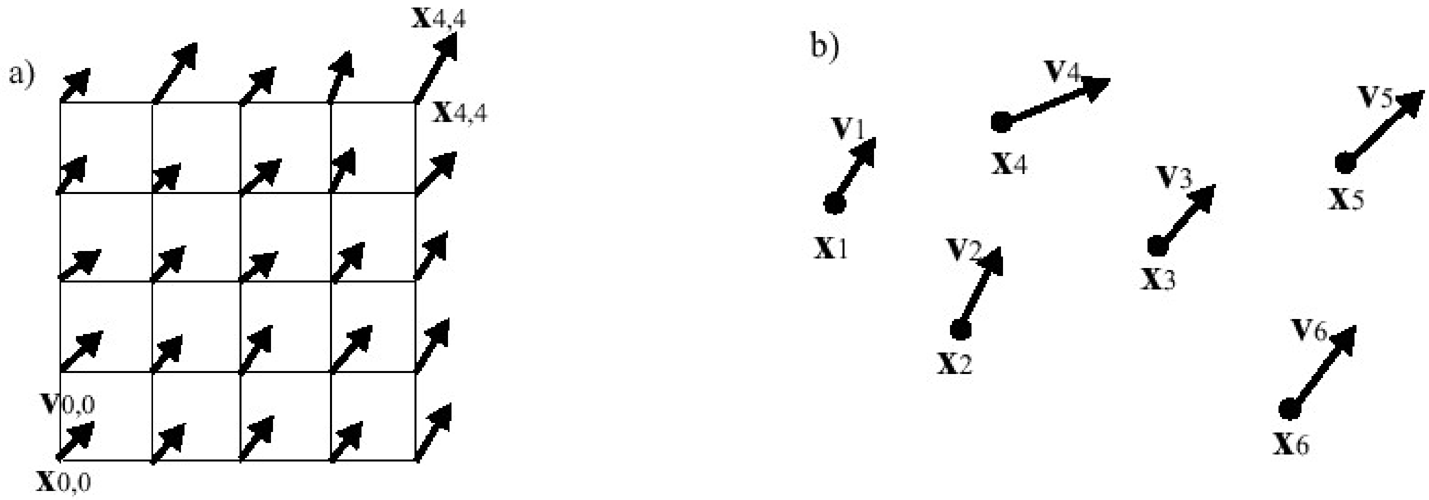
Integrating velocity over time yields spatial motion



Vector Fields

Each vector is usually thought of as a velocity vector

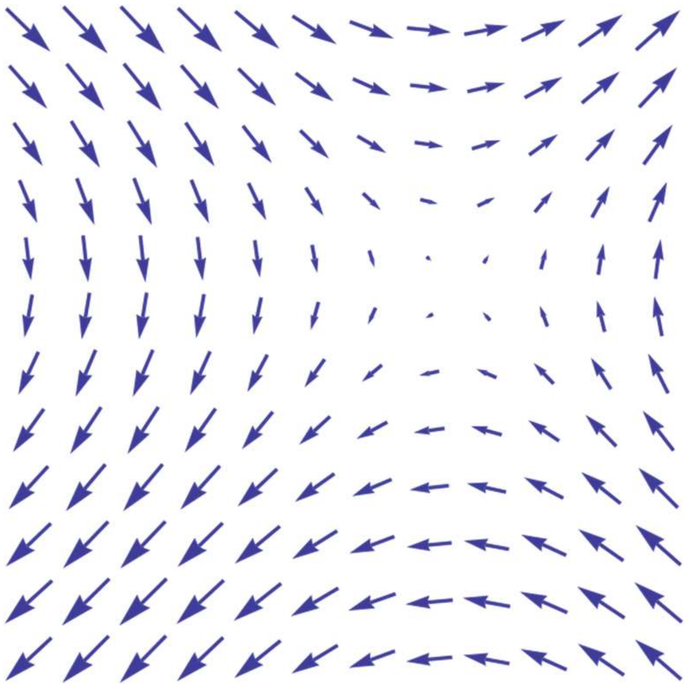
- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

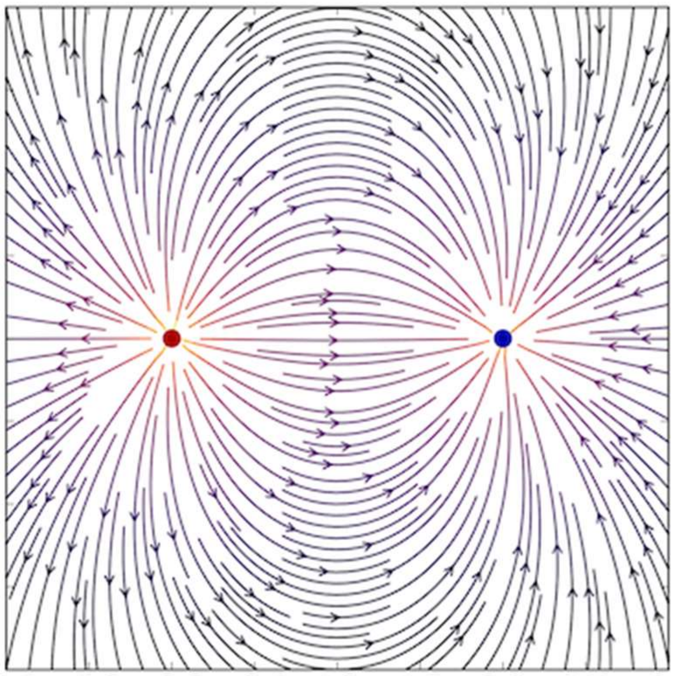


Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)





images from wikipedia

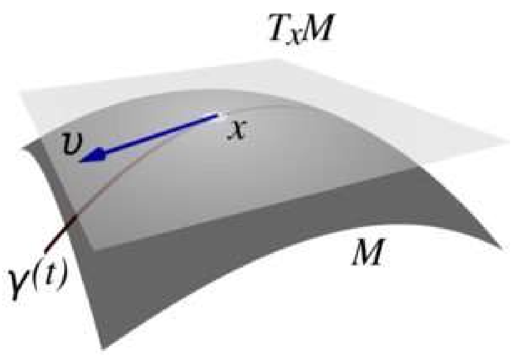
Vector Fields

Each vector is usually thought of as a velocity vector

- Example for actual velocity: fluid flow
- But also force fields, etc. (e.g., electrostatic field)

Each vector in a vector field
lives in the **tangent space**
of the manifold at that point:

Each vector is a **tangent vector**



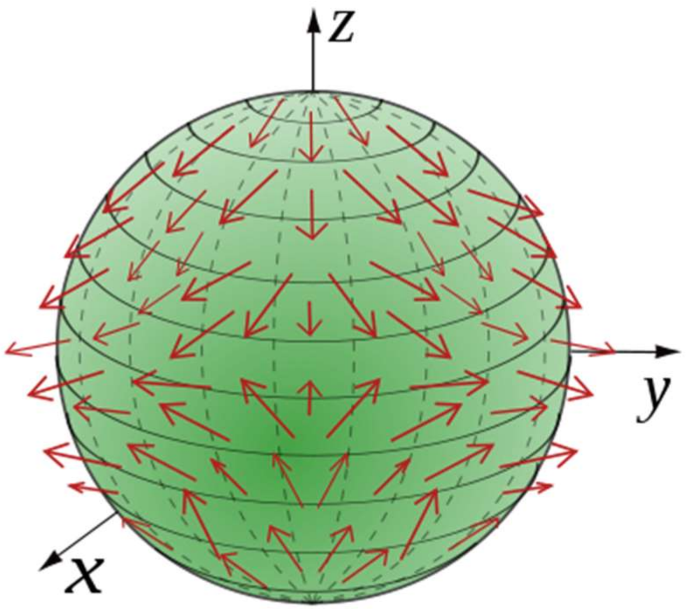


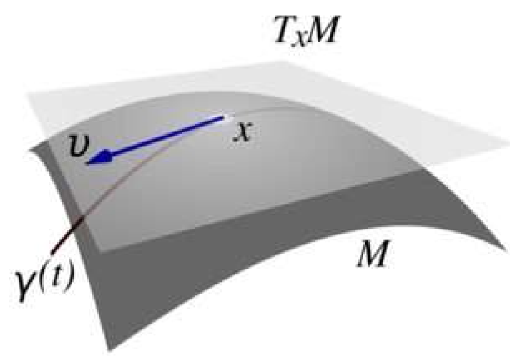
image from wikipedia

Vector Fields

Vector fields on general manifolds M (not just Euclidean space)

Tangent space at a point $x \in M$:

$$T_x M$$



Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \rightarrow M$$

Vector field: *Section* of tangent bundle

$$s : M \rightarrow TM, \quad x \mapsto s(x).$$

$$\pi(s(x)) = x$$

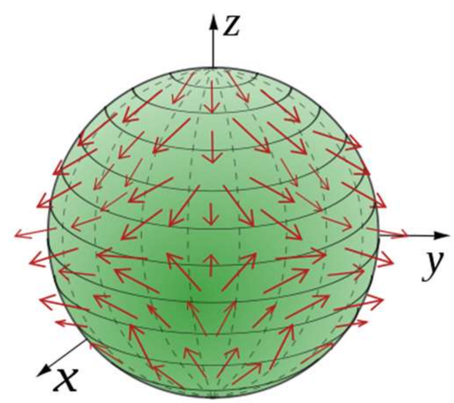


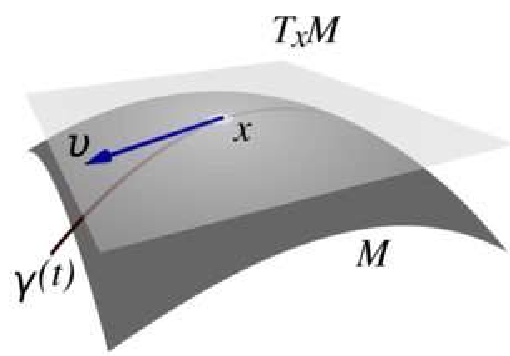
image from wikipedia

Vector Fields

Vector fields on general manifolds M (not just Euclidean space)

Tangent space at a point $x \in M$:

$$T_x M$$



Tangent bundle: Manifold of all tangent spaces over base manifold

$$\pi : TM \to M$$

Vector field: *Section* of tangent bundle

$$\mathbf{v} : M \to TM, \quad x \mapsto \mathbf{v}(x). \qquad \mathbf{v}(x) \in T_x M$$

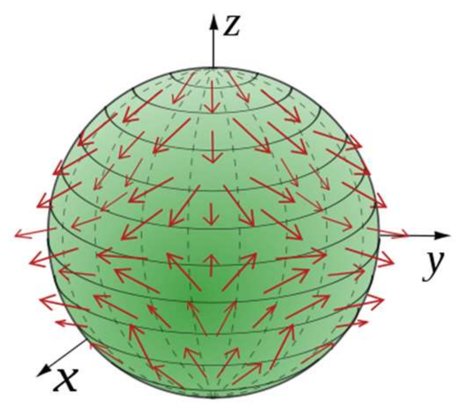
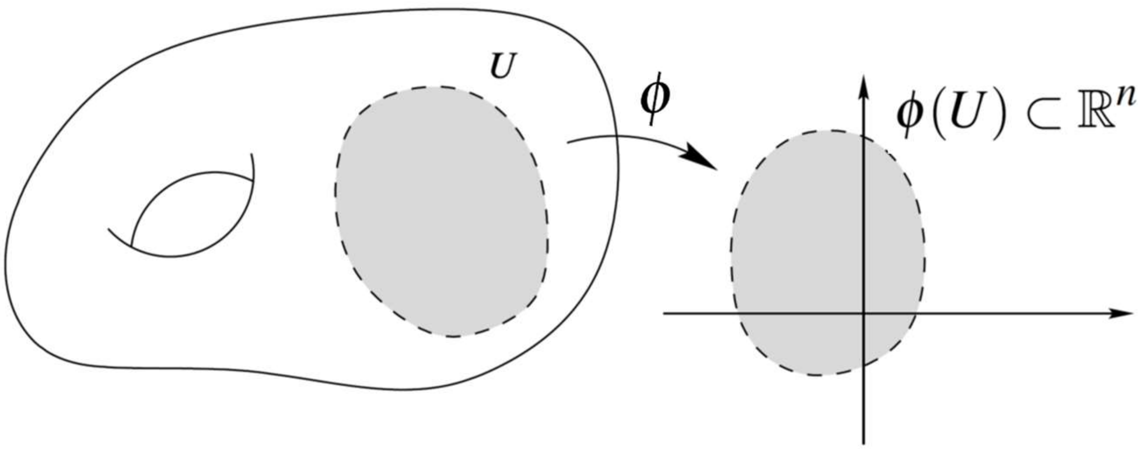


image from wikipedia

Interlude: Coordinate Charts

Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$



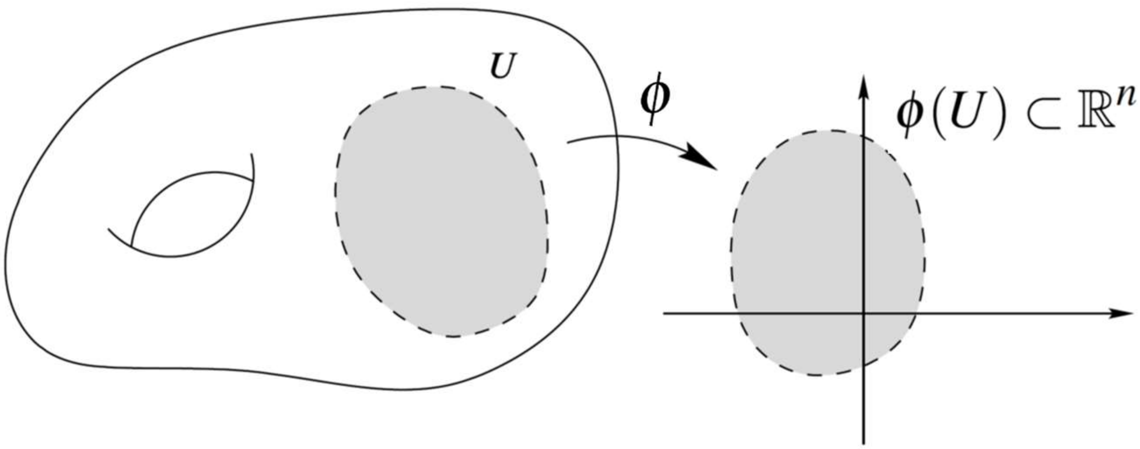
Interlude: Coordinate Charts

Coordinate chart

$$\phi : U \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Coordinate functions

$$x^i : U \subset M \to \mathbb{R},$$
$$x \mapsto x^i(x).$$



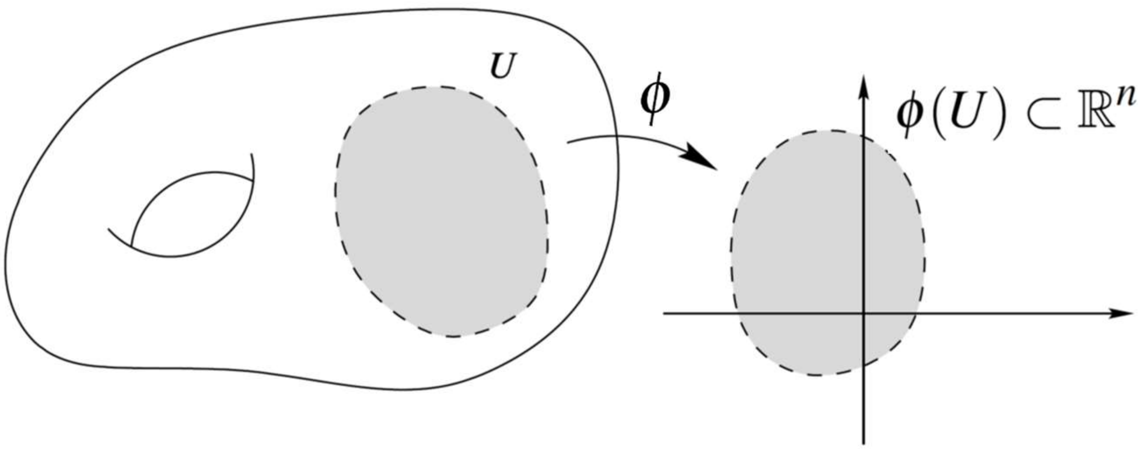
Interlude: Coordinate Charts

Coordinate charts

$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

Atlas

$$\{(U_\alpha, \phi_\alpha)\}_{\alpha \in I}$$



Interlude: Coordinate Charts

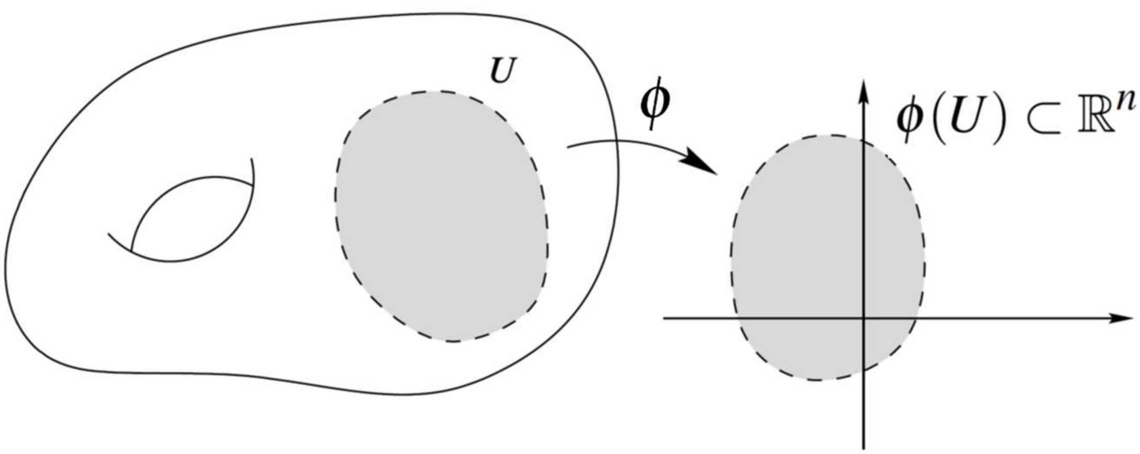
Coordinate charts

$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1, x^2, \ldots, x^n).$$

$$\phi_\alpha : U_\alpha \subset M \to \mathbb{R}^n,$$
$$x \mapsto (x^1(x), x^2(x), \ldots, x^n(x)).$$

Atlas

$$\{(U_\alpha, \phi_\alpha)\}_{\alpha \in I}$$



Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\mathbf{v} : U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$

$$(x, y) \mapsto \begin{bmatrix} u \\ v \end{bmatrix}.$$

Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\mathbf{v} : U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
$$(x, y) \mapsto \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\mathbf{v} : U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
$$(x, y) \mapsto \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix}.$$

Vector Fields vs. Vectors in Components

Because Euclidean space is most common, often slightly sloppy notation

$$\mathbf{v}: U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
$$(x,y) \mapsto \begin{bmatrix} u \\ v \end{bmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^3 \to \mathbb{R}^3,$$
$$(x,y,z) \mapsto \begin{bmatrix} u \\ v \\ w \end{bmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^2 \to \mathbb{R}^2,$$
$$(x,y) \mapsto \begin{pmatrix} u(x,y) \\ v(x,y) \end{pmatrix}.$$

$$\mathbf{v}: U \subset \mathbb{R}^3 \to \mathbb{R}^3,$$
$$(x,y,z) \mapsto \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix}.$$

Vector Fields vs. Vectors in Components

$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

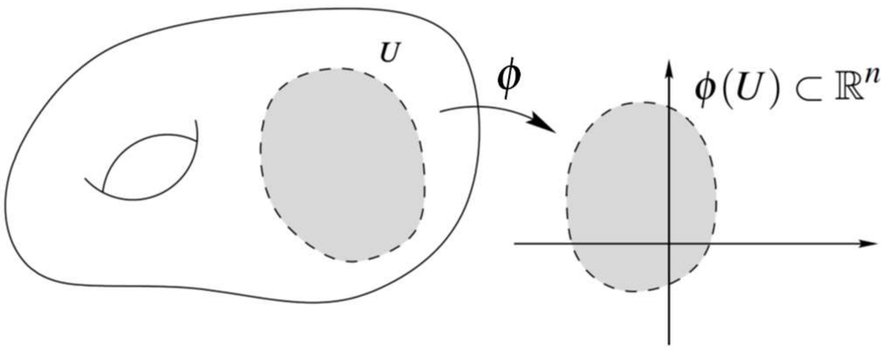
$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{pmatrix} v^1(x^1, x^2, \ldots, x^n) \\ v^2(x^1, x^2, \ldots, x^n) \\ \vdots \\ v^n(x^1, x^2, \ldots, x^n) \end{pmatrix}.$$

Vector Fields vs. Vectors in Components

$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$



$$\mathbf{v}: U \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{pmatrix} v^1(x^1, x^2, \ldots, x^n) \\ v^2(x^1, x^2, \ldots, x^n) \\ \vdots \\ v^n(x^1, x^2, \ldots, x^n) \end{pmatrix}.$$

$$\mathbf{v}\big|_U : \phi(U) \subset \mathbb{R}^n \to \mathbb{R}^n,$$

$$(x^1, x^2, \ldots, x^n) \mapsto \begin{bmatrix} v^1 \\ v^2 \\ \vdots \\ v^n \end{bmatrix}.$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\begin{aligned} \mathbf{e}_i : U \subset M \to TM, \\ x \mapsto \mathbf{e}_i(x) \end{aligned} \qquad \left\{\mathbf{e}_i(x)\right\}_{i=1}^{n} \quad \text{basis for } T_xM$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \qquad x \mapsto \mathbf{e}_i(x)$$

$$\{\mathbf{e}_i(x)\}_{i=1}^{n} \quad \text{basis for } T_xM$$

$$\mathbf{v} : U \subset M \to TM, \qquad x \mapsto v^1\mathbf{e}_1 + v^2\mathbf{e}_2 + \ldots + v^n\mathbf{e}_n.$$

$$\mathbf{v} : U \subset M \to TM, \qquad x \mapsto v^1(x)\,\mathbf{e}_1(x) + v^2(x)\,\mathbf{e}_2(x) + \ldots + v^n(x)\,\mathbf{e}_n(x).$$

Vector Fields vs. Vectors in Components

Need basis vector fields

$$\mathbf{e}_i : U \subset M \to TM, \qquad x \mapsto \mathbf{e}_i(x)$$

$$\{\mathbf{e}_i(x)\}_{i=1}^n \quad \text{basis for } T_x M$$

Coordinate basis:

$$\mathbf{e}_i := \frac{\partial}{\partial x^i}$$

$$\mathbf{v} : U \subset M \to TM, \qquad x \mapsto v^1 \mathbf{e}_1 + v^2 \mathbf{e}_2 + \ldots + v^n \mathbf{e}_n.$$

$$\mathbf{v} : U \subset M \to TM, \qquad x \mapsto v^1(x)\,\mathbf{e}_1(x) + v^2(x)\,\mathbf{e}_2(x) + \ldots + v^n(x)\,\mathbf{e}_n(x).$$

Examples of Coordinate Curves and Bases

Coordinate functions, coordinate curves, bases

- Coordinate functions are real-valued ("scalar") functions on the domain
- On each coordinate curve, *one* coordinate changes, *all others stay constant*
- Basis: n linearly independent vectors at each point of domain

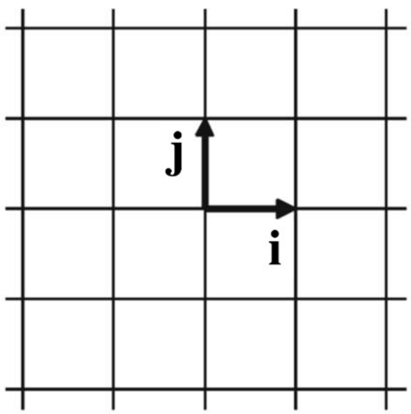
Cartesian coordinates

$$x^1 = x$$

$$x^2 = y$$

$$\mathbf{e}_1 = \frac{\partial}{\partial x} = \mathbf{i}$$

$$\mathbf{e}_2 = \frac{\partial}{\partial y} = \mathbf{j}$$



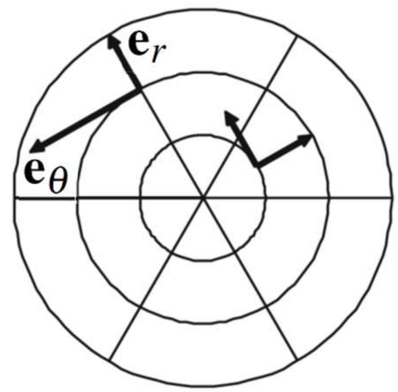
polar coordinates

$$x^1 = r$$

$$x^2 = \theta$$

$$\mathbf{e}_1 = \frac{\partial}{\partial r} = \mathbf{e}_r$$

$$\mathbf{e}_2 = \frac{\partial}{\partial \theta} = \mathbf{e}_\theta$$



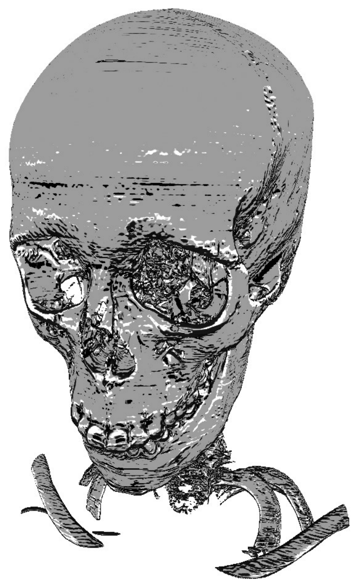
[bonus slides:]

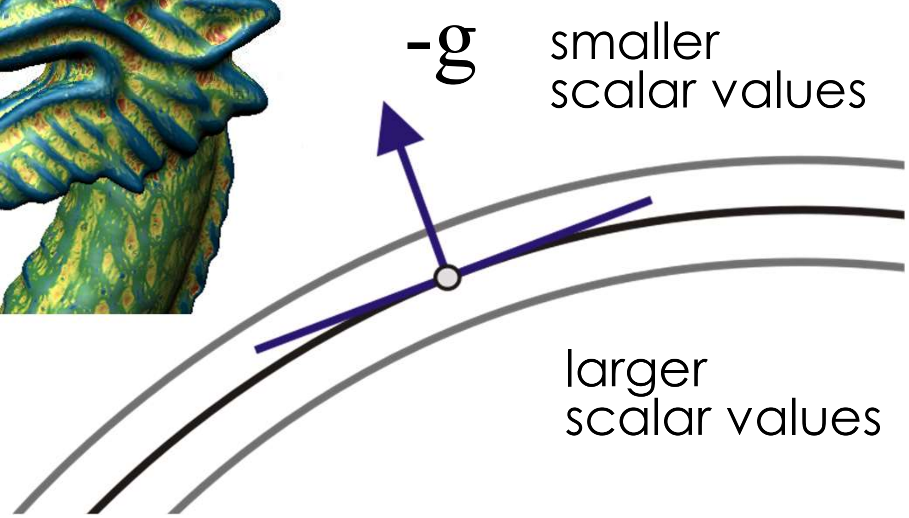
Curvature-Based Transfer Functions

Curvature-Based Isosurface Illustration

Curvature measure color mapping

Curvature directions; ridges and valleys





- Implicit surface curvature
- Isosurface through a point

(Extrinsic) Curvature

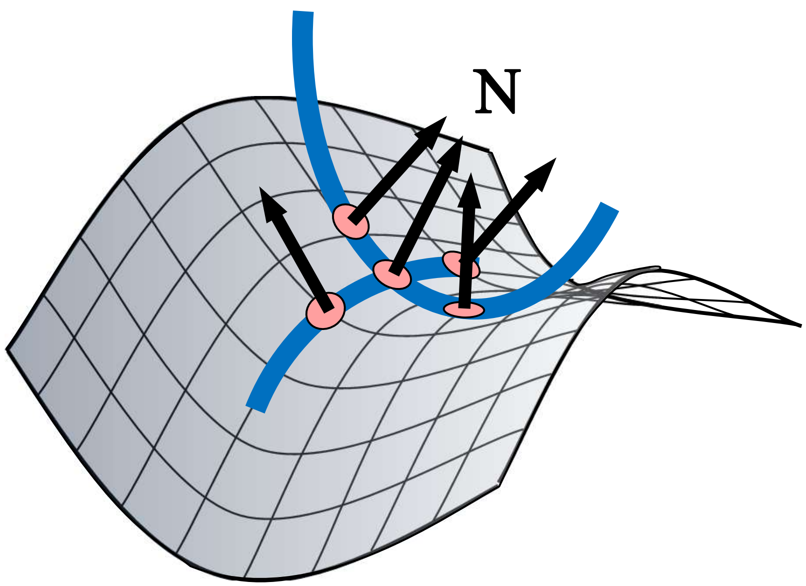
How fast do positional changes (in different directions) on the surface change the normal vector?

- Gauss map: assigns normal to each point

$$\begin{aligned} \mathrm{N} : M &\to \mathbb{S}^2, \\ x &\mapsto \mathrm{N}(x). \end{aligned}$$

- Differential of Gauss map:
  Shape operator / Weingarten map

$$\begin{aligned} \mathrm{dN} : T_x M &\to T_{\mathrm{N}(x)} \mathbb{S}^2, \\ \mathbf{v} &\mapsto \mathrm{dN}(\mathbf{v}). \end{aligned}$$

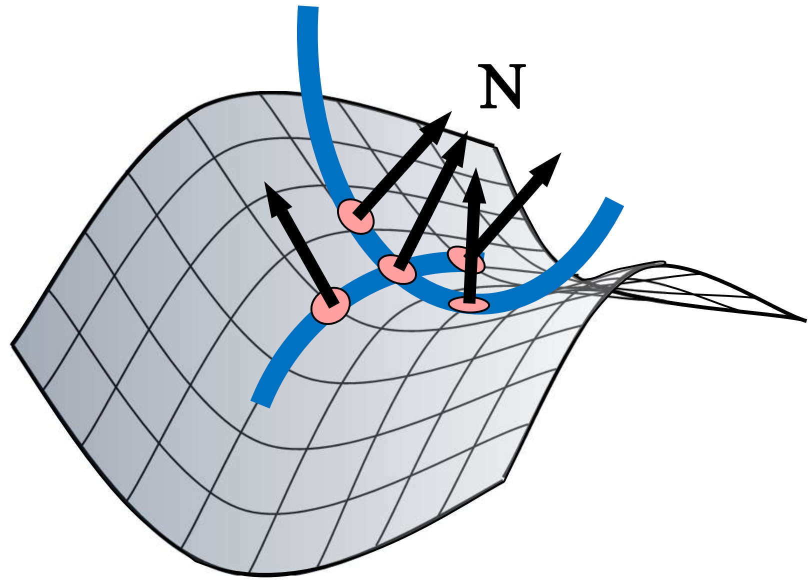


courtesy of Gordon Kindlmann

(Extrinsic) Curvature

Analyze shape operator S

- Eigenvalues: principal curvatures (magnitudes)
  - First and second principal curvature
  - Maximum: κ_1
  - Minimum: κ_2
- Eigenvectors: principal curv. directions
- Gaussian curvature (intrinsic!): $\kappa_1 \kappa_2$



courtesy of Gordon Kindlmann

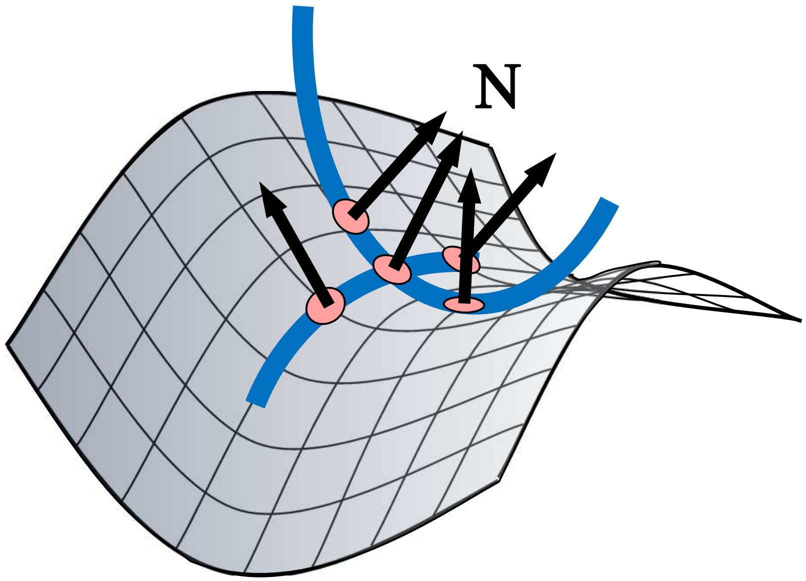
(Extrinsic) Curvature Computation

Simple recipe for implicit isosurfaces in volume

- Build on gradient and Hessian matrix
- Hessian contains curvature information

Transform Hessian into tangent space

- Curvature magnitudes:
  Eigenvalues of 2x2 matrix
- Curvature directions:
  Eigenvectors of 2x2 matrix



courtesy of Gordon Kindlmann

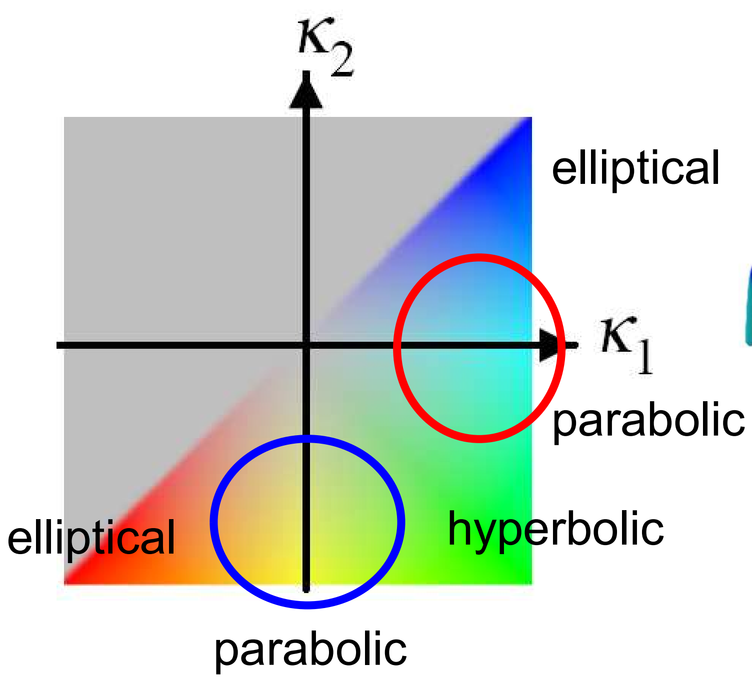
Alternative:

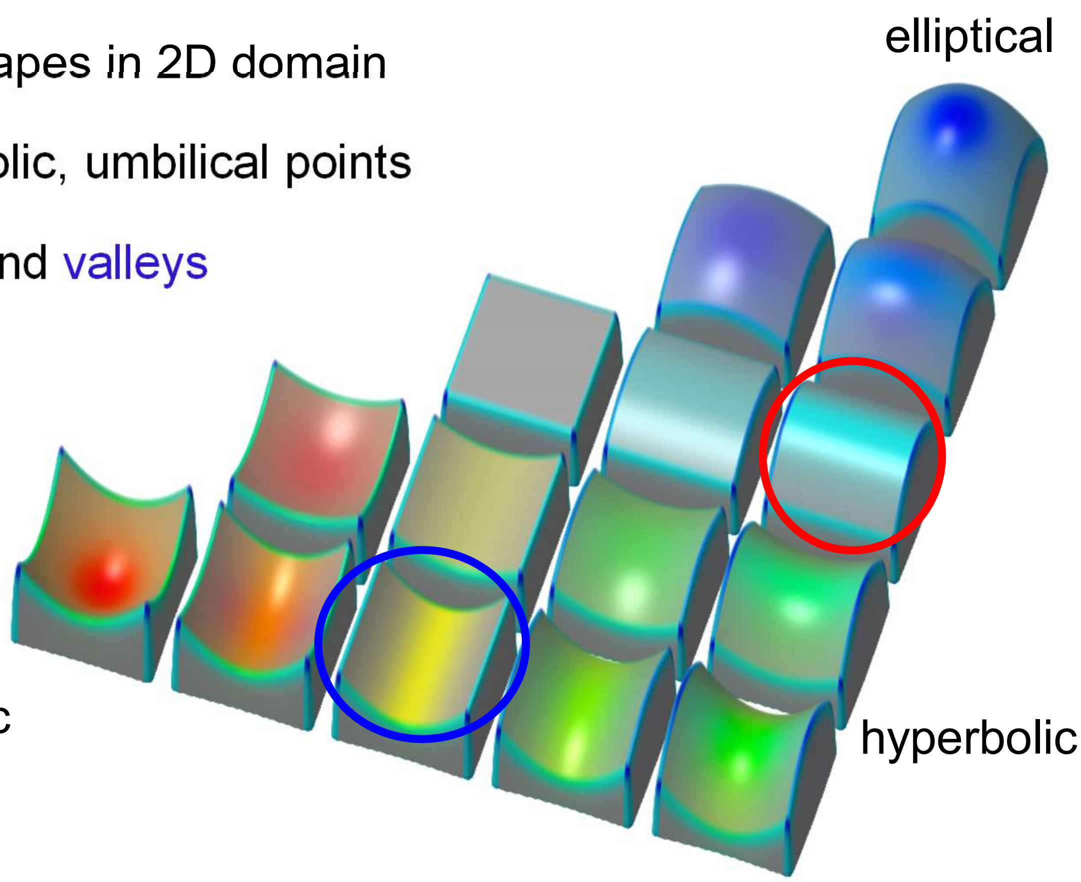
- Compute in 3D (see Real-Time Volume Graphics, 14.4.4)

The Principal Curvature Domain

Maximum/minimum principal curvature magnitude

- Identification of different shapes in 2D domain
- Elliptical, parabolic, hyberbolic, umbilical points
- Feature lines: e.g., ridges and valleys

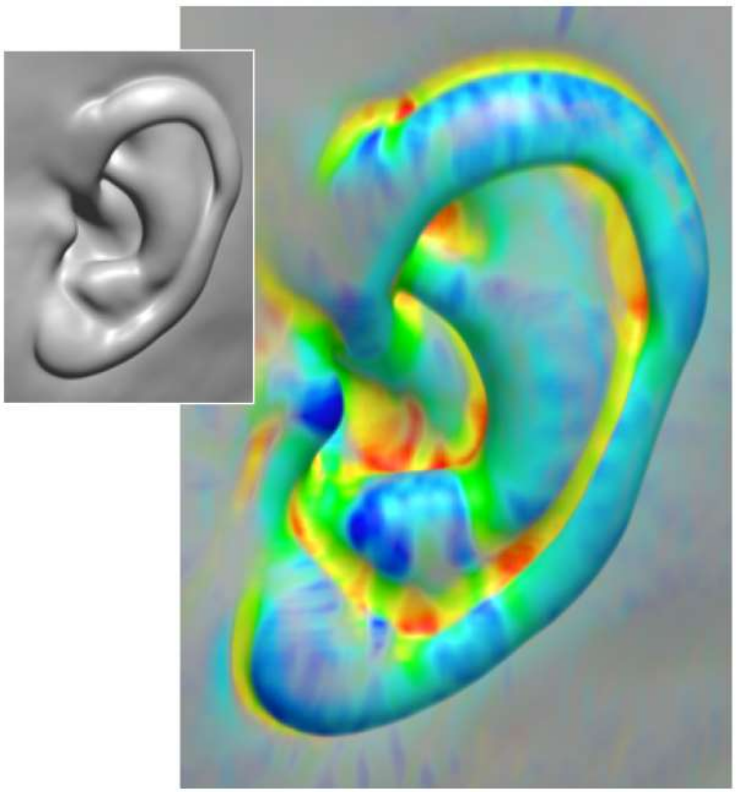


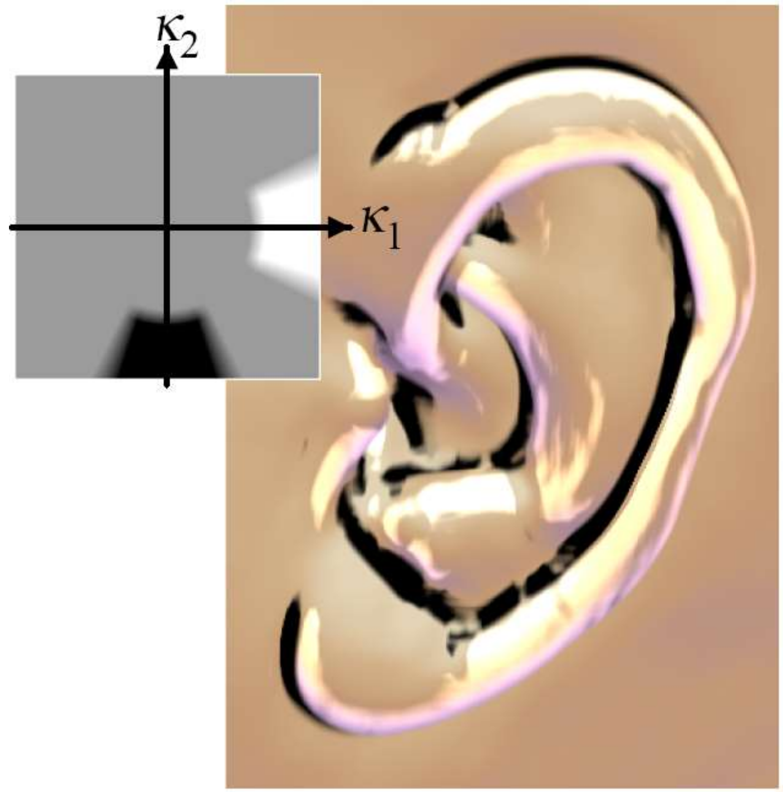


courtesy of Gordon Kindlmann

Curvature Transfer Functions

- Color coding of curvature domain
- Paint features: ridge and valley lines

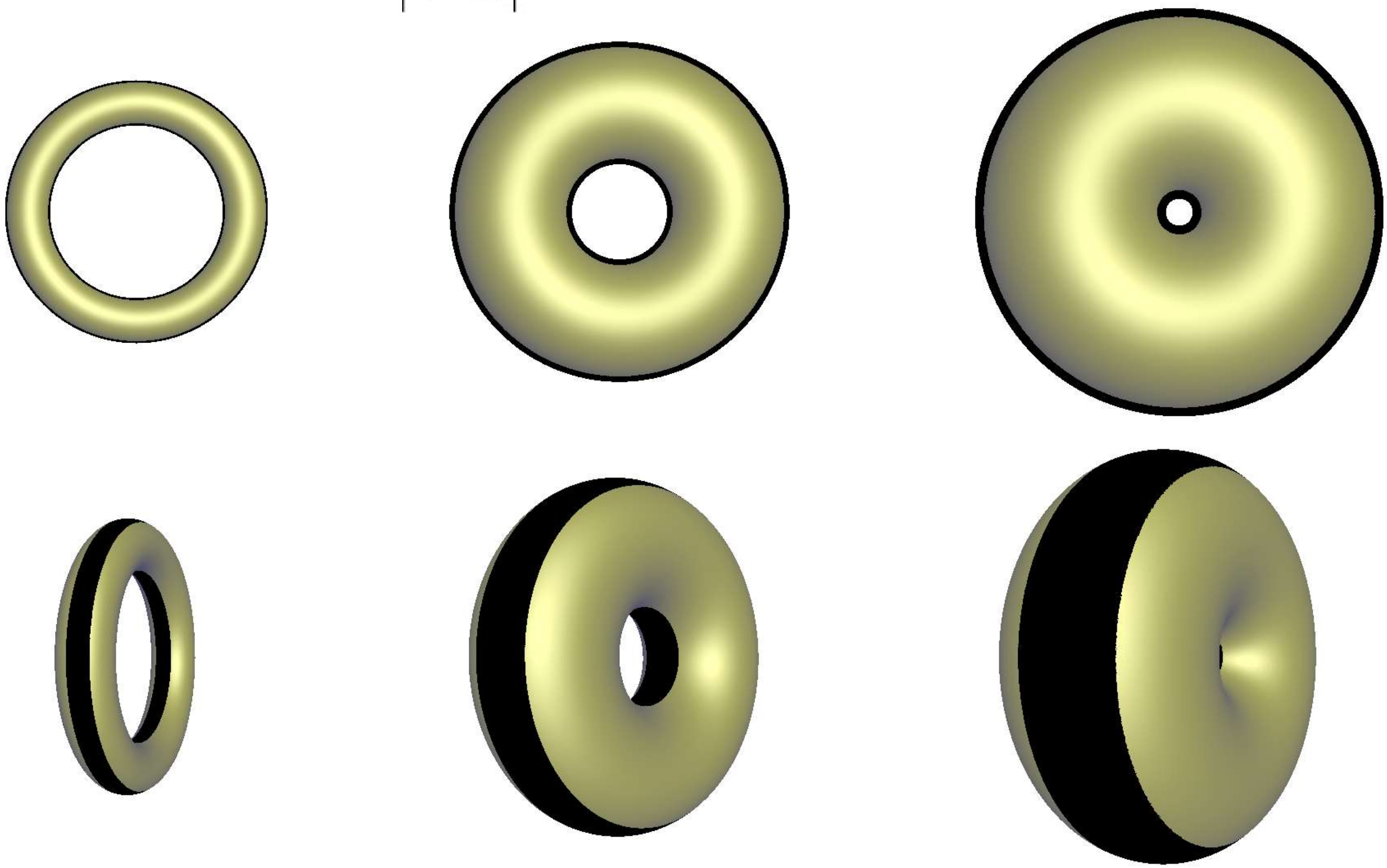




courtesy of Gordon Kindlmann

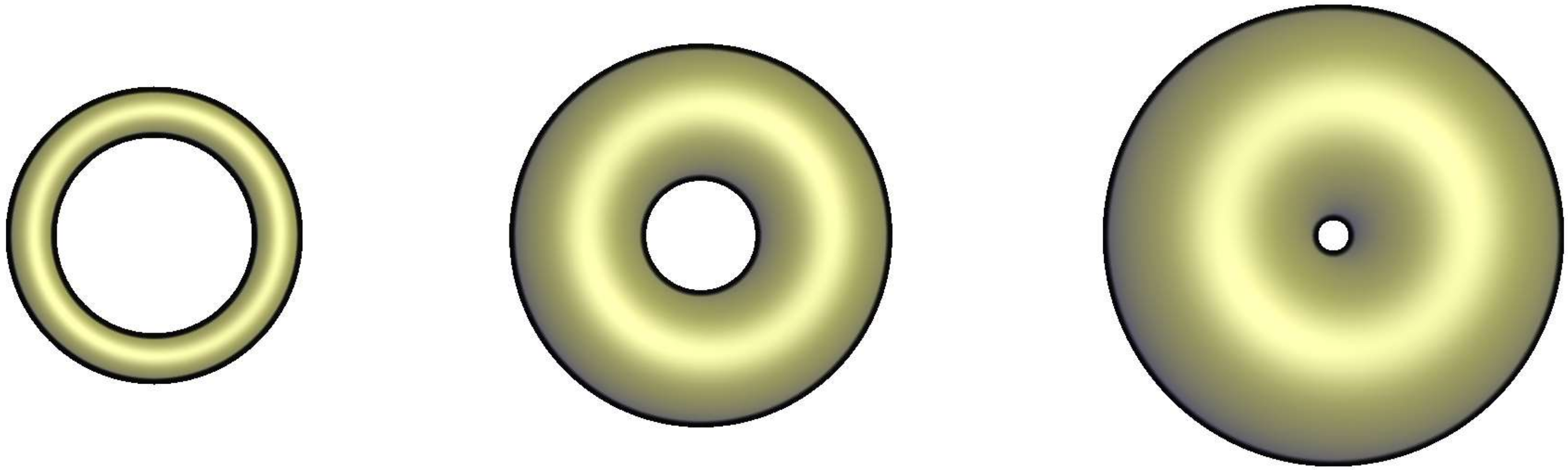
Problems of Implicit Surface Contours

Constant threshold on $|\mathbf{v} \cdot \mathbf{n}|$

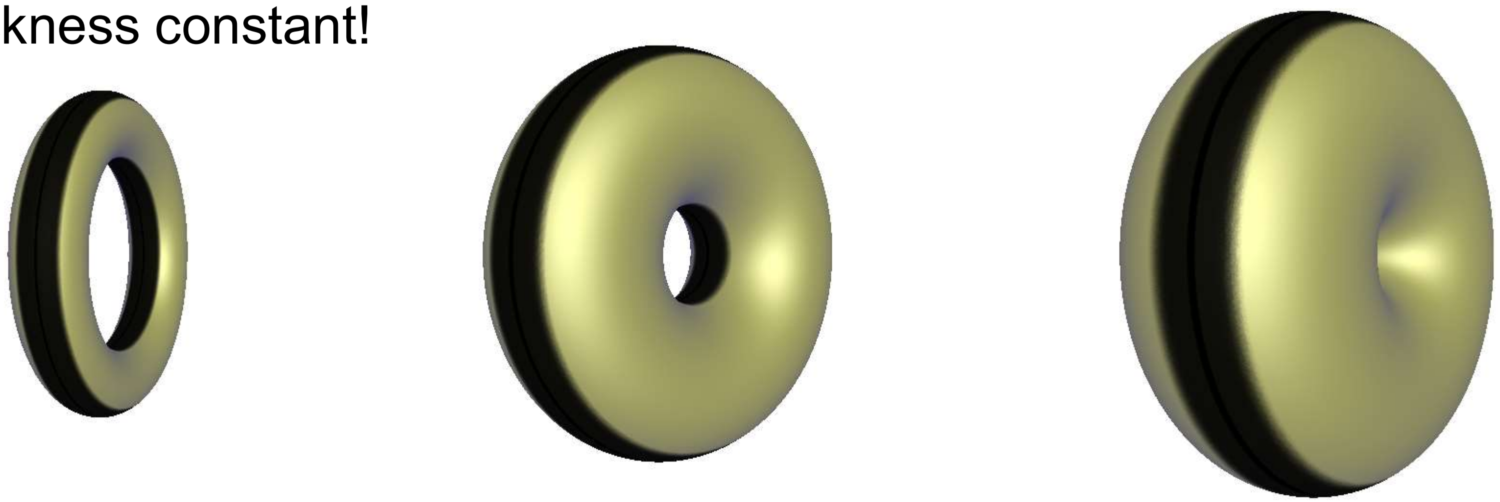


Curvature-Based Contour Threshold (1)

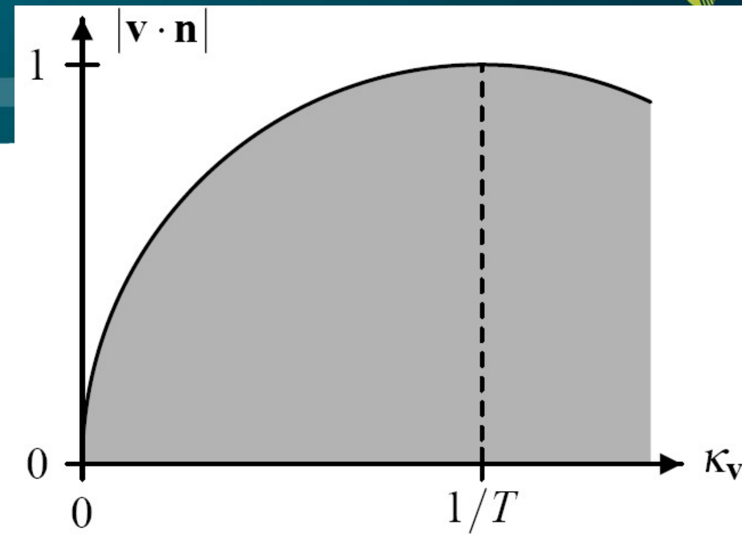
Threshold dependent on curvature in view direction



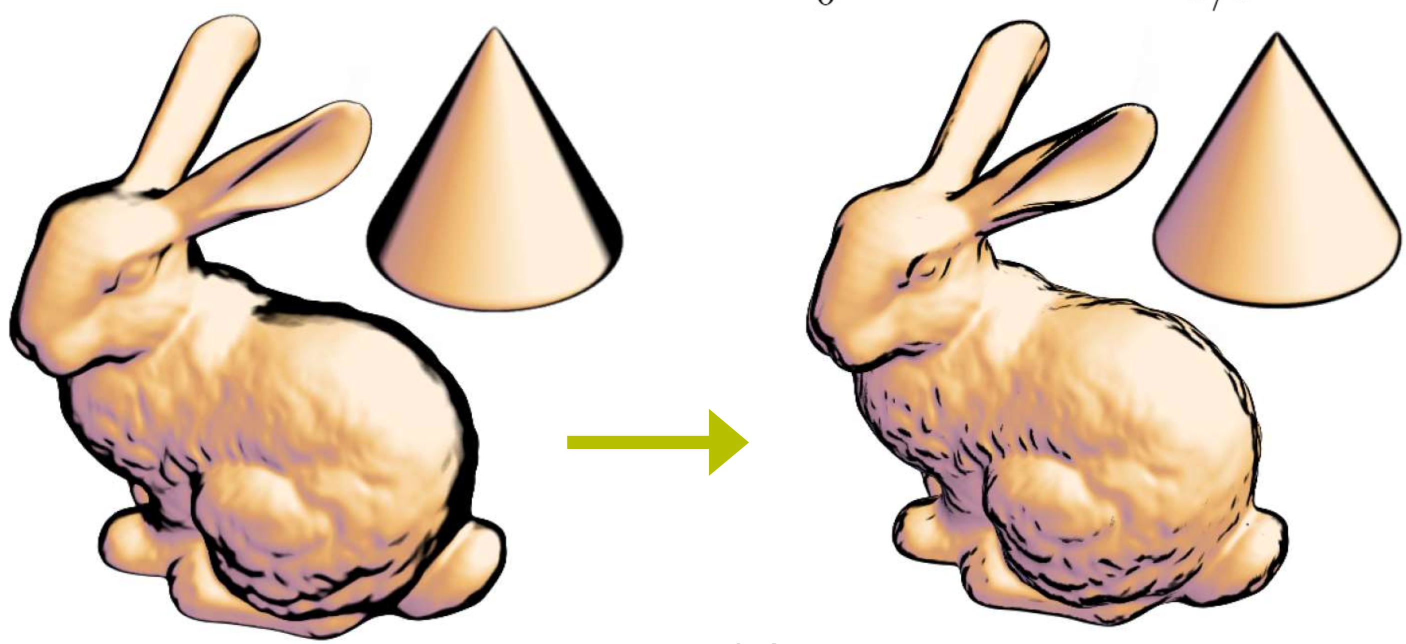
Thickness constant!



Curvature-Based Contour Threshold (2)



Higher curvature in view direction needs higher threshold



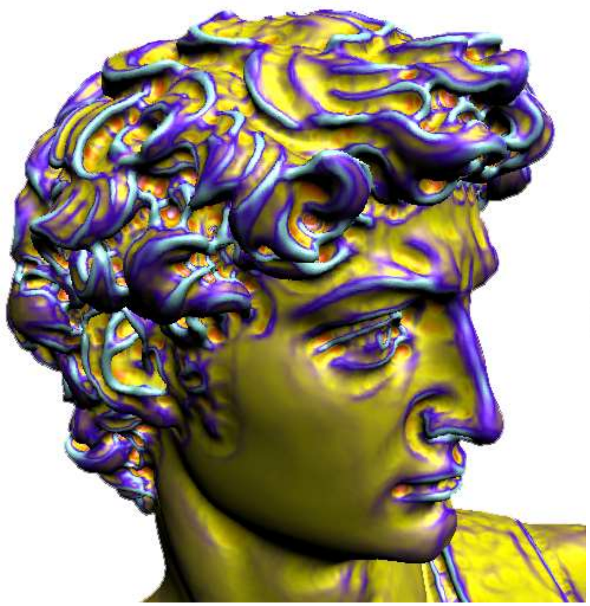
courtesy of Gordon Kindlmann

Example

Deferred Isosurface Shading

- Shading is expensive
- Compute surface intersection image from volume
- Compute derivatives and shading in image space

intersection image



curvature color coding

ridges and valleys

Implicit Curvature via Convolution

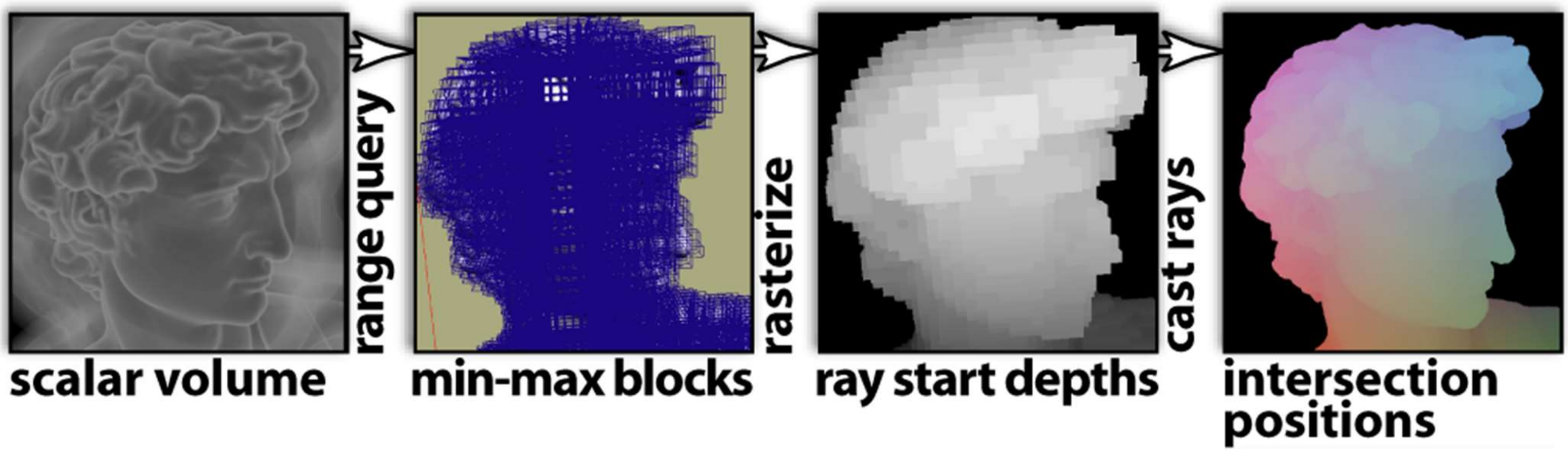
Computed from first and second derivatives

- Can use fast texture-based tri-cubic filters in shader
- Can use deferred computation and shading

first derivative maximum curvature minimum curvature

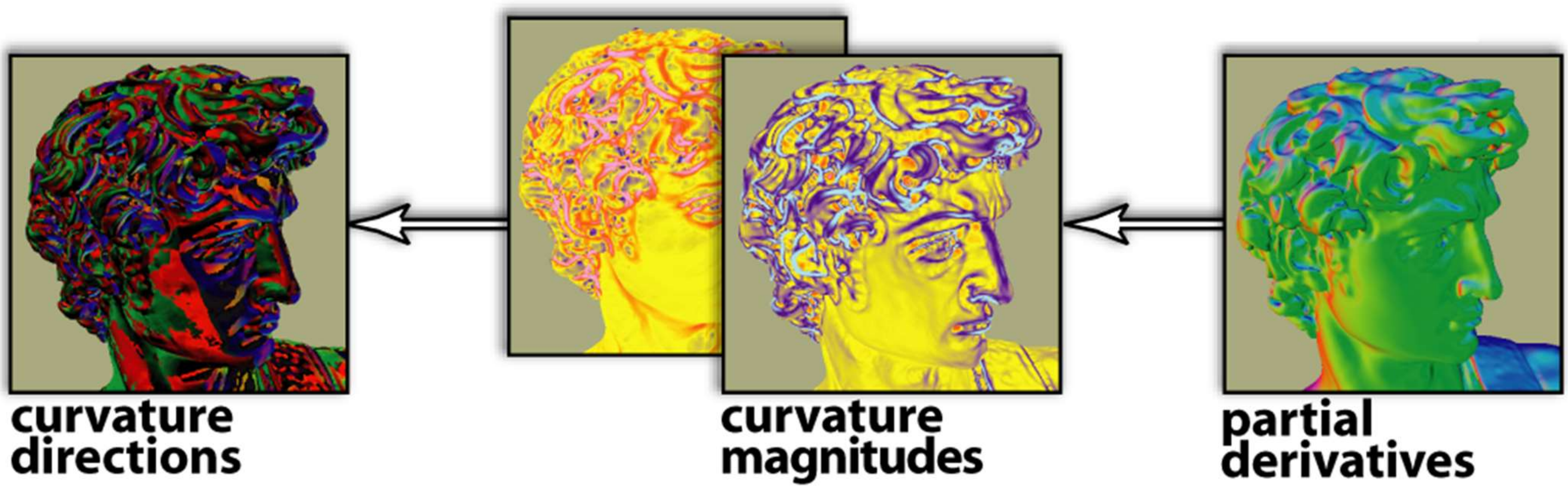
Pipeline Stage #1: Ray-Casting

- Rasterize faces of active min-max blocks
- Cast into the volume; stop when isosurface hit
- Refine isosurface hit positions (root search)



Pipeline Stage #2: Differential Properties

- Basis for visualization of surface shape
- First and second derivatives (gradient, Hessian)
- From these: curvature information, …



Pipeline Stage #3: Shading

Build on previous images

- Position in object space
- Gradient
- Principal curvature magnitudes and directions

curvature flow

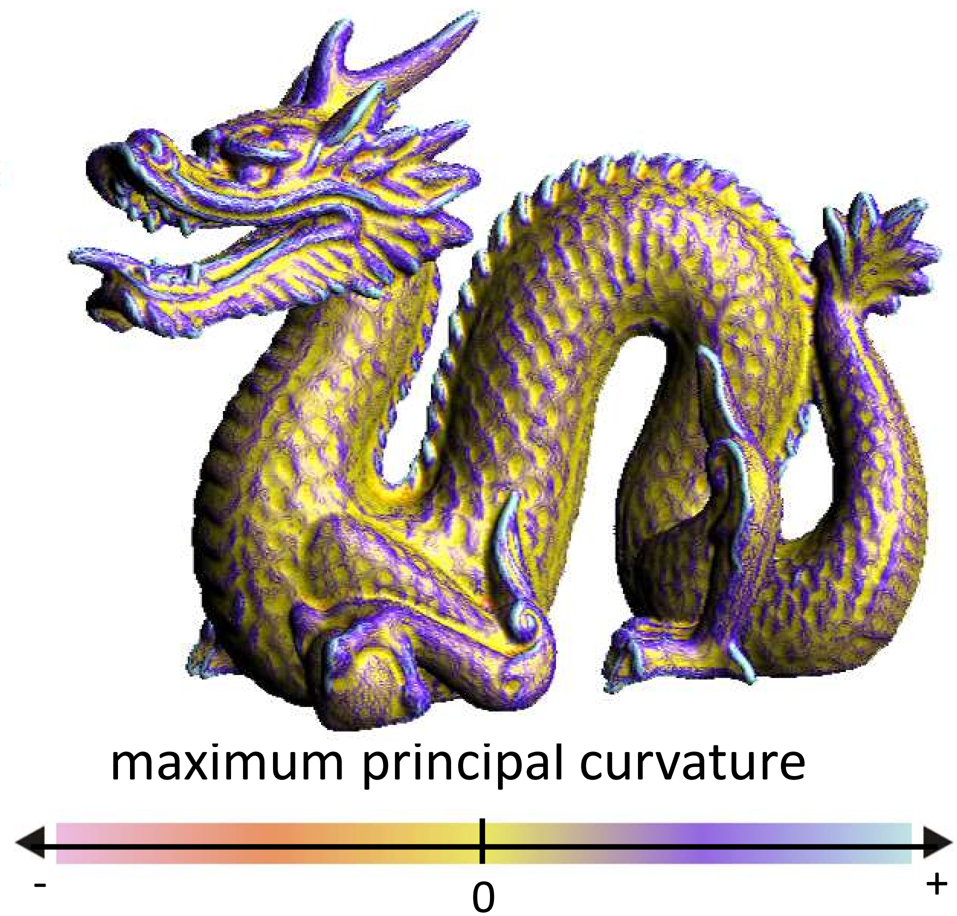
ridges+valleys

curvature mapping

tone shading

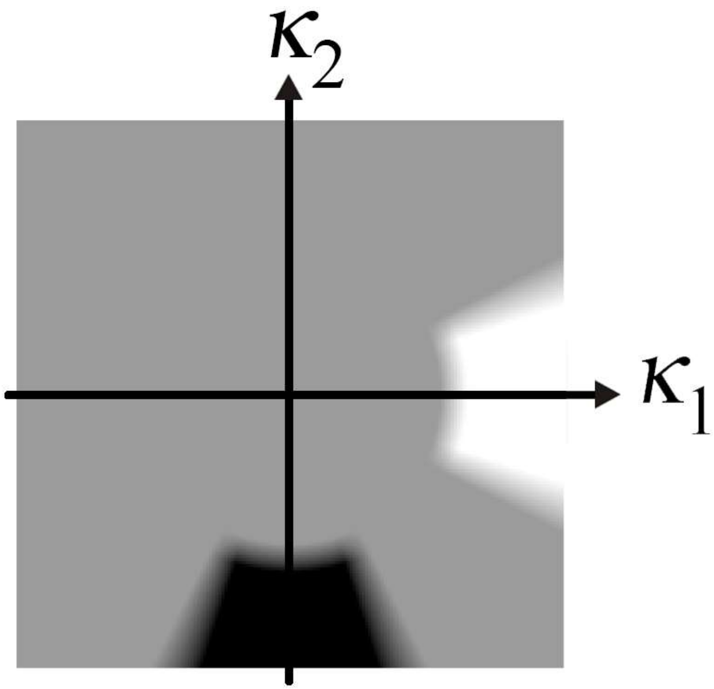
Color Coding Scalar Curvature Measures

- 1D color lookup table



2D Curvature Transfer Functions

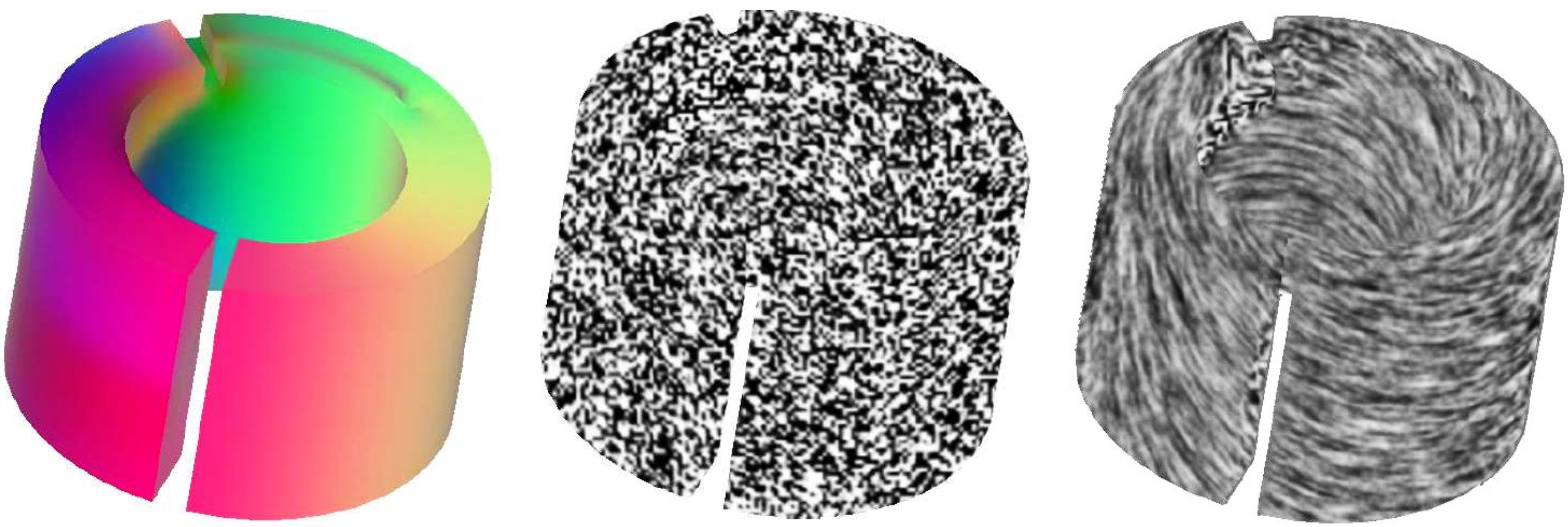
- 2D lookup table in domain of principal curvatures



ridges and valleys, plus contours:

Visualizing Curvature Directions (1)

- Use 3D vector field visualization on curved surfaces [van Wijk, 2003], [Laramee et al., 2003]
- Project 3D vectors to screen space
- Advect dense noise textures in screen space



Visualizing Curvature Directions (2)

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama