King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 15: Volume Rendering, Pt. 3

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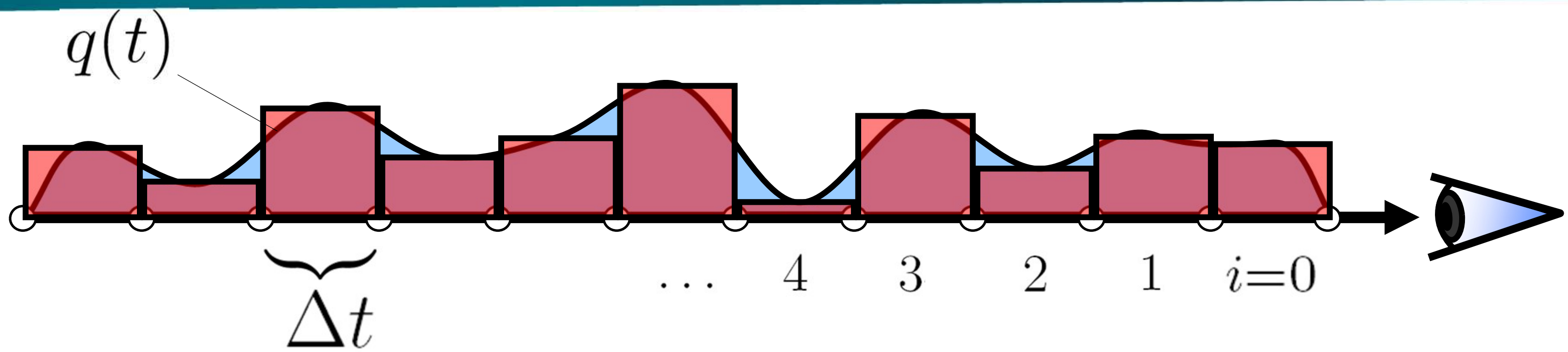
Reading Assignment #8 (until Mar 22)

Read (required):

- Real-Time Volume Graphics, Chapter 4 (Transfer Functions) until Sec. 4.4 (inclusive)
- Paper:

  Jens Krüger and Rüdiger Westermann,
  Acceleration Techniques for GPU-based Volume Rendering,
  IEEE Visualization 2003,
  `http://dl.acm.org/citation.cfm?id=1081482`

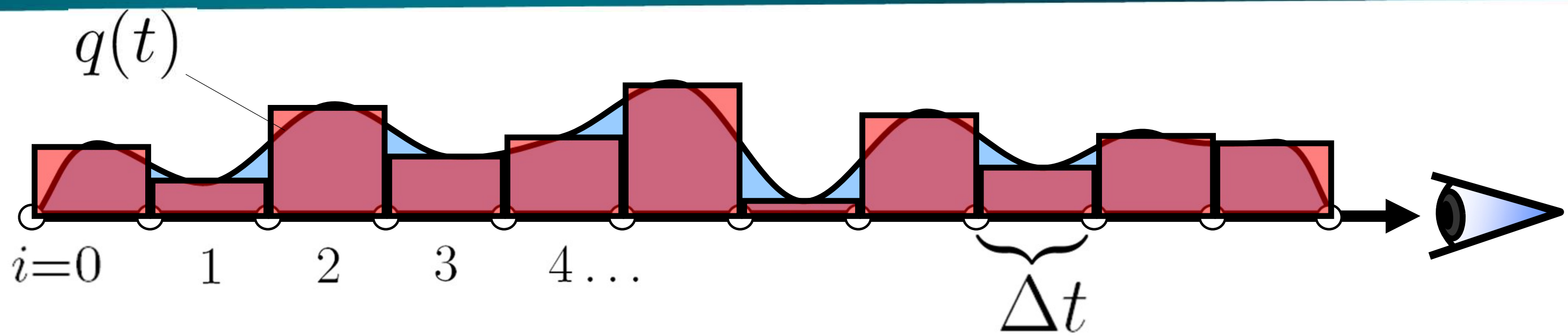
Volume Rendering Integral: Numerical Solution



$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively/iteratively!

Volume Rendering Integral: Numerical Solution

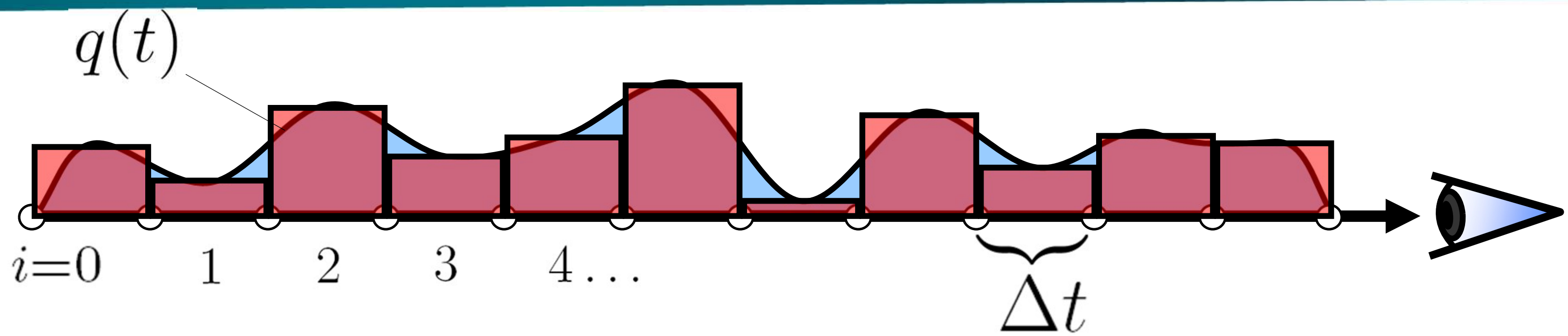


Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !

can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Volume Rendering Integral: Numerical Solution



can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

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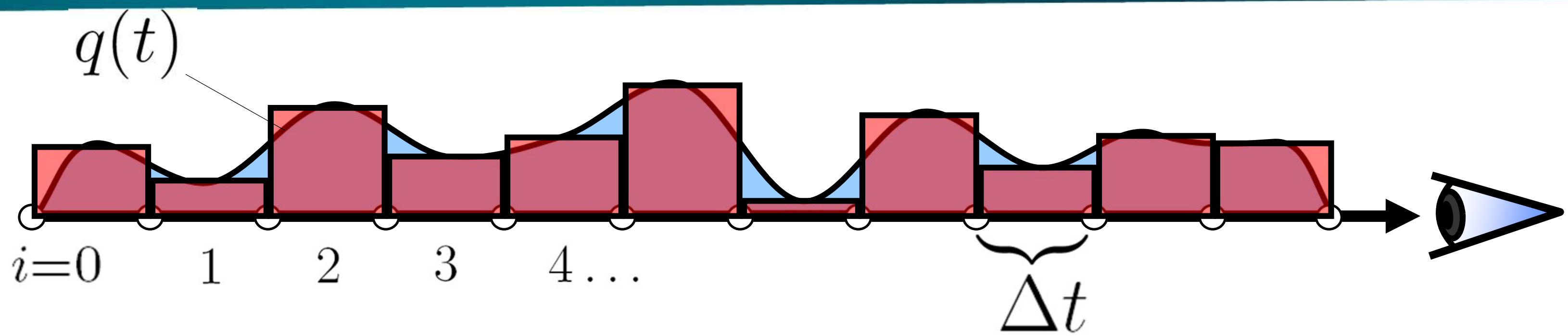


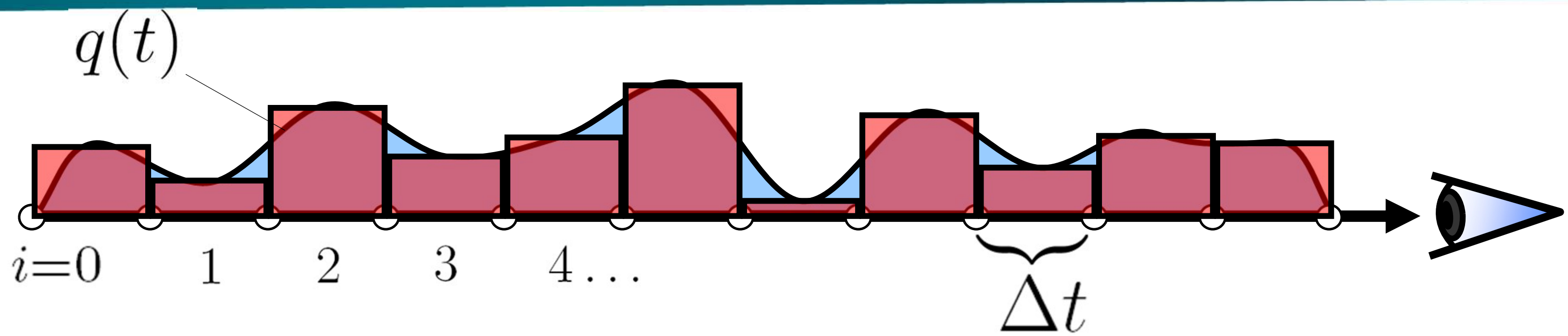
Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

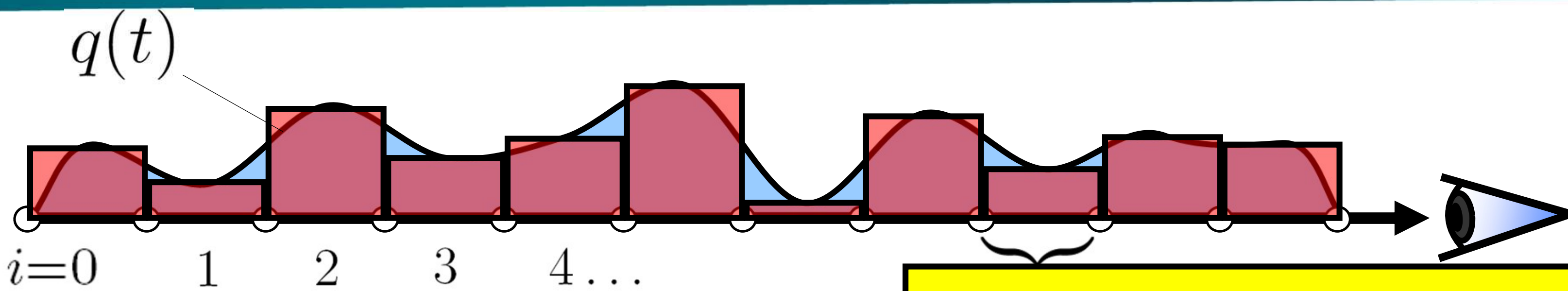
Iterate from *i*=0 (back) to *i*=max (front): *i* increases

Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1})C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1})A_i \end{aligned}$$

Iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i \;=\; C_i + (1$$

Iterate from *i*=0 (back

Early Ray Termination:
Stop the calculation when

$$A'_i \approx 1$$

Front-to-back compositing

$$\begin{aligned} C'_i &= C'_{i+1} + (1 - A'_{i+1}) C_i \\ A'_i &= A'_{i+1} + (1 - A'_{i+1}) A_i \end{aligned}$$

Iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral Summary

Volume rendering integral
for *Emission Absorption* model

$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, \mathrm{d}\tilde{s}$$

Numerical solutions:

Back-to-front compositing

$$C'_i \;=\; C_i + (1 - A_i) C'_{i-1}$$

Front-to-back compositing

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1}) C_i$$

$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1}) A_i$$

Opacity Correction

Simple compositing only works as far as the opacity values are correct… and they depend on the sample distance!

$$T_i = e^{-\int_{s_i}^{s_i+\Delta t} \kappa(t)\,dt} \approx e^{-\kappa(s_i)\Delta t} = e^{-\kappa_i \Delta t}$$

$$A_i = 1 - e^{-\kappa_i \Delta t}$$

$$\tilde{T}_i = T_i^{\left(\frac{\Delta \tilde{t}}{\Delta t}\right)}$$

$$\tilde{A}_i = 1 - (1 - A_i)^{\left(\frac{\Delta \tilde{t}}{\Delta t}\right)}$$

opacity correction formula

Beware that usually this is done *for each different scalar value* (every transfer function entry), not actually at spatial positions/intervals *i*

So… "Self-Absorption" What?

Rewrite the volume rendering integral as sum of interval computations ("slabs"); each slab is correctly integrated

$$I(s_n) = I_0 \, e^{-\int_{s_0}^{s_n} \kappa(t)\,\mathrm{d}t} + \sum_{i=0}^{n-1} I(s_i, s_{i+1}) \, e^{-\int_{s_{i+1}}^{s_n} \kappa(t)\,\mathrm{d}t}$$

Each slab is the integral (emission with "self-absorption")

$$I(s_i, s_{i+1}) = \int_{s_i}^{s_{i+1}} q(s)\, e^{-\int_{s}^{s_{i+1}} \kappa(t)\,\mathrm{d}t} \,\mathrm{d}s$$

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Approximate with constant absorption and emission in slab

$$q(s) = q_i \quad \forall s \in [s_i, s_{i+1}) \qquad\qquad \kappa(s) = \kappa_i \quad \forall s \in [s_i, s_{i+1})$$

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$$q(s) = q_i \quad \forall s \in [s_i, s_{i+1}) \qquad\qquad \kappa(s) = \kappa_i \quad \forall s \in [s_i, s_{i+1})$$

$$\bar{I}(s_i, s_{i+1}) = \int_{s_i}^{s_{i+1}} q_i\, e^{-\kappa_i \cdot (s_{i+1} - s)}\, \mathrm{d}s$$

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$$\bar{I}(s_i, s_{i+1}) = q_i \int_{s_i}^{s_{i+1}} e^{-\kappa_i\, s_{i+1} + \kappa_i\, s}\, \mathrm{d}s$$

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$$\bar{I}(s_i, s_{i+1}) = q_i e^{-\kappa_i s_{i+1}} \left(\frac{1}{\kappa_i} e^{\kappa_i s_{i+1}} - \frac{1}{\kappa_i} e^{\kappa_i s_i} \right)$$

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$$\bar{I}(s_i, s_{i+1}) = \frac{q_i}{\kappa_i}\left(1 - e^{-\kappa_i \Delta t}\right) \qquad\qquad q_i := \hat{C}_i \kappa_i$$

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$$\hat{C}_i \qquad\qquad A_i$$

Associated Colors in Volume Rendering

Standard emission-absorption optical model

- Only one kind of particle: the same particles that absorb light, emit light
- Aha! Therefore lower absorption means lower emission as well

Light observed from (in front of) segment *i* (without any light behind it):

$$C_i = \frac{q_i}{\kappa_i}\left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i A_i$$

$$q_i := \hat{C}_i \kappa_i$$

$$A_i := 1 - e^{-\kappa_i \Delta t}$$

$$\lim_{\kappa_i \to 0} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \lim_{\kappa_i \to 0} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = 0$$

$$\lim_{\kappa_i \to \infty} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \lim_{\kappa_i \to \infty} \hat{C}_i \left(1 - e^{-\kappa_i \Delta t}\right) = \hat{C}_i$$

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Interlude: What Happens Without Absorption?

If we do not keep the ratio constant, but use any *q(t)*… ??

$$\lim_{\kappa_i \to 0} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \text{ ?}$$

$$\lim_{\kappa_i \to 0} \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = \Delta t$$

This is not what is done in the standard emission-absorption model!

Standard: only opacity correction (no separate emission correction)!

Interlude: What Happens Without Absorption?

If we do not keep the ratio constant, but use any *q(t)*… ??

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$$\lim_{\kappa_i \to 0} q_i \frac{\left(1 - e^{-\kappa_i \Delta t}\right)}{\kappa_i} = q_i \Delta t$$

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Standard: only opacity correction (no separate emission correction)!

Associated Colors

Associated (or “opacity-weighted” colors) are often used in compositing equations

Every color is *pre-multiplied* by its corresponding opacity

$$\begin{bmatrix} R \\ G \\ B \\ A \end{bmatrix} \Rightarrow \begin{pmatrix} R*A \\ G*A \\ B*A \\ A \end{pmatrix}$$

Our compositing equations assume associated colors!

Important:

After opacity-correction, all associated colors must be updated!

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama