King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 14: Volume Rendering, Pt. 2

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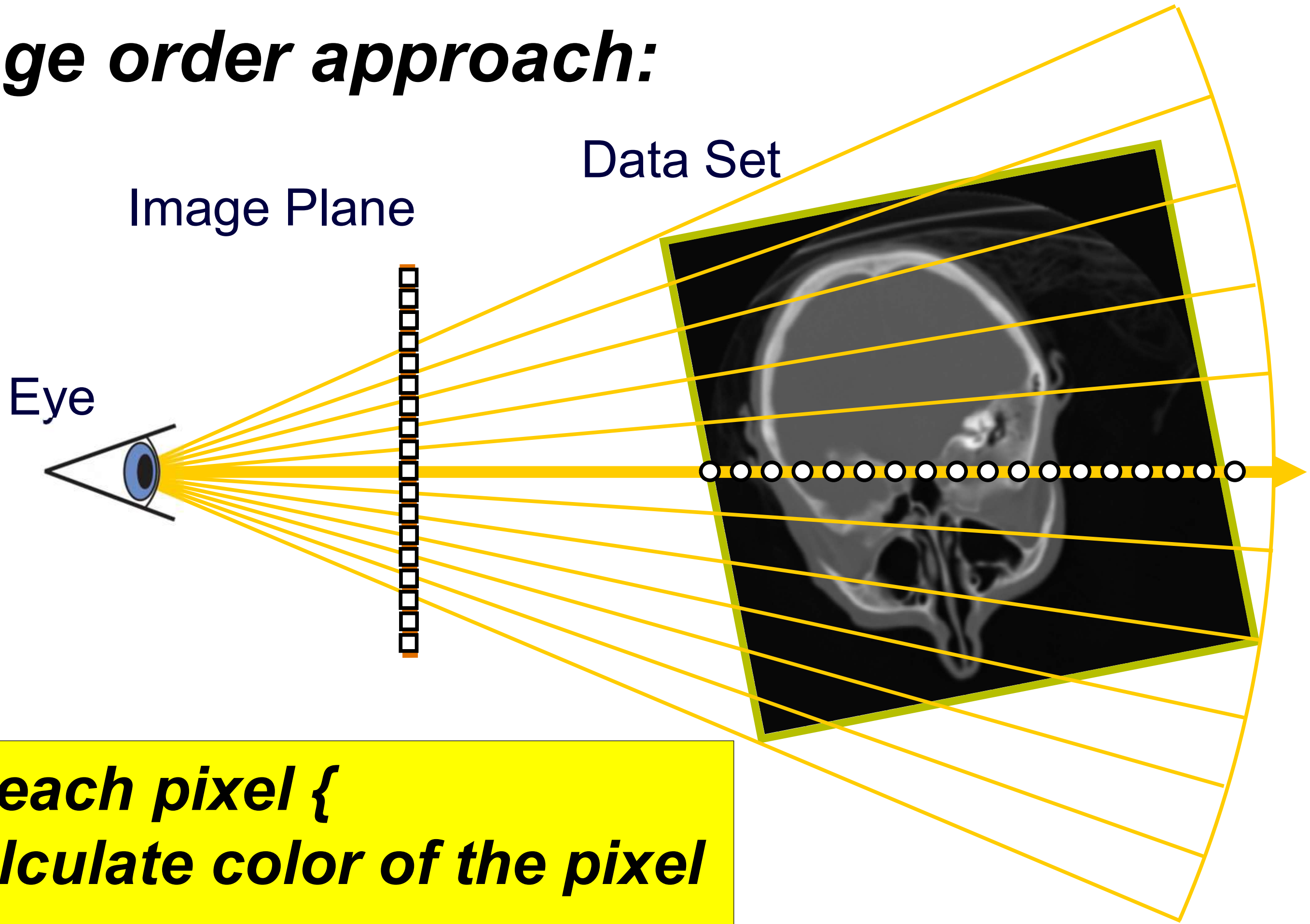
Reading Assignment #7 (until Mar 17)

Read (required):

- Real-Time Volume Graphics, Chapter 1
  (*Theoretical Background and Basic Approaches*),
  from beginning to 1.4.4 (inclusive)
- Paper:
  Nelson Max, Optical Models for Direct Volume Rendering,
  IEEE Transactions on Visualization and Computer Graphics, 1995
  http://dx.doi.org/10.1109/2945.468400

Direct Volume Rendering

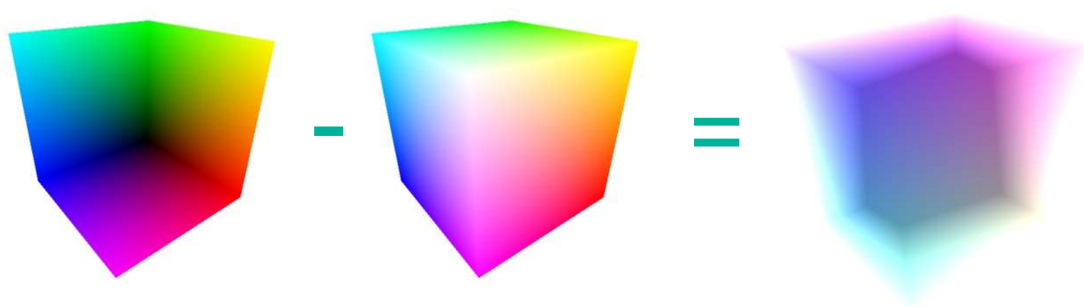
Image order approach:

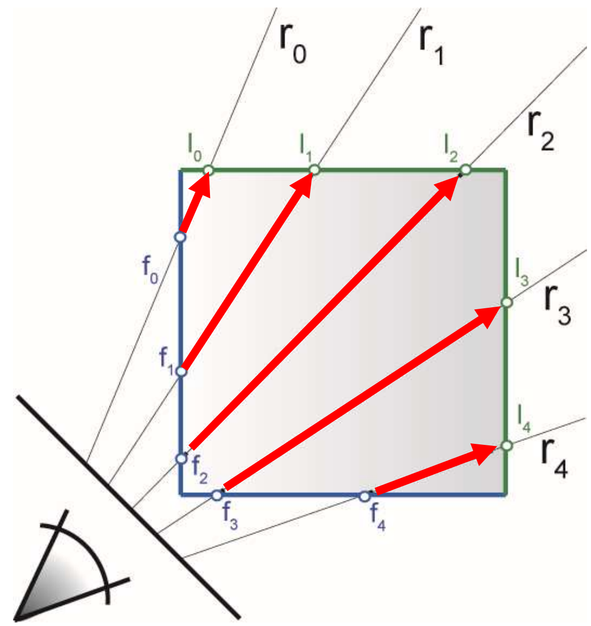


For each pixel {
calculate color of the pixel
}

Rasterization-Based Ray Setup

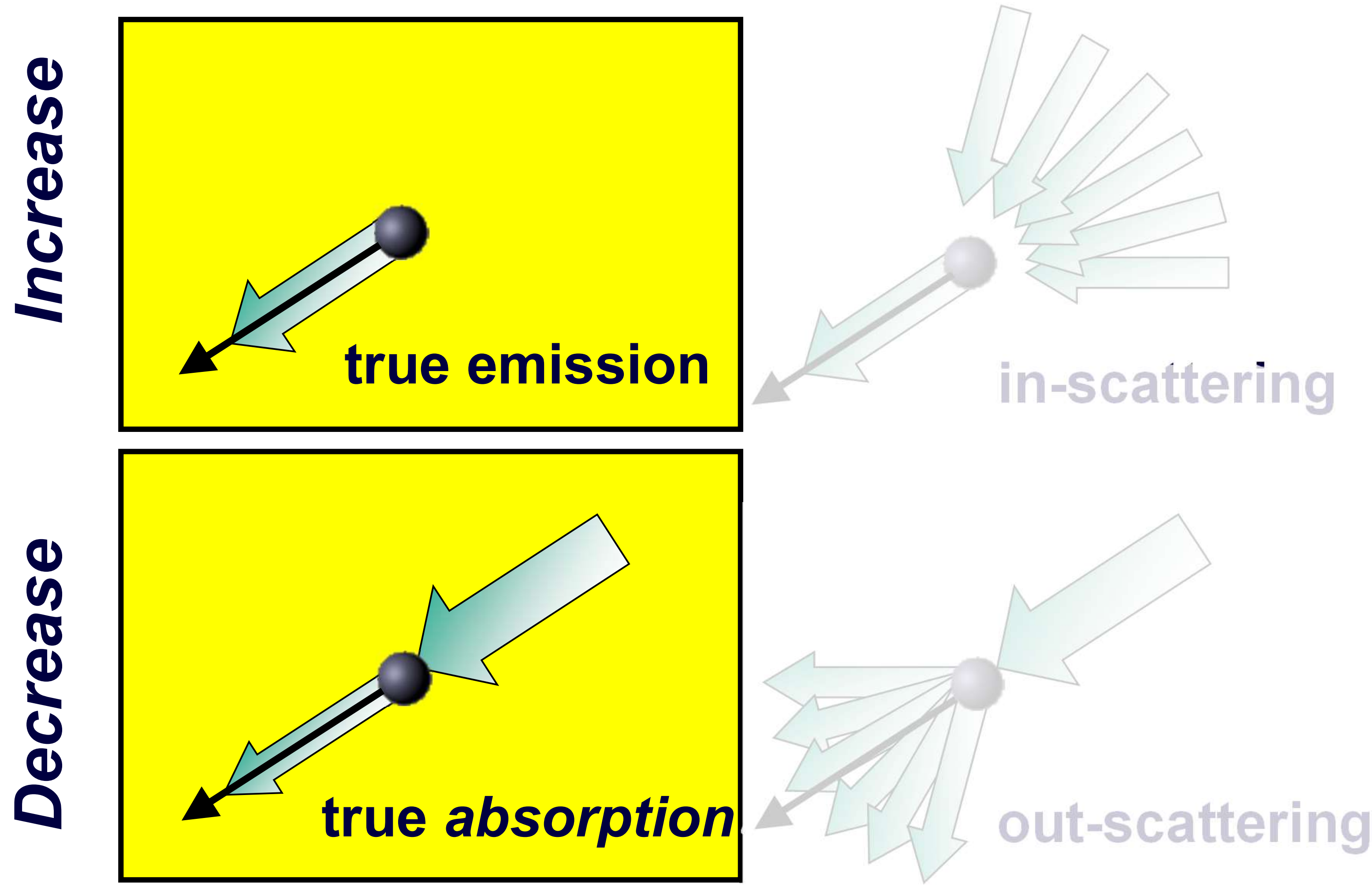
- Fragment == ray
- Need ray start pos, direction vector
- Rasterize bounding box





- Identical for orthogonal and perspective projection!

Physical Model of Radiative Transfer



Volume Rendering Integral

Volume rendering integral
for *Emission Absorption* model

$$I(s) = I(s_0)\, e^{-\tau(s_0, s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s}, s)}\, \mathrm{d}\tilde{s}$$

Numerical solutions:

Back-to-front compositing

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1}) C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1}) A_i$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

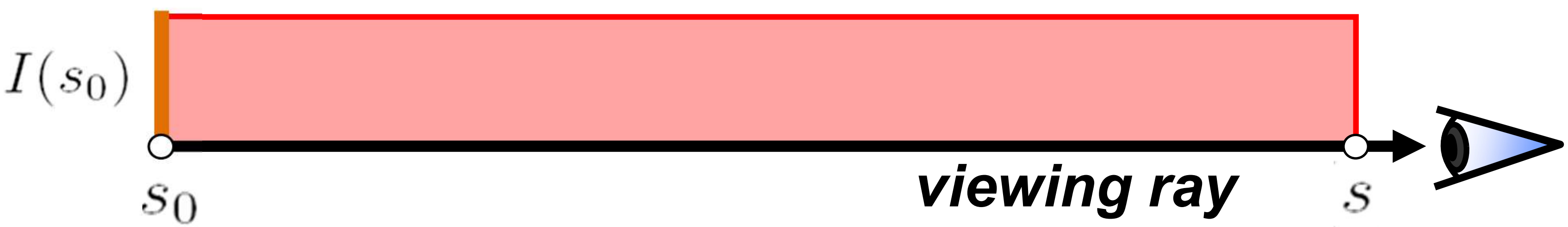
Initial intensity at s_0

$$I(s) = \boxed{I(s_0)}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



Initial intensity at s_0

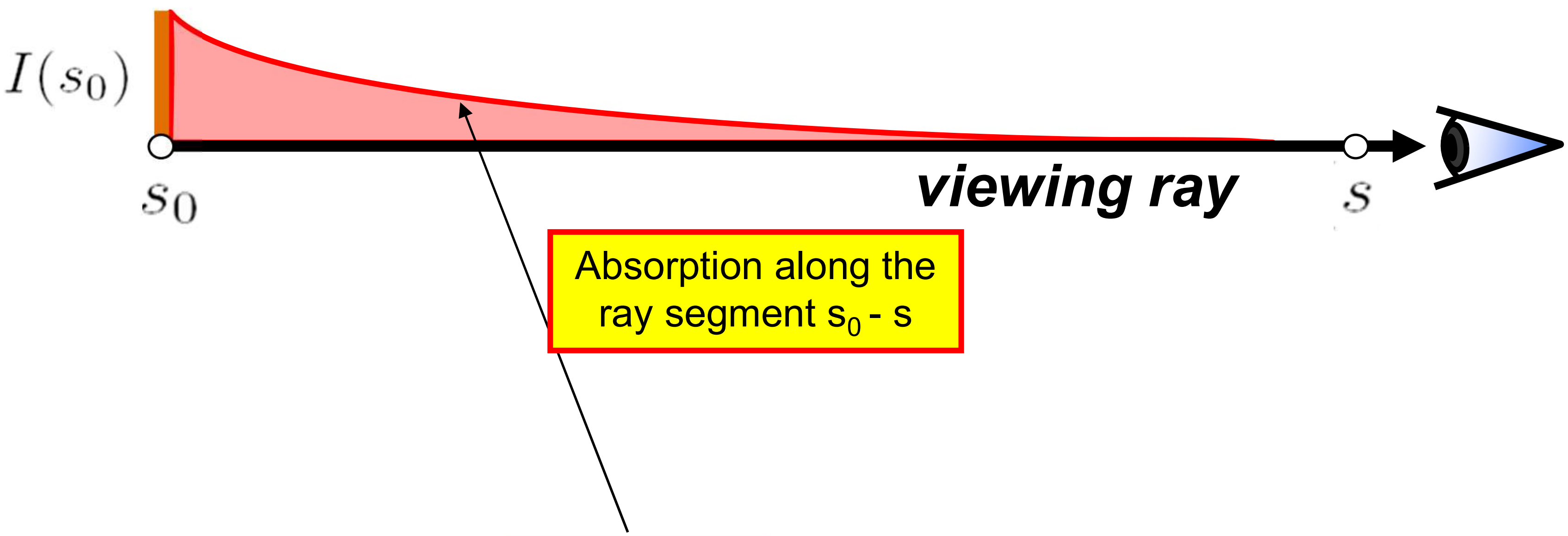
$$I(s) = I(s_0)$$

Without absorption all the initial radiant energy would reach the point ***s.***

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering

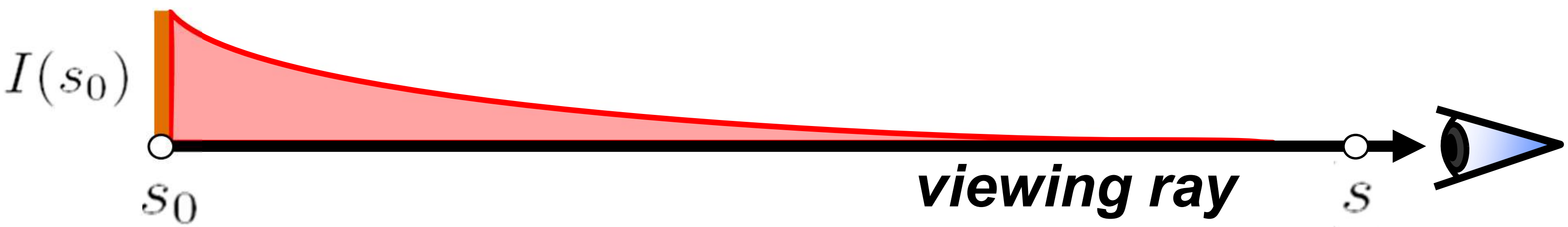


$$I(s) = I(s_0)\, e^{-\tau(s_0,s)}$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



$$I(s) = I(s_0)\, e^{-\tau(s_0, s)}$$

Optical depth τ

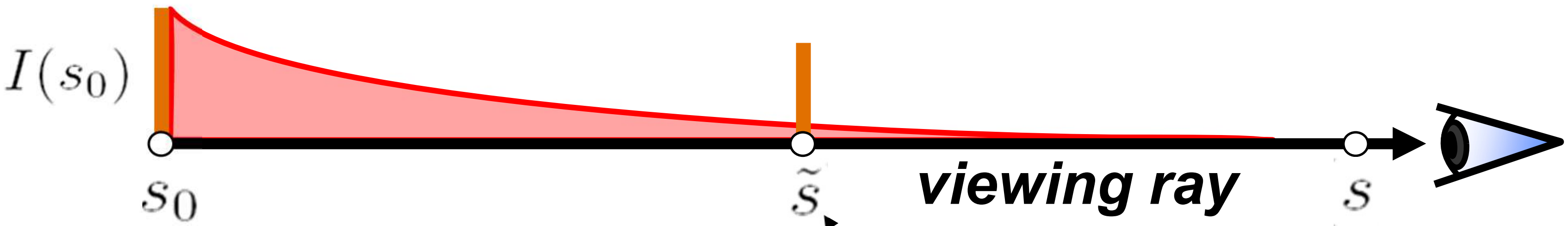
Absorption κ

$$\tau(s_1, s_2) = \int_{s_1}^{s_2} \kappa(s)\, ds.$$

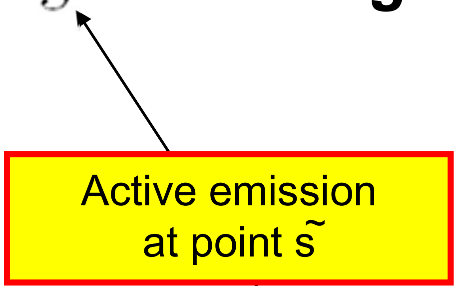
Volume Rendering Integral

How do we determine the radiant energy along the ray?

Physical model: emission and absorption, no scattering



One point $\tilde{s}$ along the viewing ray emits additional radiant energy.

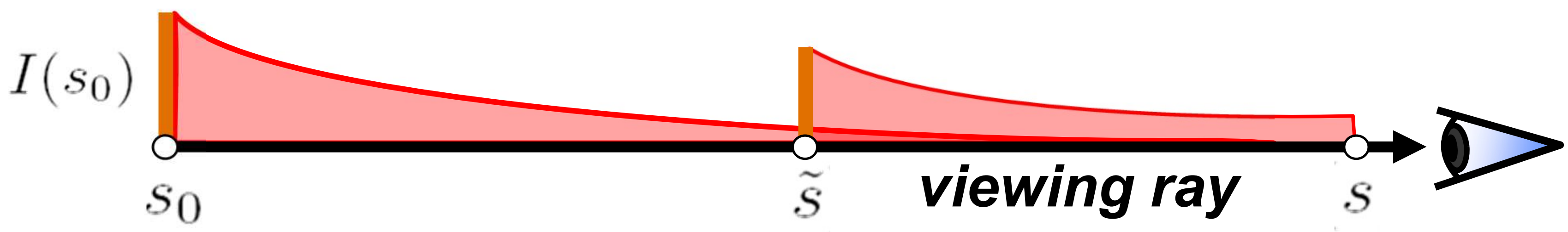


$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + q(\tilde{s})$$

Volume Rendering Integral

How do we determine the radiant energy along the ray?

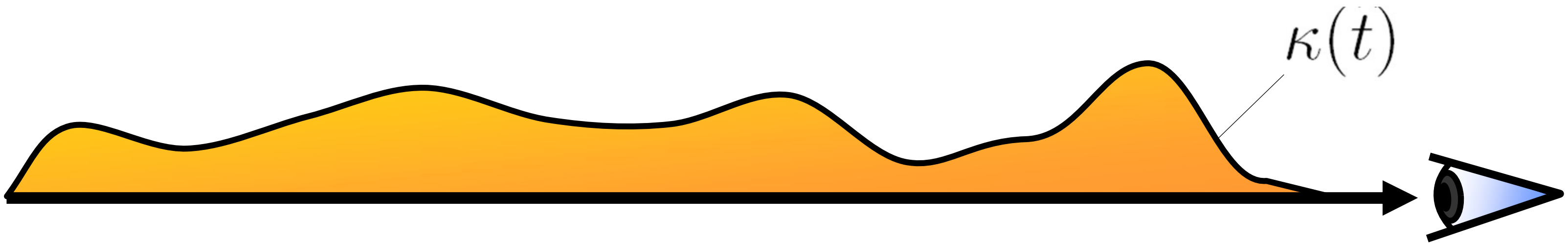
Physical model: emission and absorption, no scattering



Every point $\tilde{s}$ along the viewing ray emits additional radiant energy

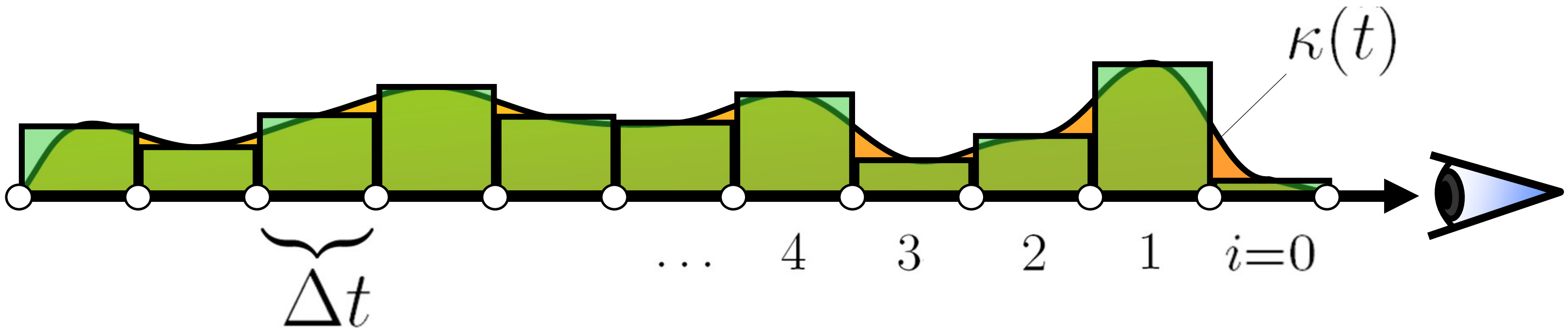
$$I(s) = I(s_0)\, e^{-\tau(s_0,s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s},s)}\, d\tilde{s}$$

Volume Rendering Integral: Numerical Solution



Optical depth: $\tau(0, t) \quad = \quad \int_0^t \kappa(\hat{t})\, d\hat{t}$

Volume Rendering Integral: Numerical Solution

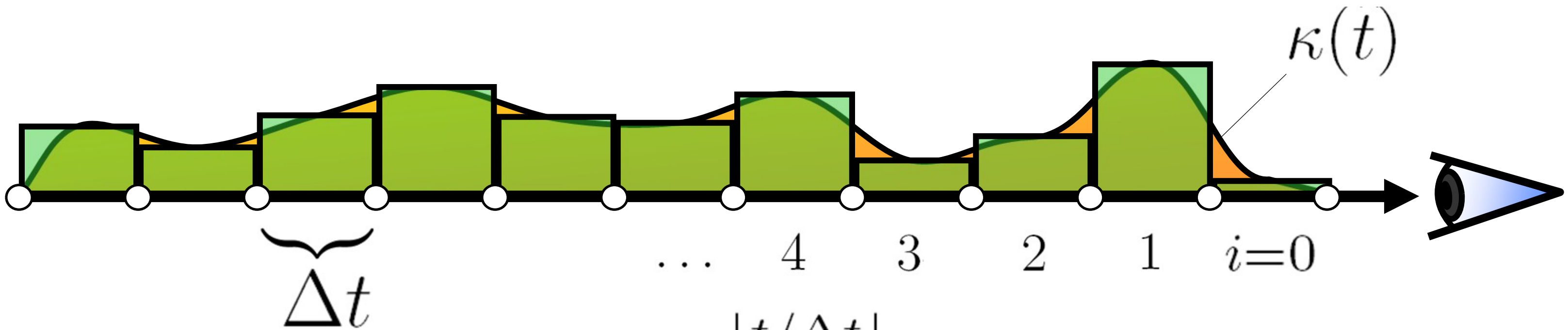


Optical depth: $\tau(0,t) \quad = \quad \int_0^t \kappa(\hat{t})\, d\hat{t}$

Approximate Integral by Riemann sum:

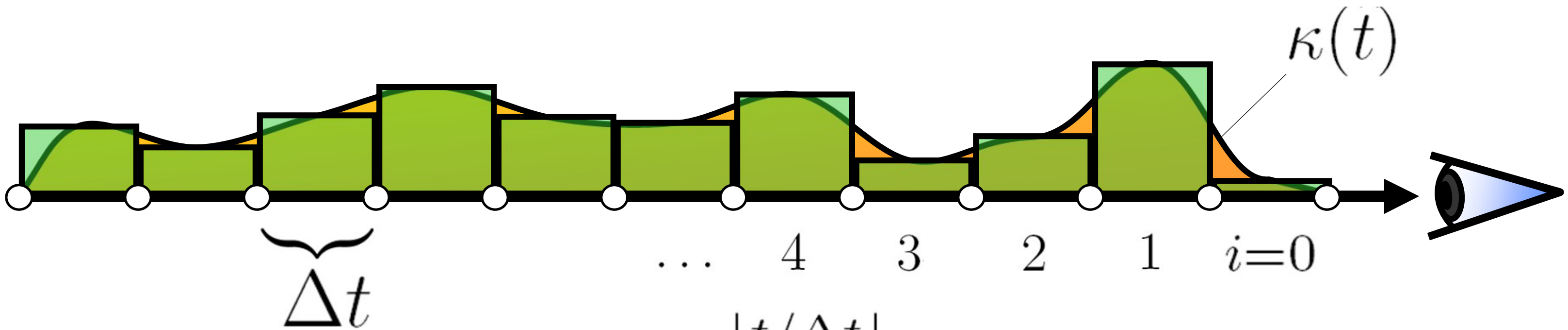
$$\tau(0,t) \quad \approx \quad \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

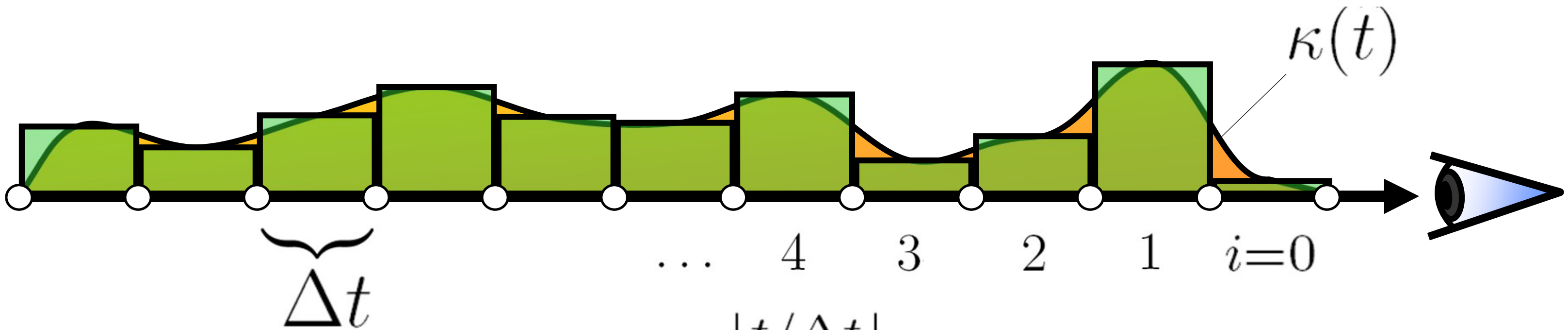
Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

$$e^{-\tilde{\tau}(0,t)} = e^{-\sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



$$\tau(0,t) \approx \tilde{\tau}(0,t) = \sum_{i=0}^{\lfloor t/\Delta t \rfloor} \kappa(i \cdot \Delta t)\, \Delta t$$

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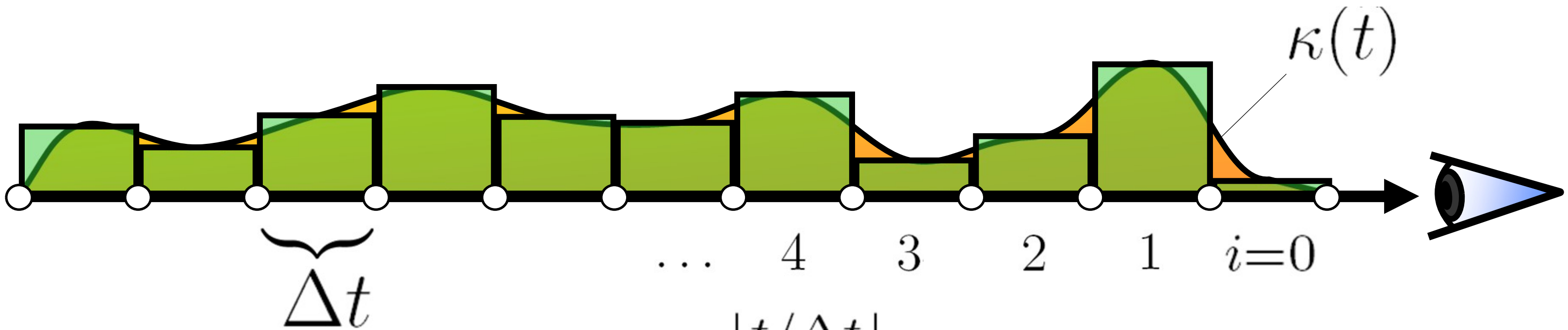
Volume Rendering Integral: Numerical Solution



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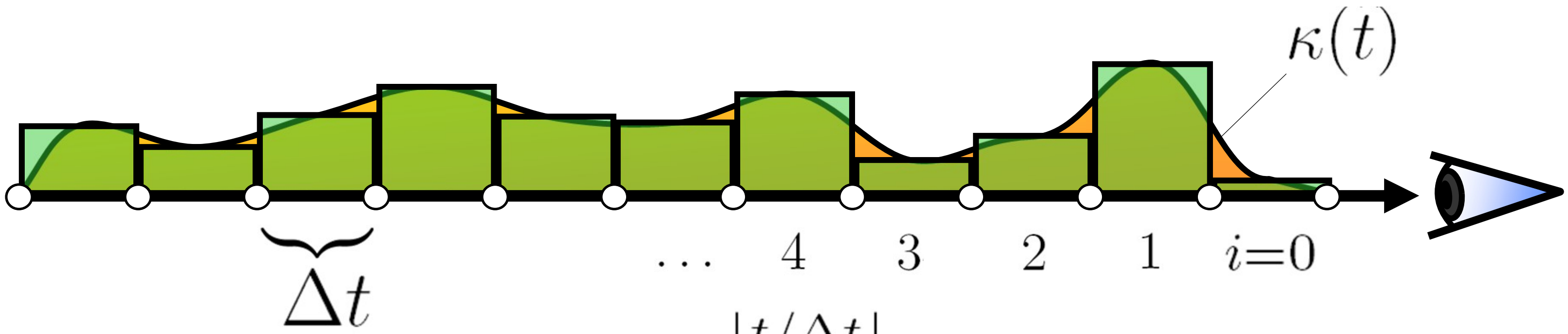
Now we introduce *opacity*:

$$A_i = 1 - e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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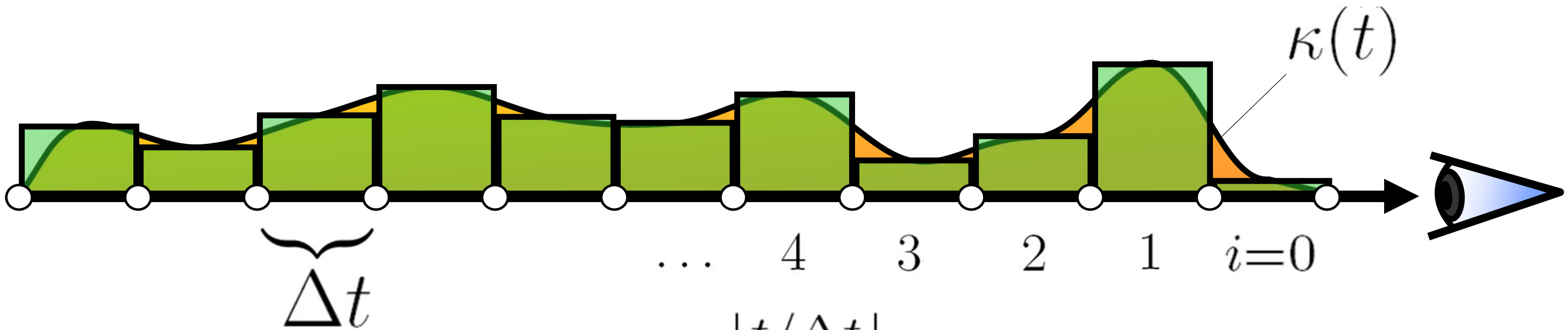
$$1 - A_i = e^{-\kappa(i \cdot \Delta t)\, \Delta t}$$

Volume Rendering Integral: Numerical Solution



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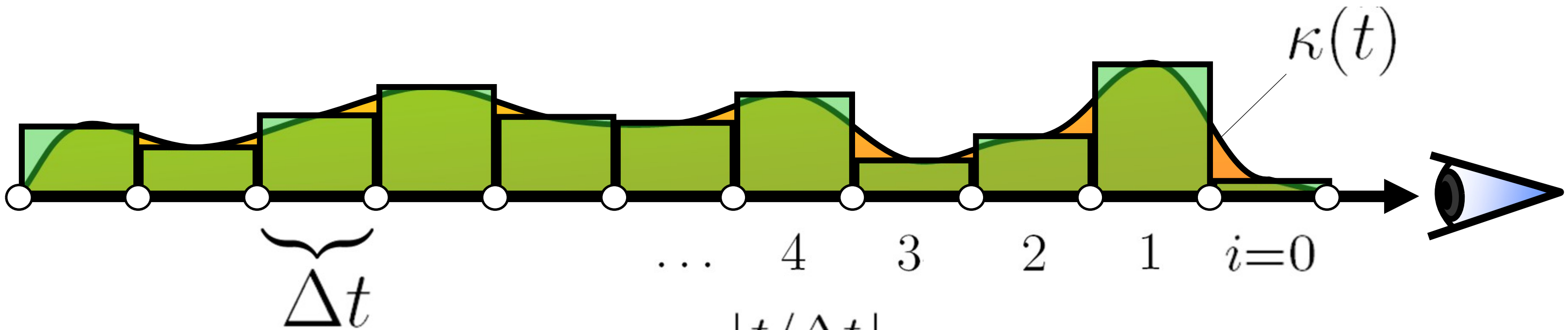
Volume Rendering Integral: Numerical Solution



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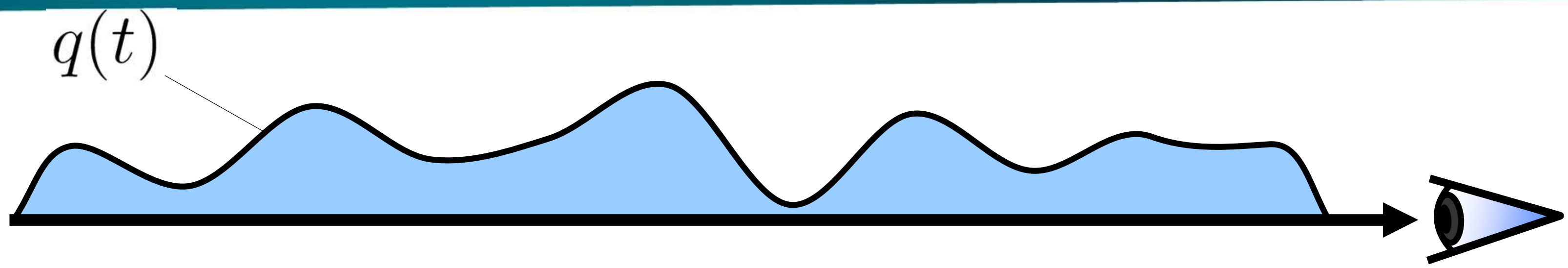
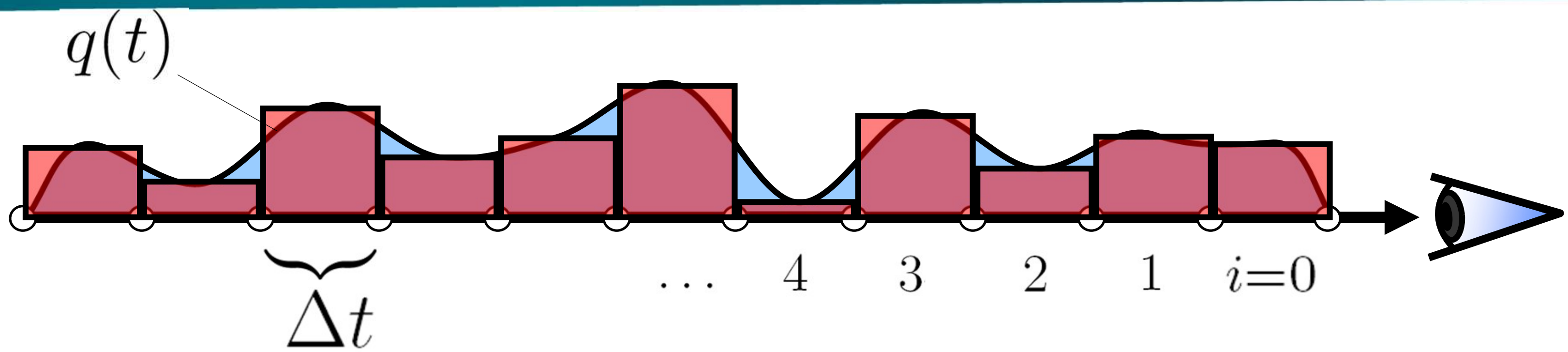
Volume Rendering Integral: Numerical Solution



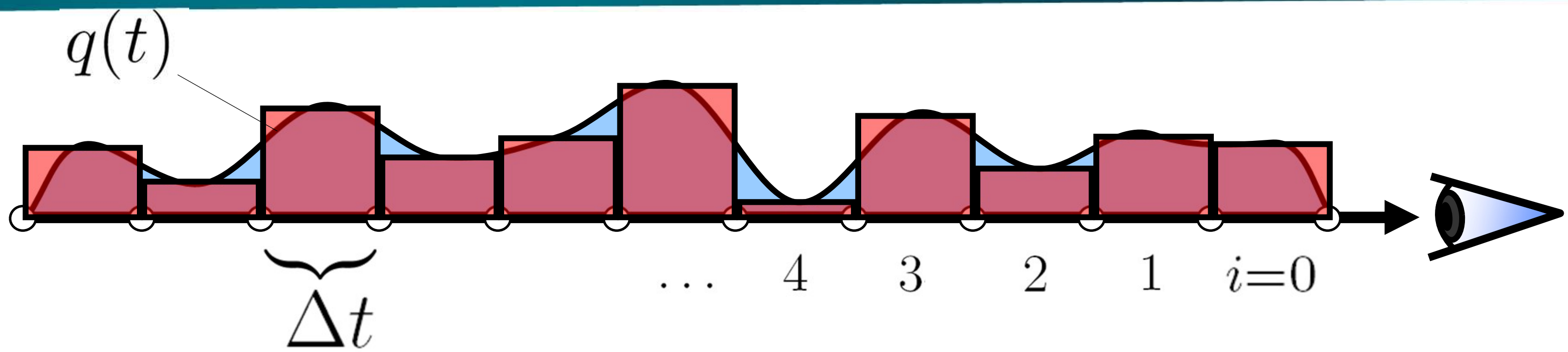
Volume Rendering Integral: Numerical Solution



$$e^{-\tilde{\tau}(0,t)} = \prod_{i=0}^{\lfloor t/\Delta t \rfloor} (1 - A_i)$$

$$q(t) \approx C_i = c(i \cdot \Delta t)\, \Delta t$$

Volume Rendering Integral: Numerical Solution



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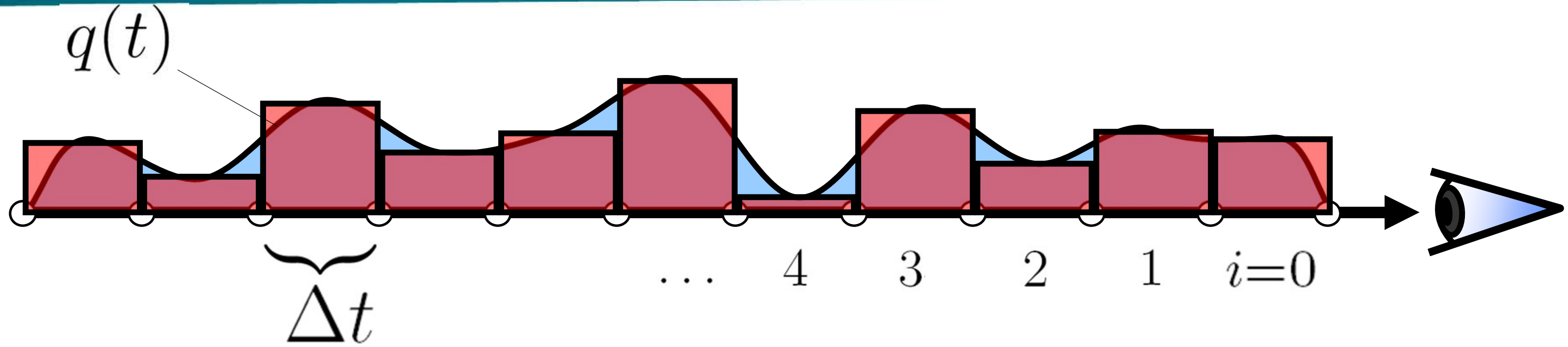
$$q(t) \approx C_i = c(i \cdot \Delta t)\,\Delta t$$

$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \, e^{-\tilde{\tau}(0,t)}$$

Volume Rendering Integral: Numerical Solution



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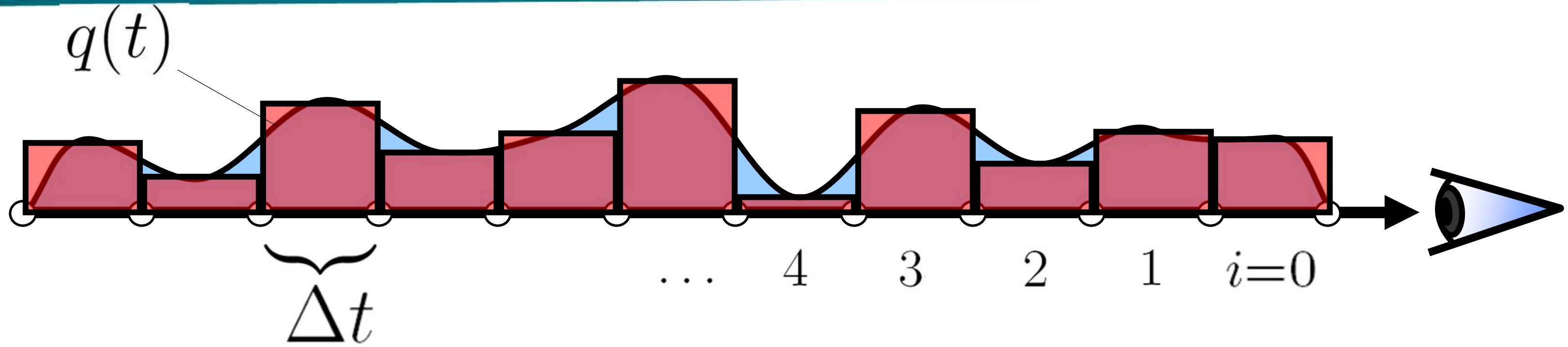

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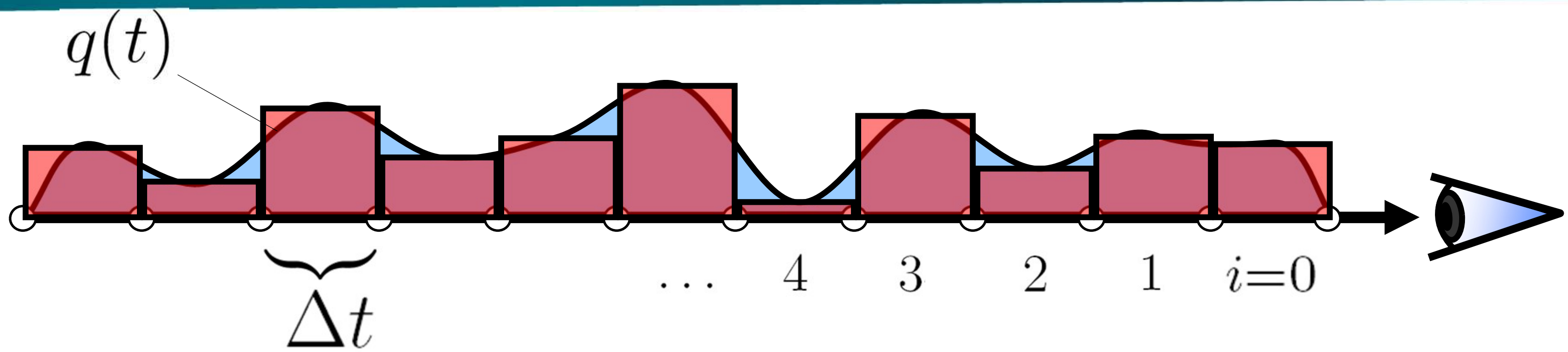
Volume Rendering Integral: Numerical Solution



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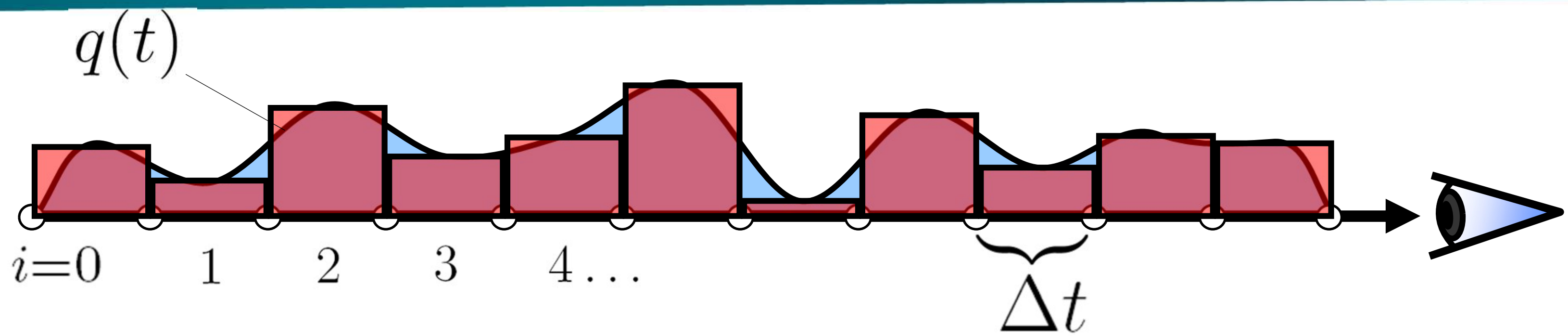
$$\tilde{C} = \sum_{i=0}^{\lfloor T/\Delta t \rfloor} C_i \prod_{j=0}^{i-1} (1 - A_j)$$

can be computed recursively/iteratively!

Volume Rendering Integral: Numerical Solution

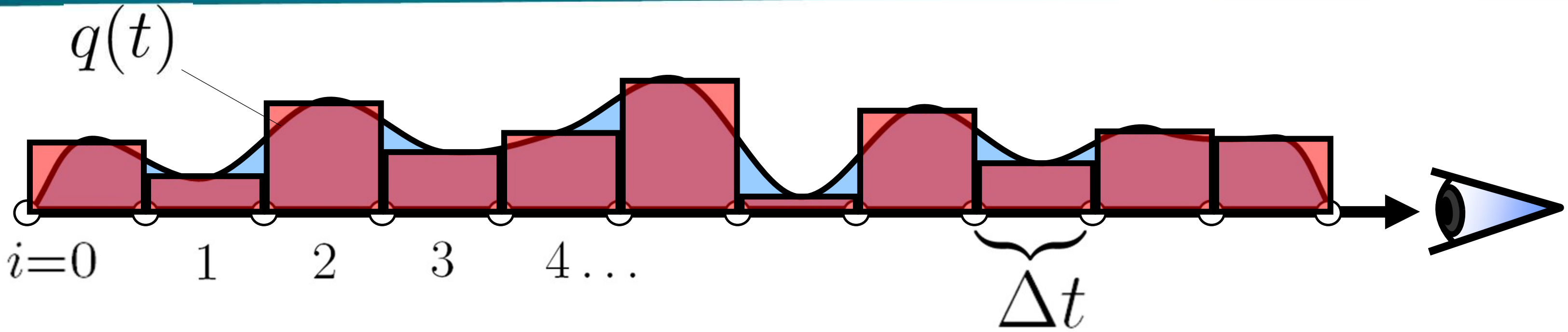


Note: we just changed the convention from i=0 is at the front of the volume (previous slides) to i=0 is at the back of the volume !

can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Volume Rendering Integral: Numerical Solution



can be computed recursively/iteratively:

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

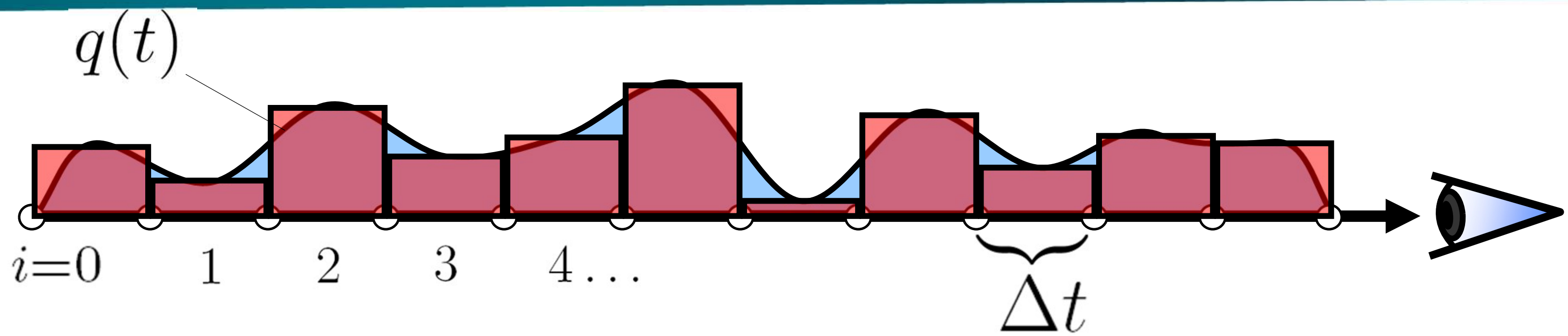
Radiant energy observed at position ***i***

Radiant energy emitted at position ***i***

Absorption at position ***i***

Radiant energy observed at position ***i–1***

Volume Rendering Integral: Numerical Solution



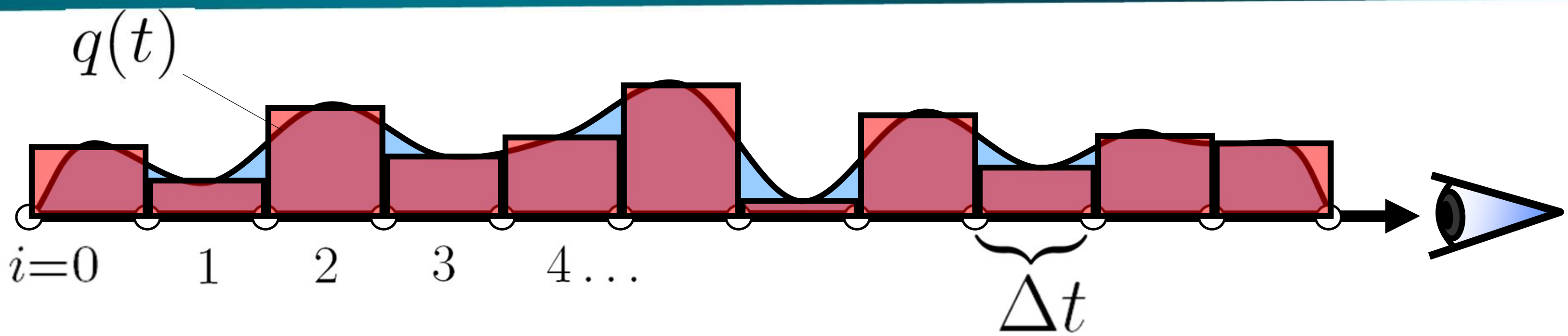
Back-to-front compositing

$$C'_i = C_i + (1 - A_i) C'_{i-1}$$

Front-to-back compositing

$$C'_i = C'_{i+1} + (1 - A'_{i+1}) C_i$$
$$A'_i = A'_{i+1} + (1 - A'_{i+1}) A_i$$

Volume Rendering Integral: Numerical Solution



Back-to-front compositing

$$C'_i = C_i + (1 - A_i)C'_{i-1}$$

Iterate from *i*=0 (back) to *i*=max (front): *i* increases

Front-to-back compositing

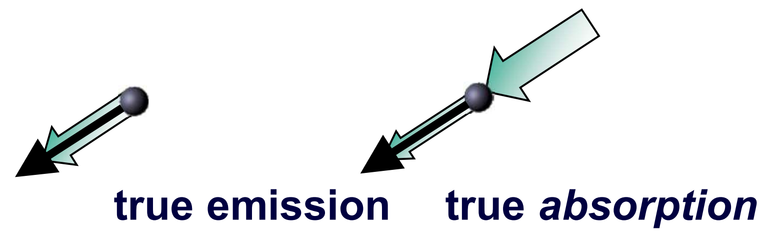
$$C'_i = C'_{i+1} + (1 - A'_{i+1})C_i$$

$$A'_i = A'_{i+1} + (1 - A'_{i+1})A_i$$

Iterate from *i*=max (front) to *i*=0 (back) : *i* decreases

Volume Rendering Integral Summary

Volume rendering integral
for *Emission Absorption* model



$$I(s) \;=\; I(s_0)\, e^{-\tau(s_0, s)} + \int_{s_0}^{s} q(\tilde{s})\, e^{-\tau(\tilde{s}, s)}\, \mathrm{d}\tilde{s}$$

Numerical solutions:

Back-to-front compositing

$$C'_i \;=\; C_i + (1 - A_i) C'_{i-1}$$

Front-to-back compositing

$$C'_i \;=\; C'_{i+1} + (1 - A'_{i+1}) C_i$$

$$A'_i \;=\; A'_{i+1} + (1 - A'_{i+1}) A_i$$

Opacity Correction

Simple compositing only works as far as the opacity values are correct… and they depend on the sample distance!

$$T_i = e^{-\int_{s_i}^{s_i+\Delta x} \kappa \, \mathrm{d}s} \approx e^{-\kappa(s_i)\Delta x} \qquad \tilde{T} = T^{\left(\frac{\Delta \tilde{x}}{\Delta x}\right)}$$

Opacity correction formula:

$$A_i = 1 - e^{-\kappa(s_i)\Delta x} \qquad \boxed{\tilde{A}_i = 1 - (1 - A_i)^{\left(\frac{\Delta \tilde{x}}{\Delta x}\right)}}$$

Beware that usually this is done *for each different scalar value* (every transfer function entry), not actually at spatial positions/intervals *i*

Associated Colors

Associated (or “opacity-weighted” colors) are often used in compositing equations

Every color is *pre-multiplied* by its corresponding opacity

$$\begin{bmatrix} R \\ G \\ B \\ A \end{bmatrix} \Rightarrow \begin{pmatrix} R*A \\ G*A \\ B*A \\ A \end{pmatrix}$$

Our compositing equations assume associated colors!

Important:

After opacity-correction, all associated colors must be updated!

Associated Colors in Volume Rendering

(Standard) emission-absorption optical model

- Only one kind of particle: the same particles that absorb light, emit light
- Aha! Therefore lower absorption means lower emission as well

Light observed from (in front of) segment i:

$$\frac{q_i}{\kappa_i}\left(1 - e^{-\kappa_i \Delta t}\right)$$

$$\lim_{\kappa \to 0}(1 - e^{-\kappa \Delta t})/\kappa = \Delta t$$

$$\lim_{\kappa \to \infty}(1 - e^{-\kappa \Delta t})/\kappa = 0$$

$$= C_i A_i \qquad A_i := 1 - e^{-\kappa_i \Delta t}$$

$$q_i := C_i \kappa_i$$

Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama