King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 9: Scalar Fields, Pt. 5

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Reading Assignment #5 (until Mar 1)

Read (required):

- Gradients of scalar-valued functions

  `https://en.wikipedia.org/wiki/Gradient`

- Critical points

  `https://en.wikipedia.org/wiki/Critical_point_(mathematics)`

- Multivariable derivatives and differentials

  `https://en.wikipedia.org/wiki/Total_derivative`

  `https://en.wikipedia.org/wiki/Differential_of_a_function#Differentials_in_several_variables`

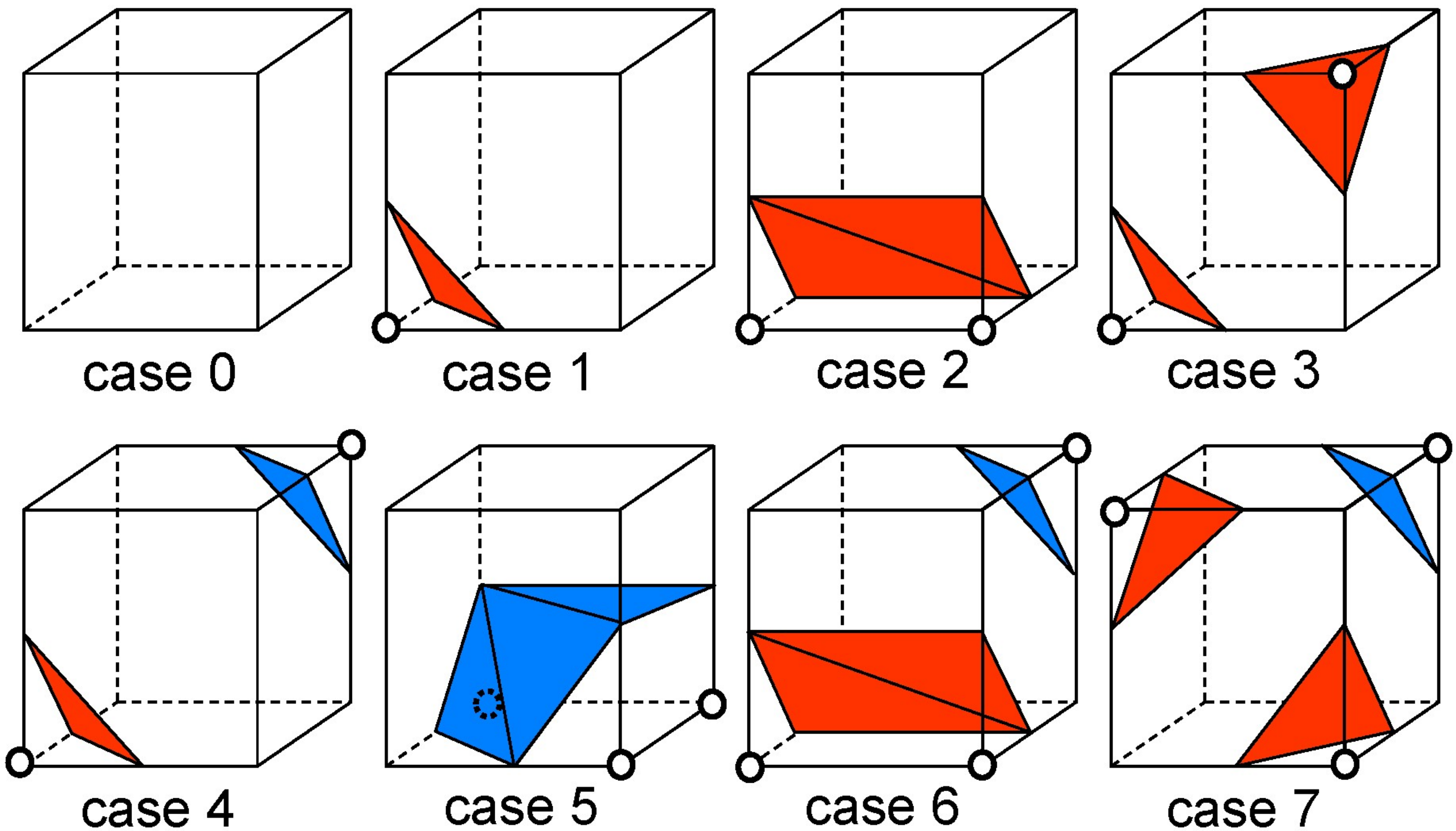
  `https://en.wikipedia.org/wiki/Hessian_matrix`

- Dot product, inner product (more general)

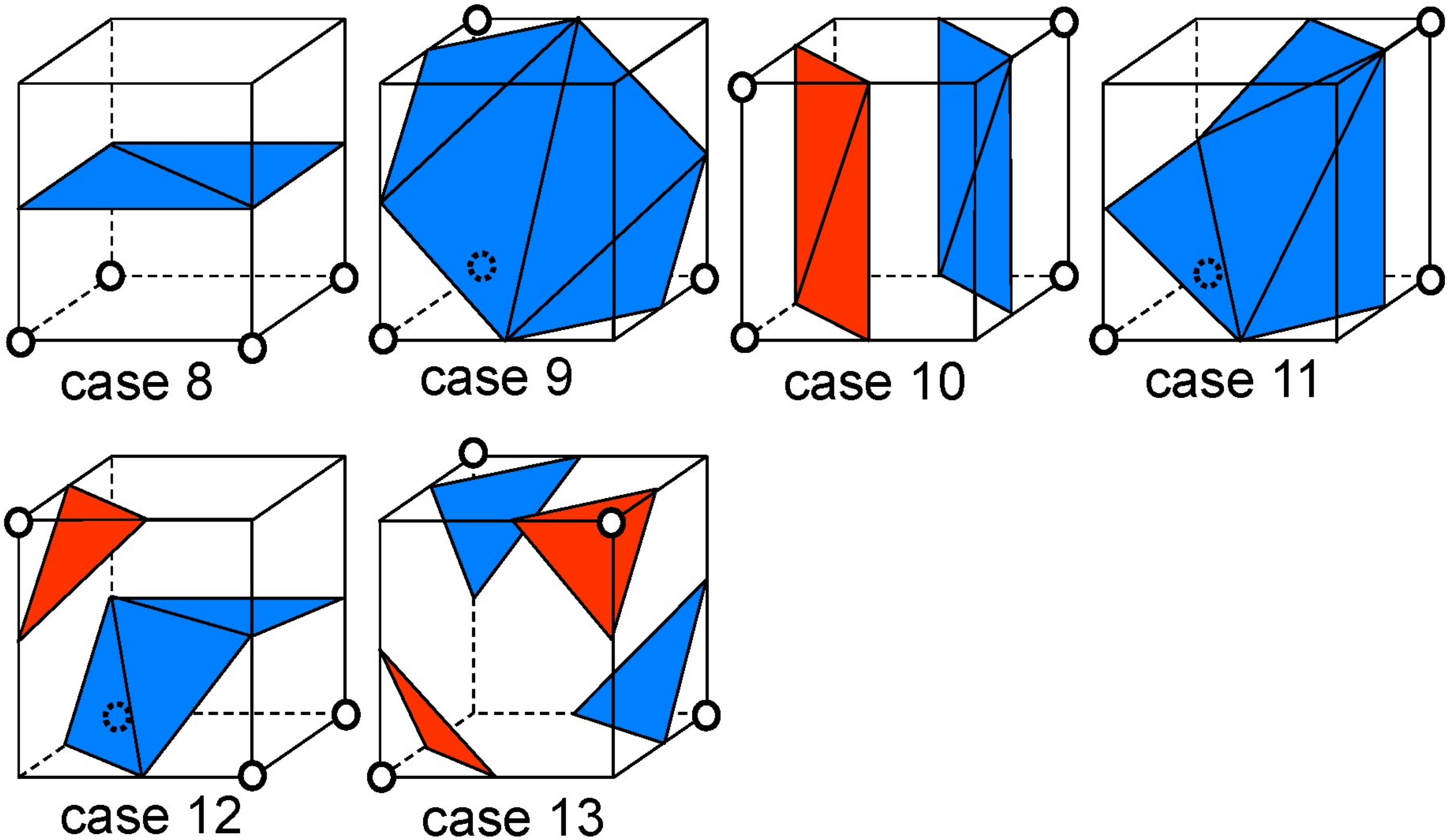
  `https://en.wikipedia.org/wiki/Dot_product`

  `https://en.wikipedia.org/wiki/Inner_product_space`

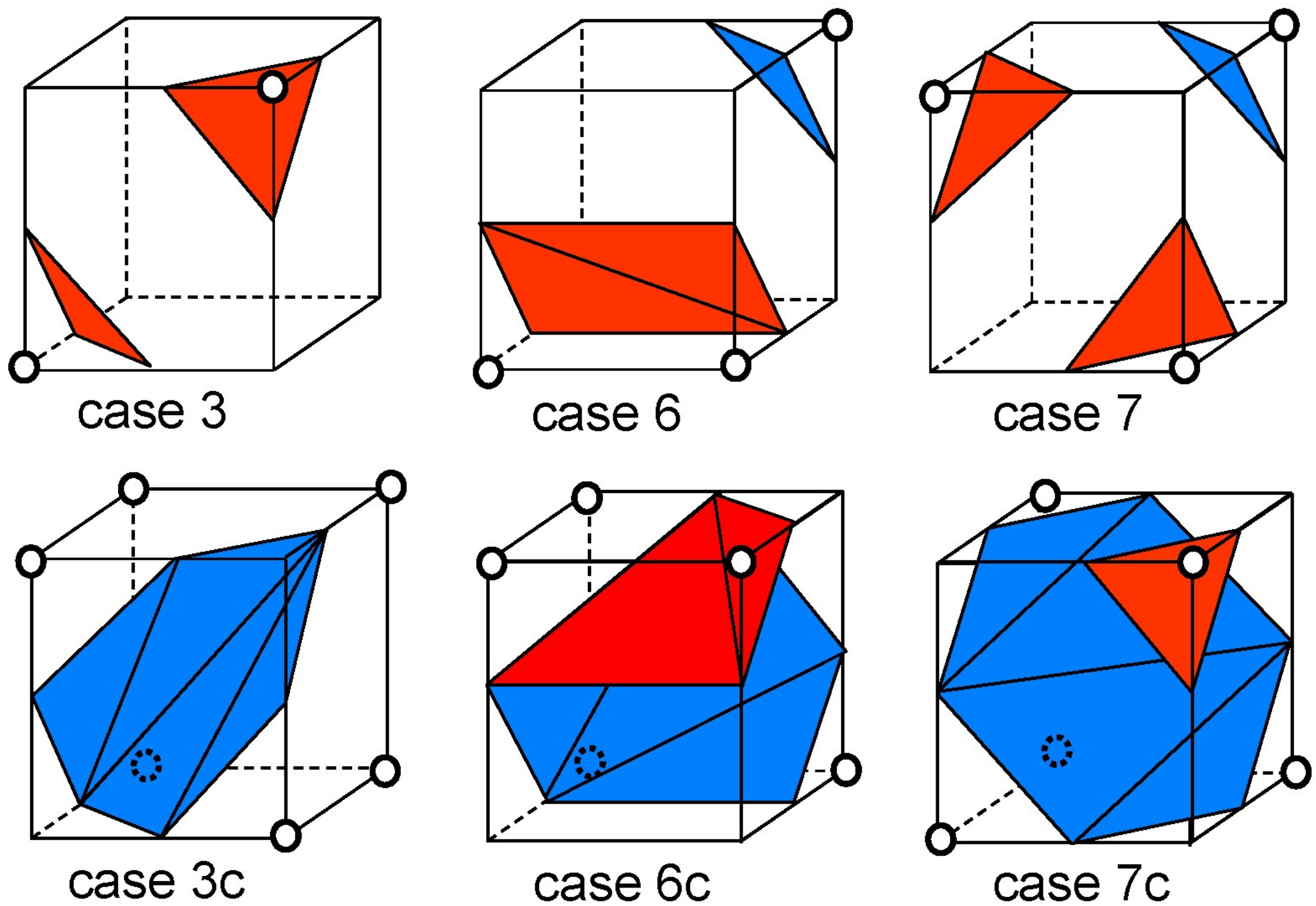
The marching cubes algorithm



The marching cubes algorithm



The marching cubes algorithm



The marching cubes algorithm

Loop over cells:

- find sign of $\tilde{f}(x_i)$ for the 8 corner nodes, giving 8-bit integer
- use as index into (256 case) table
- find intersection points on edges listed in table, using linear interpolation
- generate triangles according to table

Post-processing steps:

- connect triangles (share vertices)
- compute normal vectors
  - by averaging triangle normals (problem: thin triangles!)
  - by estimating the gradient of the field $f(x_i)$ (better)

Triangle Mesh Data Structures

- Typical implementations of unstructured grids
  - Indirect form

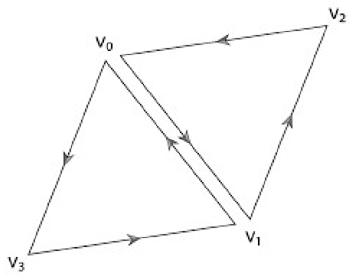
vertex list

```
v[0]: x0,y0,z0
v[1]: x1,y1,z1
v[2]: x2,y2,z2
v[3]: x3,y3,z3
```

coords for vertex 0

face list

```
f[0]: 0,1,2
f[1]: 1,0,3
f[2]: 2,1,4
...
```



  - Indexed face set
  - More efficient than direct approach in terms of memory requirements; geometry and topology separated
  - But still have to do global search to find local information (i.e. what faces share an edge)
  - More neighborhood information: half-edge data structure, …

Orientability (2-manifold embedded in 3D)

not orientable

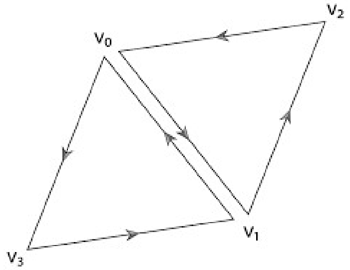
Moebius strip
(only one side!)

Orientability of 2-manifold:

Possible to assign consistent normal vector orientation

Triangle meshes

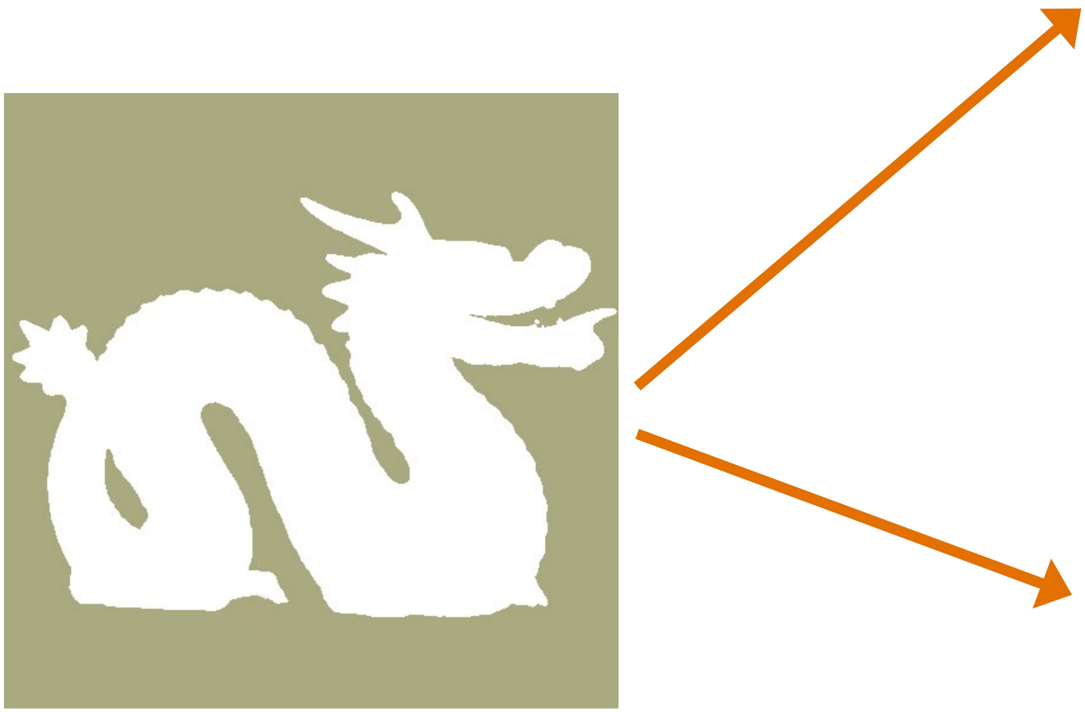
- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise) (e.g., (0,1,2) on one side of edge, (1,0,3) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”

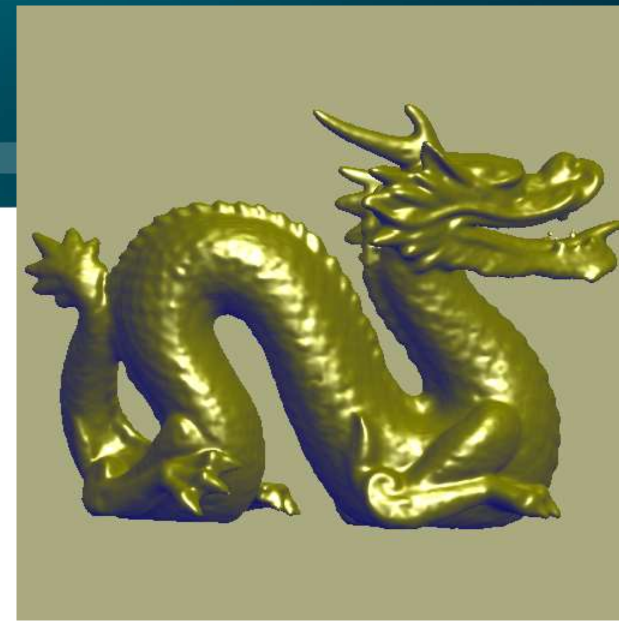


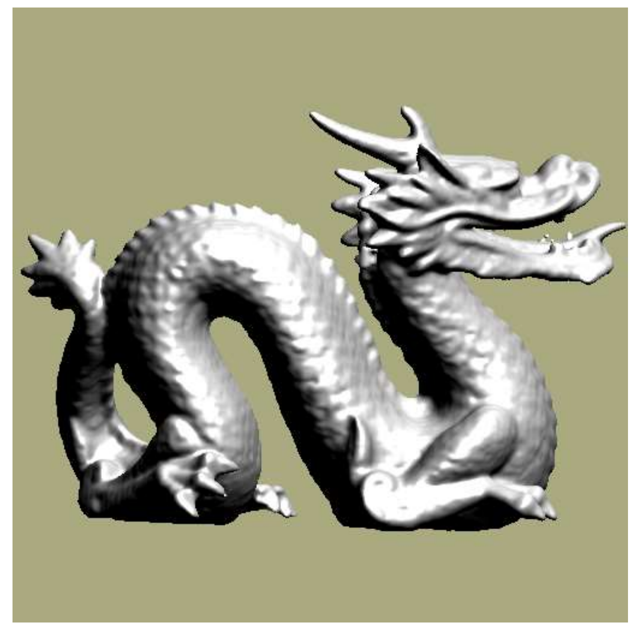
Iso-Surface / Volume Illumination

What About Volume Illumination?

Crucial for perceiving shape and depth relationships







this is a scalar volume (3D distance field)!

Local Illumination in Volumes

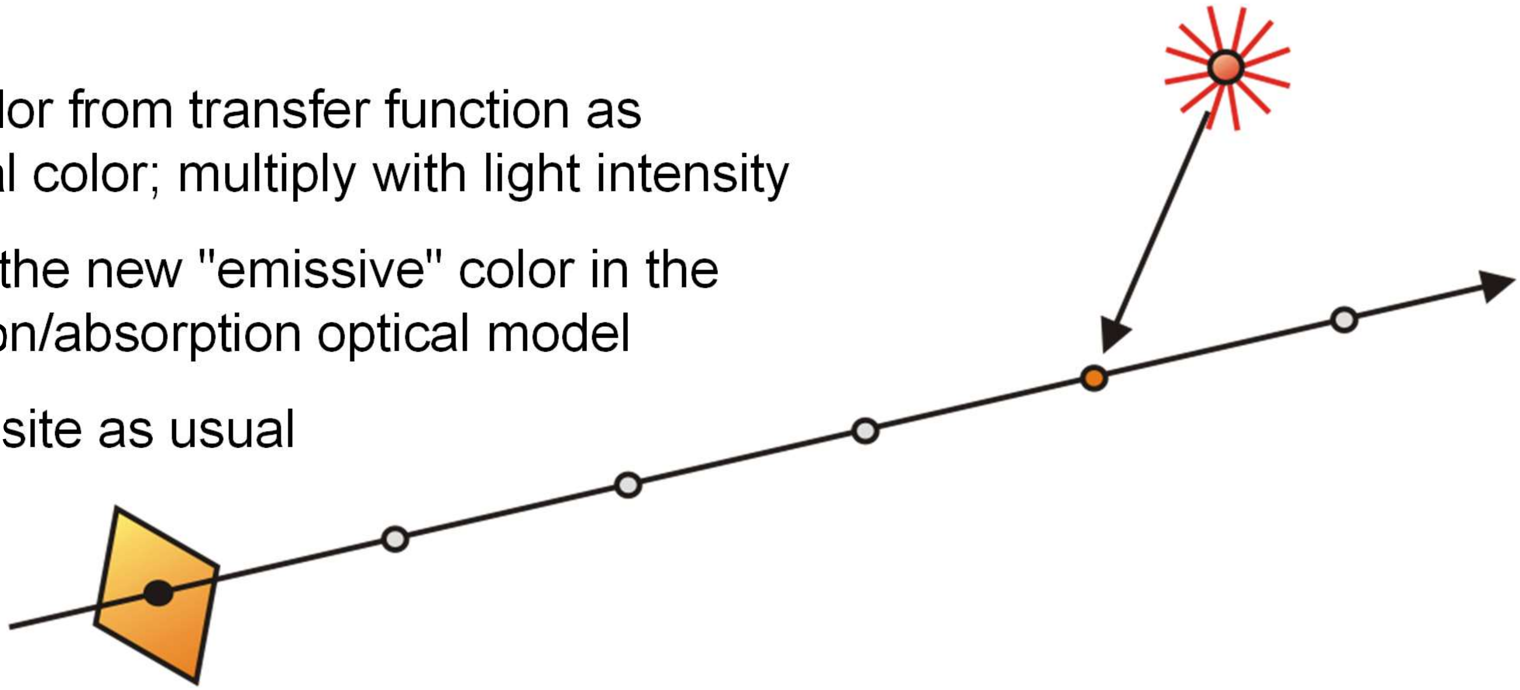
Interaction between light source and point in the volume

Local shading equation; evaluate at each point along a ray

Use color from transfer function as material color; multiply with light intensity

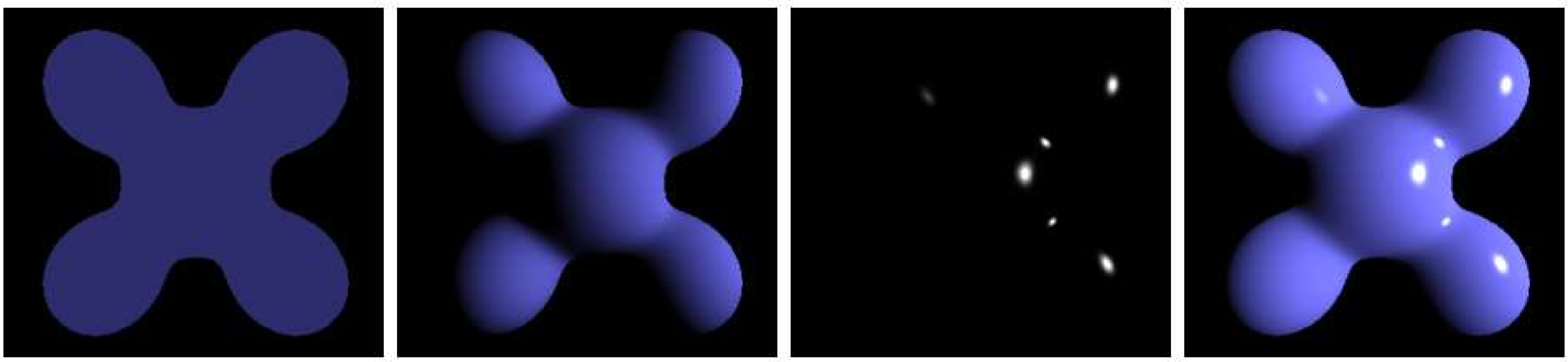
This is the new "emissive" color in the emission/absorption optical model

Composite as usual



Local Illumination Model: Phong Lighting Model

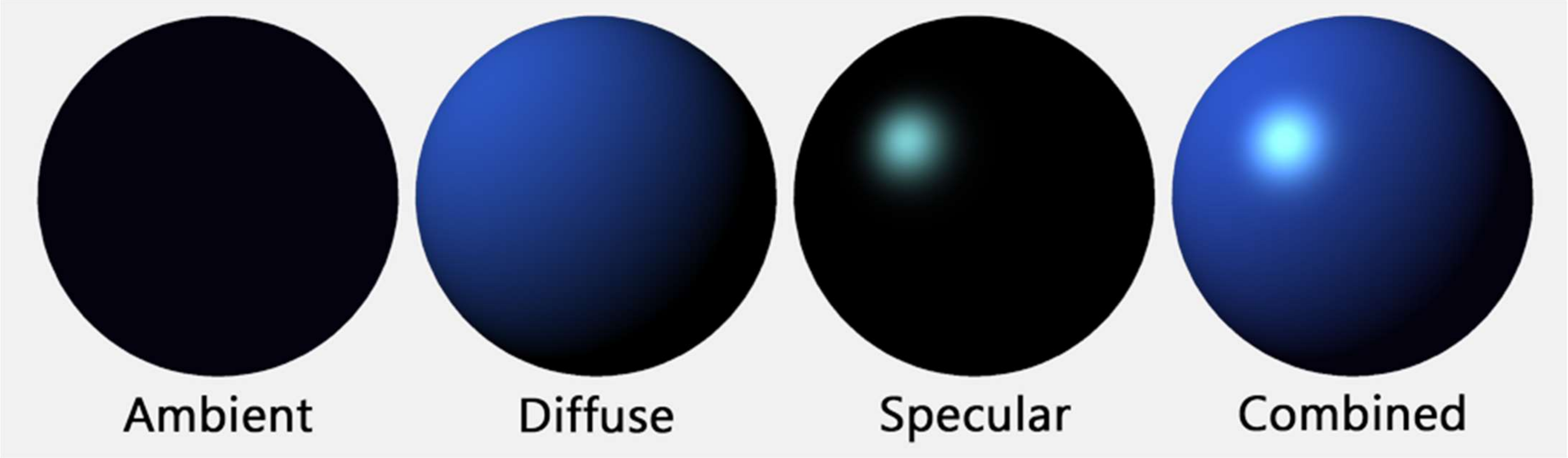
$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



Ambient + Diffuse + Specular = Phong Reflection

Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$

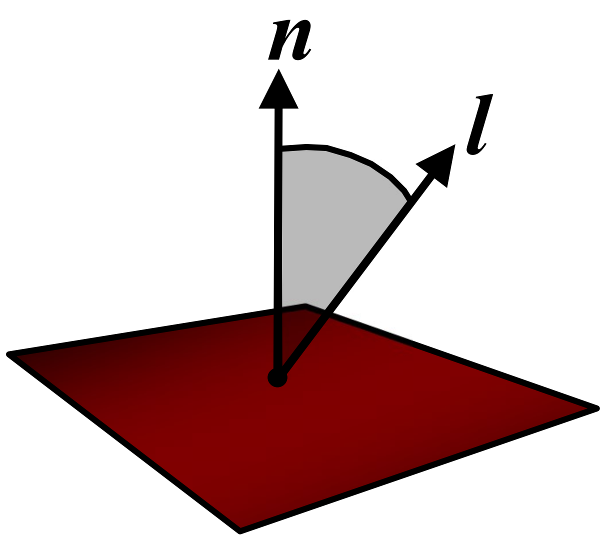


Local Shading Equations

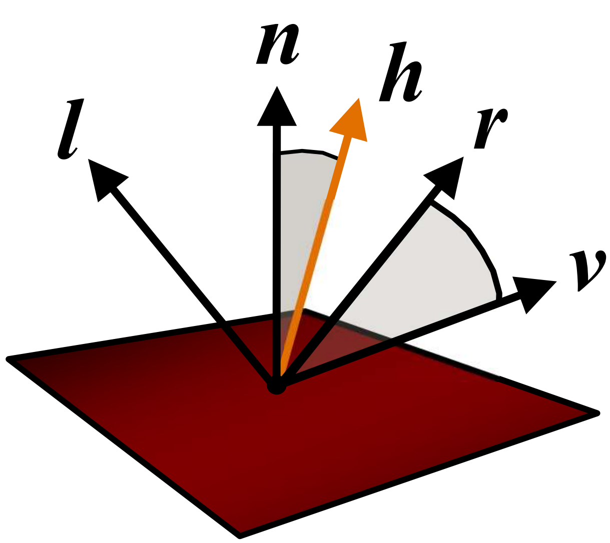
Standard volume shading adapts surface shading

Most commonly Blinn/Phong model

But what about the "surface" normal vector?



diffuse reflection



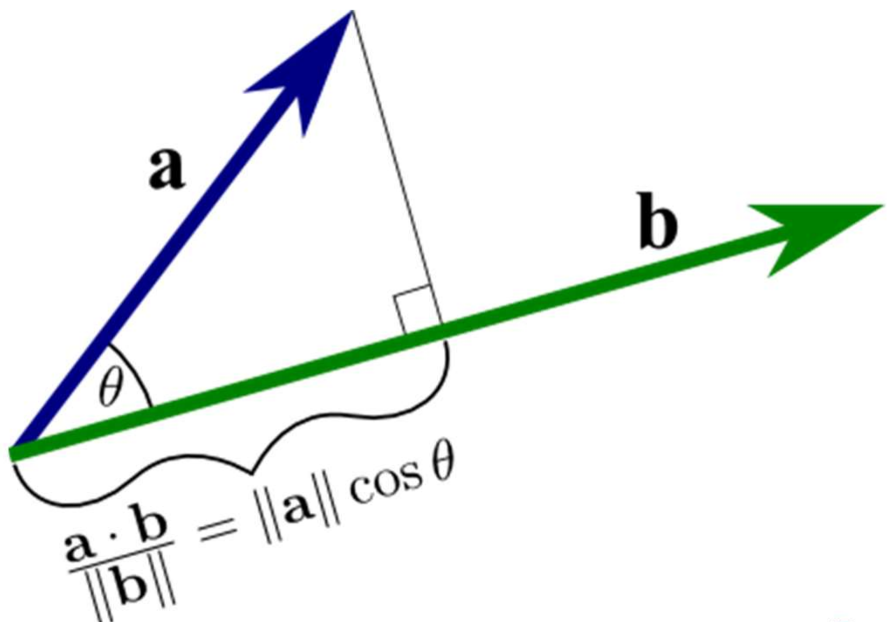
specular reflection

The Dot Product (Scalar / Inner Product)

Cosine of angle between two vectors times their lengths

$$\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{n} a_i b_i \qquad \mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\theta$$

(standard inner product in Cartesian coordinates)



Many uses:

- Project vector onto another vector,
  project into basis,
  project into tangent plane,
  ...

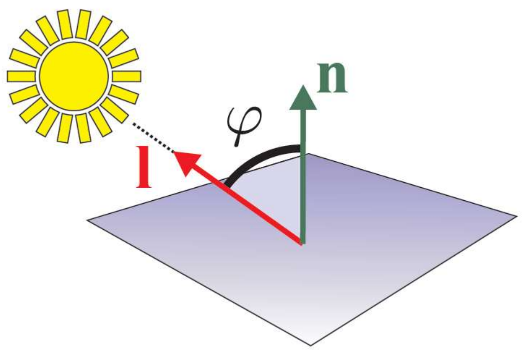
Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$

$$\mathbf{I}_{\text{ambient}} = k_a \, \mathbf{M}_a \, \mathbf{I}_a$$

Local Illumination Model: Phong Lighting Model

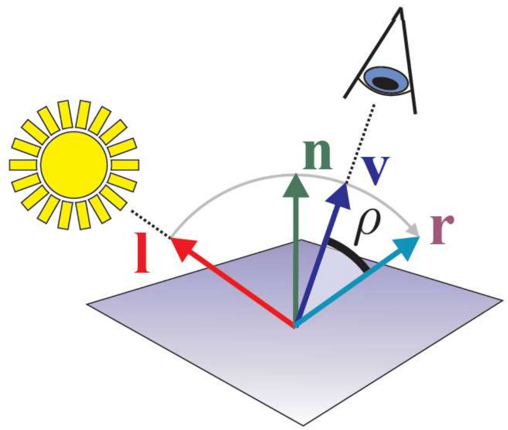
$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



$$\begin{aligned}\mathbf{I}_{\text{diffuse}} &= k_d\,\mathbf{M}_d\,\mathbf{I}_d \cos\varphi \qquad \text{if } \varphi \leq \frac{\pi}{2} \\ &= k_d\,\mathbf{M}_d\,\mathbf{I}_d \max((\mathbf{n}\cdot\mathbf{l}), 0)\end{aligned}$$

Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



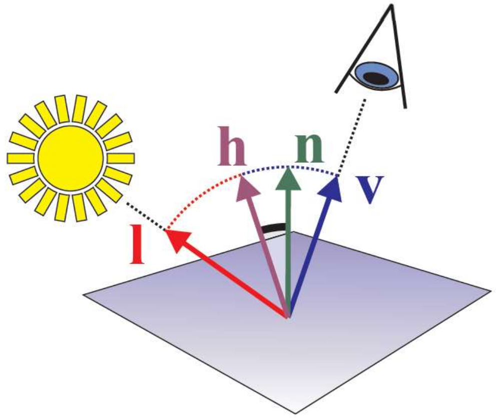
$$\mathbf{I}_{\text{specular}} = k_s\,\mathbf{M}_s\,\mathbf{I}_s\cos^n\rho\,,\quad \text{if } \rho \le \frac{\pi}{2}$$

$$= k_s\,\mathbf{M}_s\,\mathbf{I}_s\,(\mathbf{r}\cdot\mathbf{v})^n$$

must also clamp!

Local Illumination Model: Phong Lighting Model

$$\mathbf{I}_{\text{Phong}} = \mathbf{I}_{\text{ambient}} + \mathbf{I}_{\text{diffuse}} + \mathbf{I}_{\text{specular}}$$



$$\mathbf{I}_{\text{specular}} \approx k_s\, \mathbf{M}_s\, \mathbf{I}_s\, (\mathbf{h} \cdot \mathbf{n})^n$$

must also clamp!

$$\mathbf{h} = \frac{\mathbf{v} + \mathbf{l}}{\|\mathbf{v} + \mathbf{l}\|}$$

half-way vector

Gradient and Directional Derivative

Gradient $\nabla f(x,y,z)$ of scalar function $f(x,y,z)$: (in Cartesian coordinates)

$$\nabla f(x,y,z) = \left(\frac{\partial f(x,y,z)}{\partial x}, \frac{\partial f(x,y,z)}{\partial y}, \frac{\partial f(x,y,z)}{\partial z} \right)^T$$

Directional derivative in direction $\mathbf{u}$:

$$D_{\mathbf{u}} f(x,y,z) = \nabla f(x,y,z) \cdot \mathbf{u}$$

And therefore also:

$$D_{\mathbf{u}} f(x,y,z) = ||\nabla f|| \; ||\mathbf{u}|| \; \cos\theta$$

The Gradient as Normal Vector

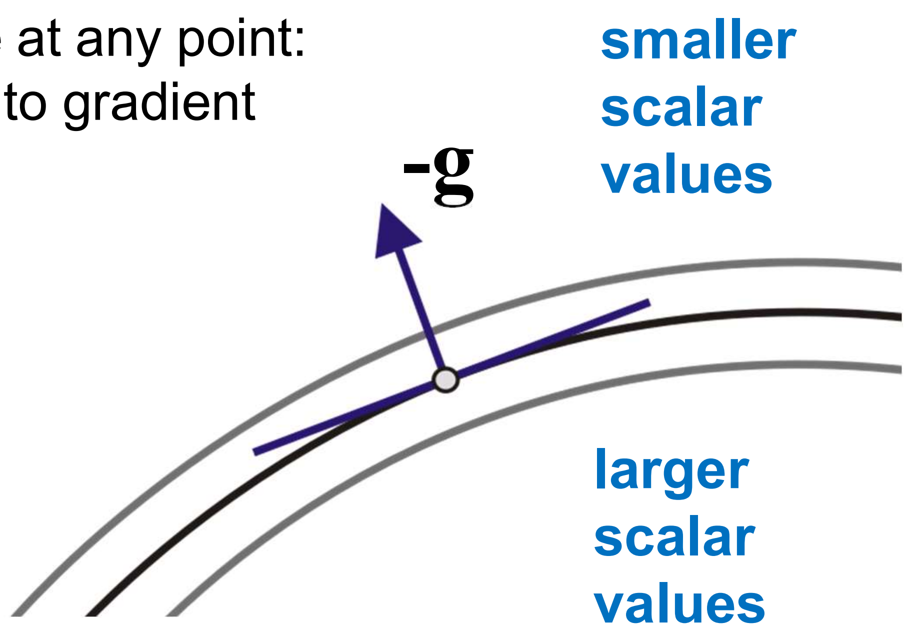
Gradient of the scalar field gives direction+magnitude of fastest change

$$\mathbf{g} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)^{\mathbf{T}}$$

Local approximation to isosurface at any point:
tangent plane = plane orthogonal to gradient

Normal of this isosurface:
normalized gradient vector
(negation is common convention)

$$\mathbf{n} = -\mathbf{g}/|\mathbf{g}|$$



Thank you.

Thanks for material

- Helwig Hauser
- Eduard Gröller
- Daniel Weiskopf
- Torsten Möller
- Ronny Peikert
- Philipp Muigg
- Christof Rezk-Salama