King Abdullah University of Science and Technology KAUST

CS 247 – Scientific Visualization
Lecture 7: Scalar Fields, Pt. 3

Markus Hadwiger, KAUST

Reading Assignment #4 (until Feb 22)

Read (required):

- Real-Time Volume Graphics book, Chapter 5 until 5.4 inclusive (*Terminology, Types of Light Sources, Gradient-Based Illumination, Local Illumination Models*)
- Paper:
  Marching Cubes: A high resolution 3D surface construction algorithm, Bill Lorensen & Harvey Cline, ACM SIGGRAPH 1987
  [> 16,000 citations and counting...]

  `http://dl.acm.org/citation.cfm?id=37422`

Quiz #1: Feb 17

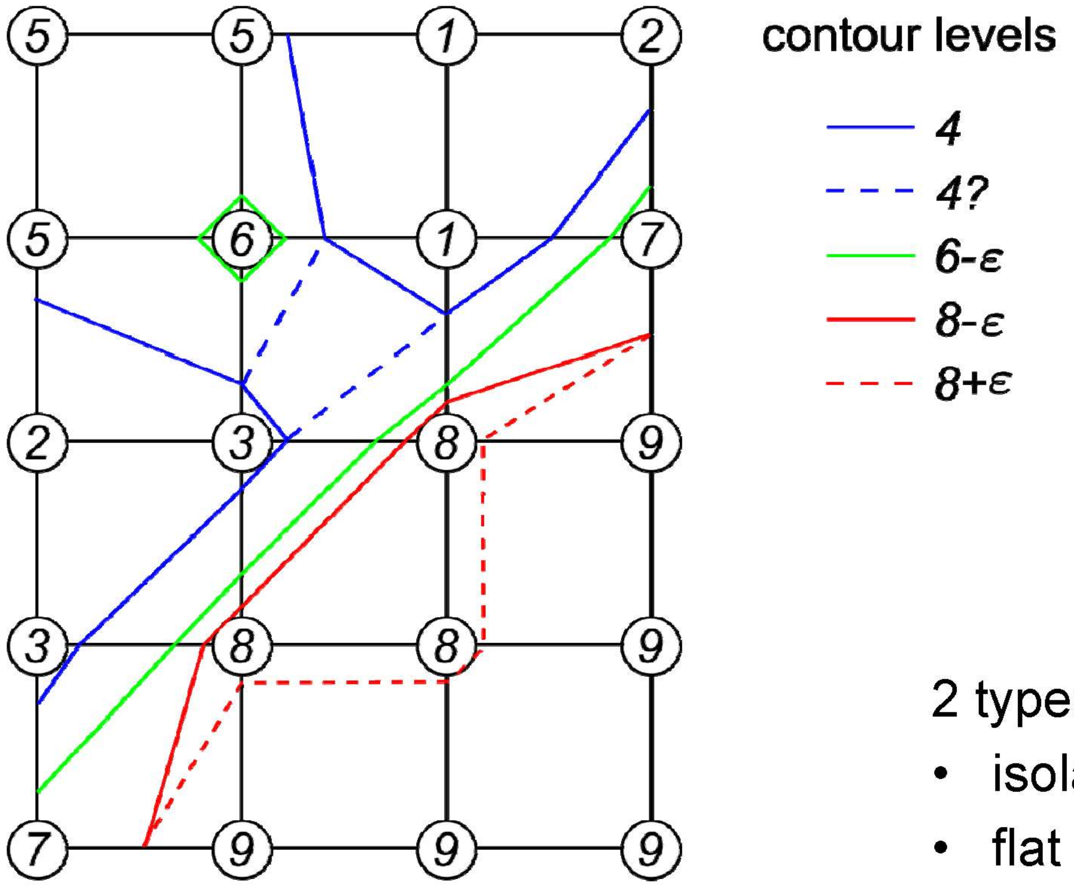
Organization

- First 30 min of lecture
- No material (book, notes, ...) allowed

Content of questions

- Lectures (both actual lectures and slides)
- Reading assignments (except optional ones)
- Programming assignments (algorithms, methods)
- Solve short practical examples

Example



2 types of degeneracies:

- isolated points (*c*=6)
- flat regions (*c*=8)

Contours in a quadrangle cell

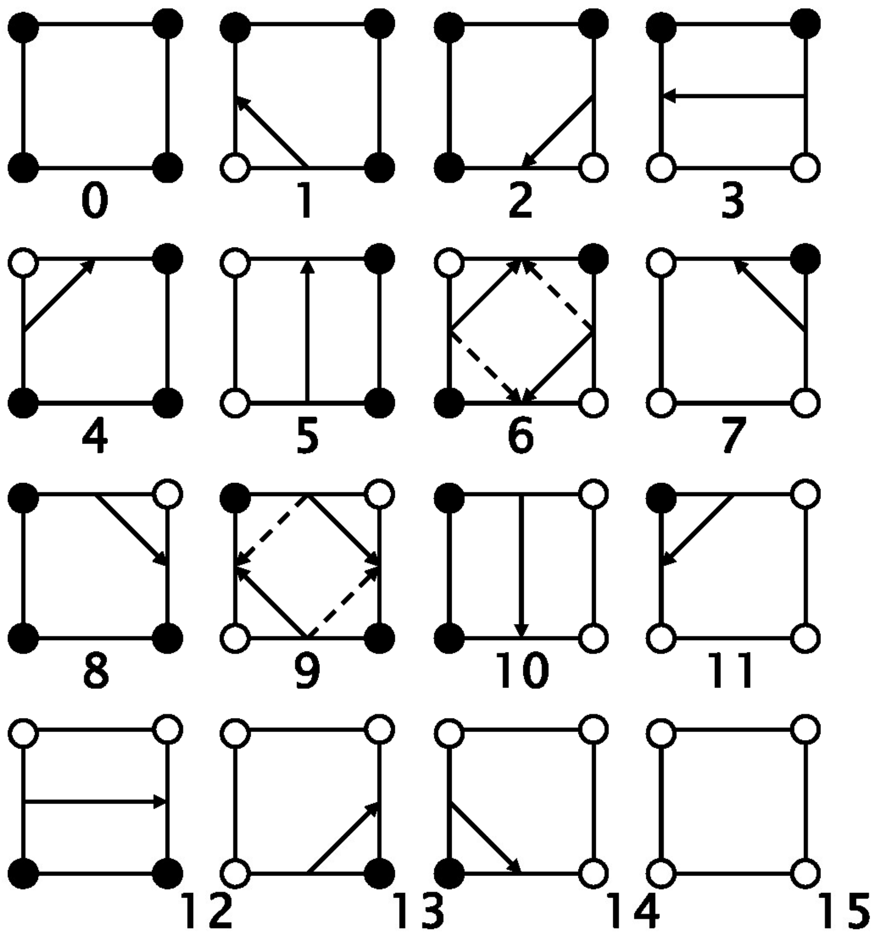
Basic contouring algorithms:

- cell-by-cell algorithms: simple structure, but generate disconnected segments, require post-processing
- contour propagation methods: more complicated, but generate connected contours

"Marching squares" algorithm (systematic cell-by-cell):

- process nodes in ccw order, denoted here as x_0, x_1, x_2, x_3
- compute at each node $\mathbf{x}_i$ the reduced field $\tilde{f}(x_i) = f(x_i) - (c - \varepsilon)$ (which is forced to be nonzero)
- take its sign as the i^{th} bit of a 4-bit integer
- use this as an index for lookup table containing the connectivity information:

Contours in a quadrangle cell



● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

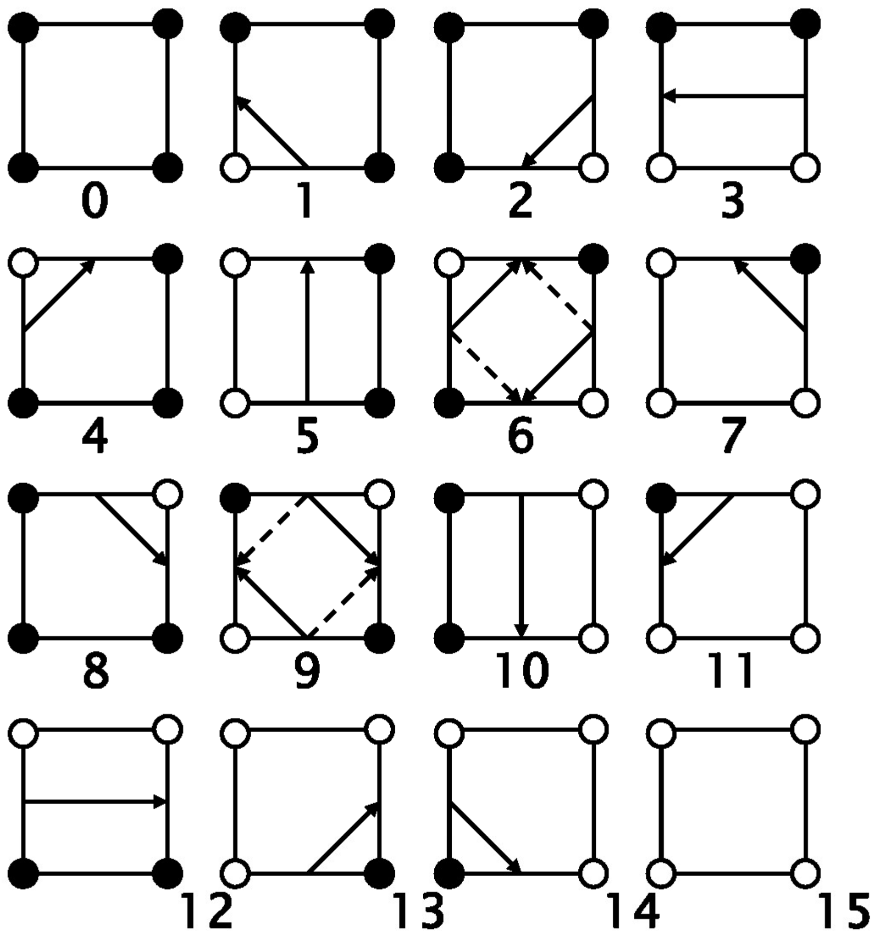
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



- ● $f(x_i) < c$
- ○ $f(x_i) \geq c$

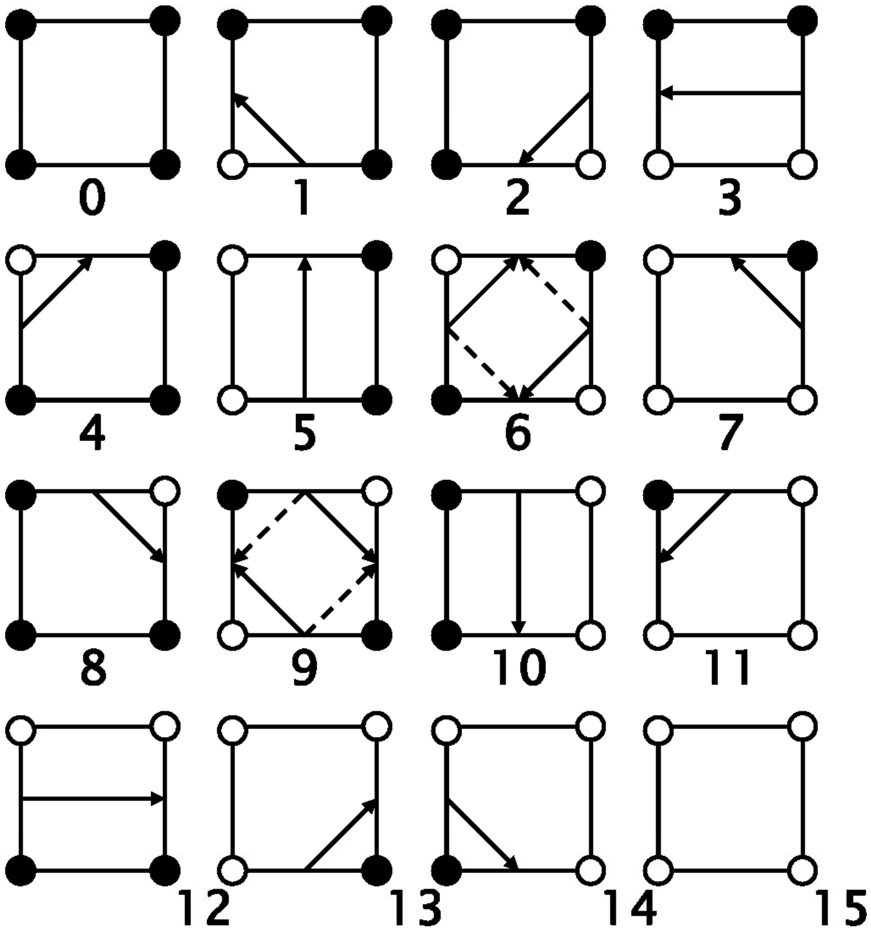
Alternating signs exist in cases 6 and 9.

Choose the solid or dashed line?

Both are possible for topological consistency.

This allows to have a fixed table of 16 cases.

Contours in a quadrangle cell



● $f(x_i) \leq c$

○ $f(x_i) > c$

- Alternating signs exist in cases 6 and 9.
- Choose the solid or dashed line?
- Both are possible for topological consistency.
- This allows to have a fixed table of 16 cases.

Orientability (1-manifold embedded in 2D)

not orientable

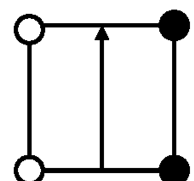
Moebius strip
(only one side!)

Orientability of 1-manifold:

Possible to assign consistent left/right orientation

Iso-contours

- Consistent side for scalar values…
  - greater than iso-value (e.g, *left* side)
  - less than iso-value (e.g., *right* side)
- Use consistent ordering of vertices
  (e.g., larger vertex index is "tip" of arrow;
  if (0,1) points "up", "left" is left, …)



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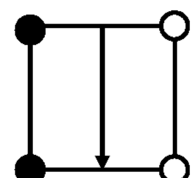

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● $\tilde{f}(x_i) < 0$

○ $\tilde{f}(x_i) > 0$

Orientability (2-manifold embedded in 3D)

Orientability of 2-manifold:

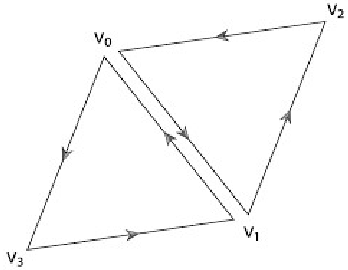
Possible to assign consistent normal vector orientation

not orientable

Moebius strip
(only one side!)

Triangle meshes

- Edges
  - Consistent ordering of vertices: CCW (counter-clockwise) or CW (clockwise)
    (e.g., (0,1,2) on one side of edge, (1,0,3) on the other side)
- Triangles
  - Consistent front side vs. back side
  - Normal vector; or ordering of vertices (CCW/CW)
  - See also: “right-hand rule”



Topological consistency

To avoid degeneracies, use symbolic perturbations:

If level c is found as a node value, set the level to c-ε where ε is a symbolic infinitesimal.

Then:

- contours intersect edges at some (possibly infinitesimal) distance from end points
- flat regions can be visualized by pair of contours at c-ε and c+ε
- contours are topologically consistent, meaning:

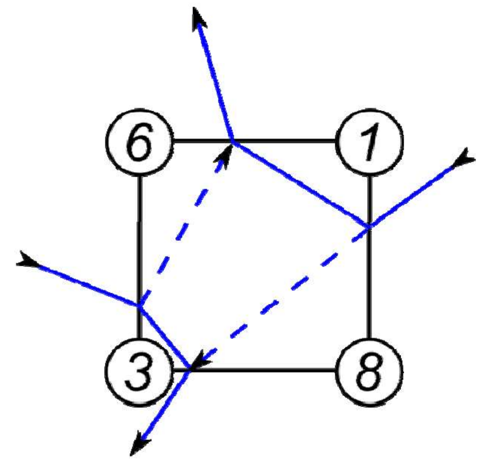
Contours are closed, orientable, nonintersecting lines.

Ambiguities of contours

What is the correct contour of c=4?

Two possibilities, both are orientable:

- connect high values ———
- connect low values - - - - - -



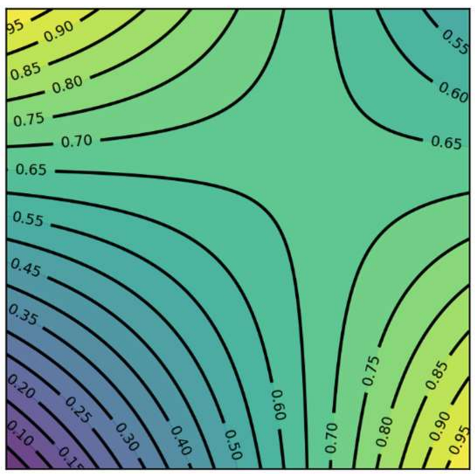
Answer: correctness depends on interior values of $f(x)$.

But: different interpolation schemes are possible.

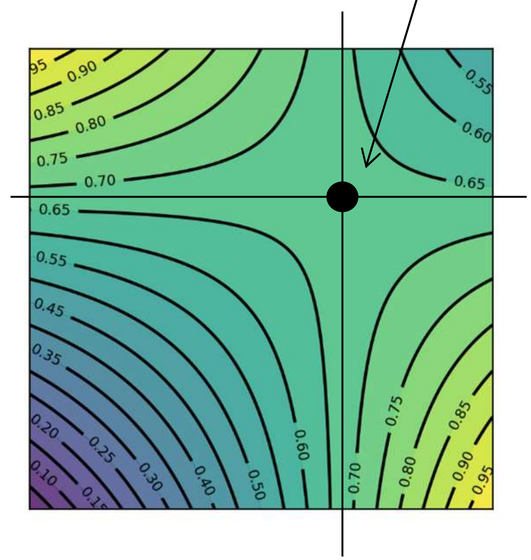
Better question: What is the correct contour with respect to bilinear interpolation?

Bi-Linear Interpolation: Critical Points

Critical points are where the gradient vanishes (i.e., is the zero vector)



critical point
(saddle point)



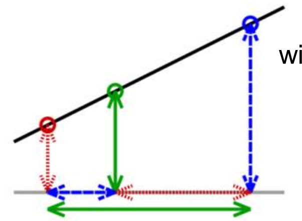
here, the critical
value is 2/3=0.666…

“Asymptotic decider”: resolve ambiguous configurations (6 and 9) by comparing specific iso-value with critical value (scalar value at critical point)

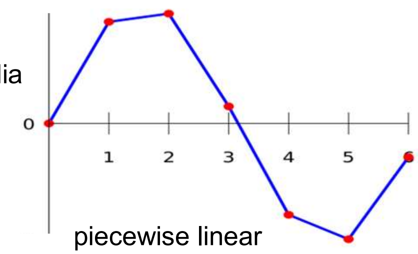
Linear Interpolation / Convex Combinations

Linear interpolation in 1D:

$$f(\alpha) = (1-\alpha)v_1 + \alpha v_2$$



wikipedia



Line embedded in 2D (linear interpolation of vertex coordinates/attributes):

$$f(\alpha_1, \alpha_2) = \alpha_1 v_1 + \alpha_2 v_2 \qquad \alpha_1 + \alpha_2 = 1$$

$$f(\alpha) = v_1 + \alpha(v_2 - v_1) \qquad \alpha = \alpha_2$$

Line segment: $\alpha_1, \alpha_2 \geq 0$ (→ convex combination)

Compare to line parameterization with parameter t:

$$v(t) = v_1 + t(v_2 - v_1)$$

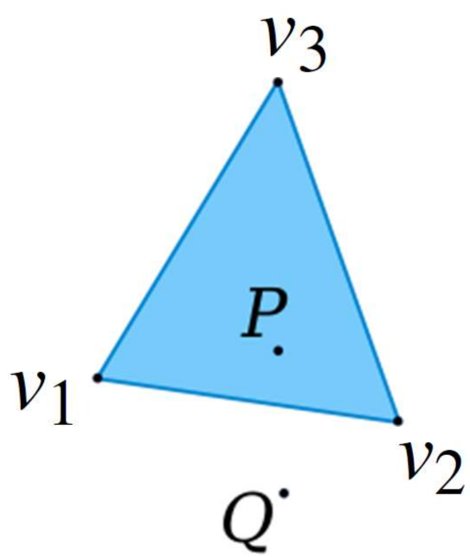
Linear Interpolation / Convex Combinations

Linear combination (n-dim. space):

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

Affine combination: Restrict to $(n-1)$-dim. subspace:

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$



Convex combination: $\alpha_i \geq 0$

(restrict to simplex in subspace)

Linear Interpolation / Convex Combinations

The weights α_i are the n normalized ***barycentric*** coordinates

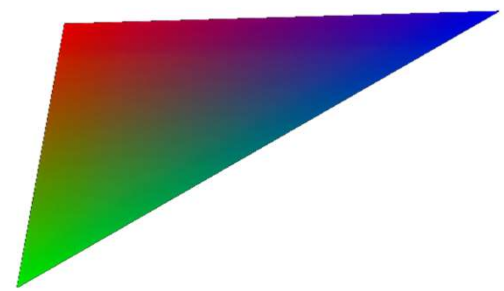
→ linear attribute interpolation in simplex

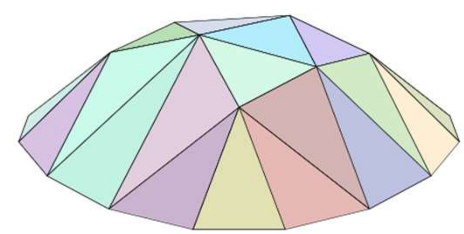
$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

$$\alpha_i \geq 0$$

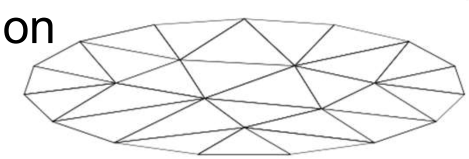
attribute interpolation





spatial position interpolation

wikipedia

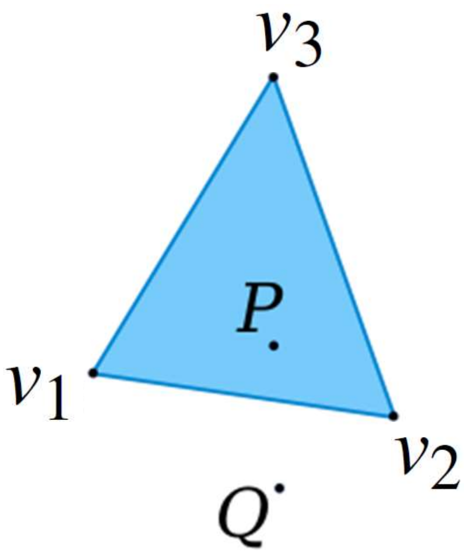


Linear Interpolation / Convex Combinations

$$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n = \sum_{i=1}^{n} \alpha_i v_i$$

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = \sum_{i=1}^{n} \alpha_i = 1$$

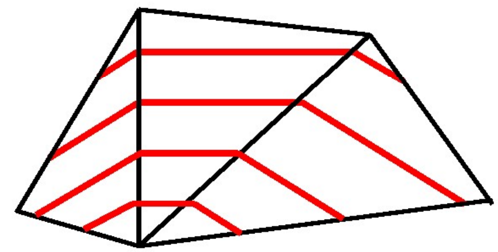
Can re-parameterize to get $(n-1)$ ***affine*** coordinates:



$$\begin{aligned} \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 &= \\ \tilde{\alpha}_1 (v_2 - v_1) + \tilde{\alpha}_2 (v_3 - v_1) + v_1 & \\ \tilde{\alpha}_1 &= \alpha_2 \\ \tilde{\alpha}_2 &= \alpha_3 \end{aligned}$$

Contours in triangle/tetrahedral cells

Linear interpolation of cells implies piece-wise linear contours.

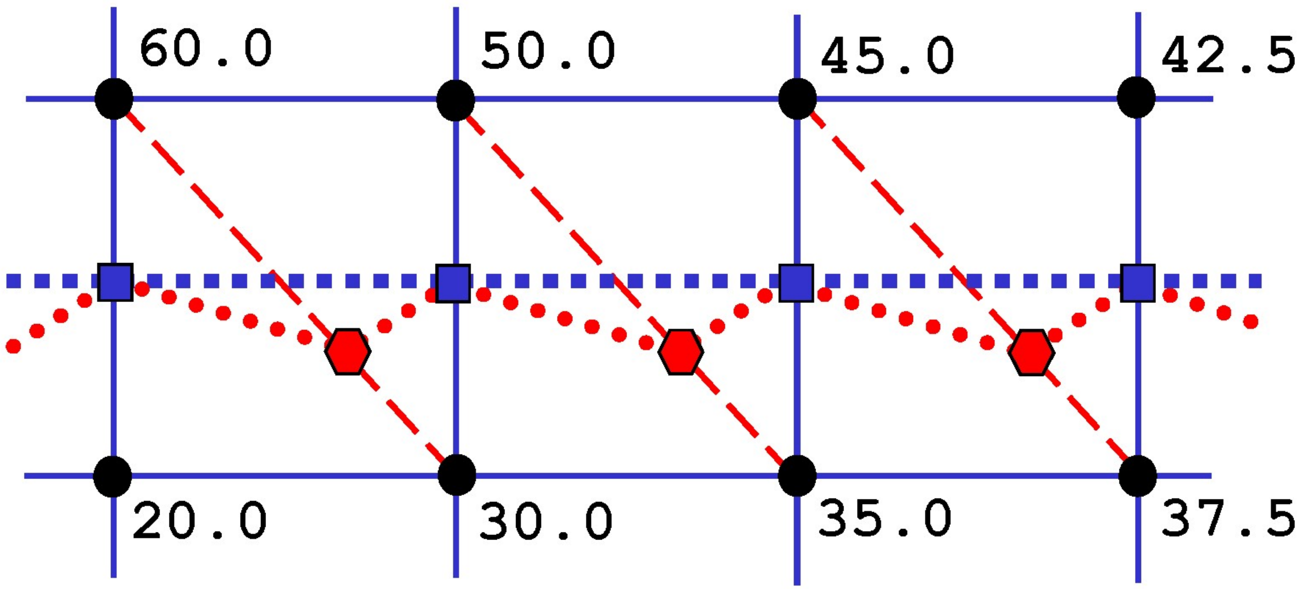


Contours are unambiguous, making "marching triangles" even simpler than "marching squares".

Question: Why not split quadrangles into two triangles (and hexahedra into five or six tetrahedra) and use marching triangles (tetrahedra)?

Answer: This can introduce periodic artifacts!

Illustrative example: Find contour at level *c*=40.0 !



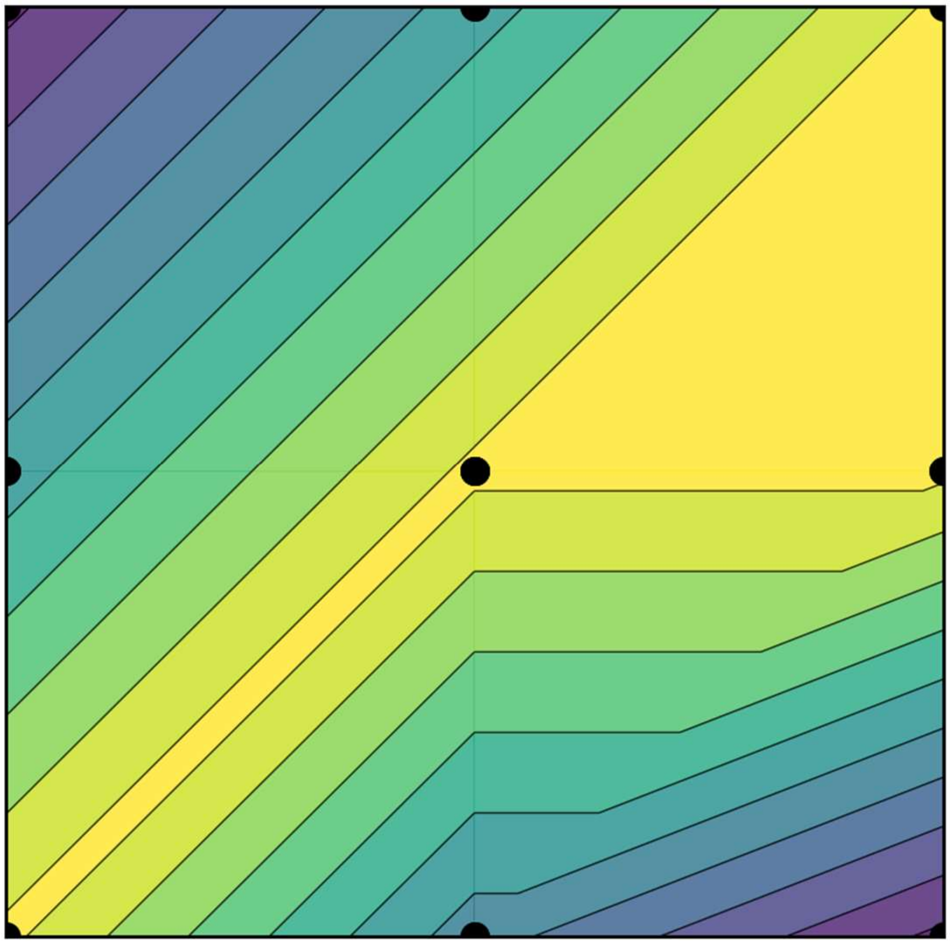
original quad grid, yielding vertices and contour

triangulated grid, yielding vertices and contour

Bi-Linear Interpolation: Comparisons

linear

(2 triangles per quad;
diagonal:
bottom-left,
top-right)



Bi-Linear Interpolation: Comparisons

linear

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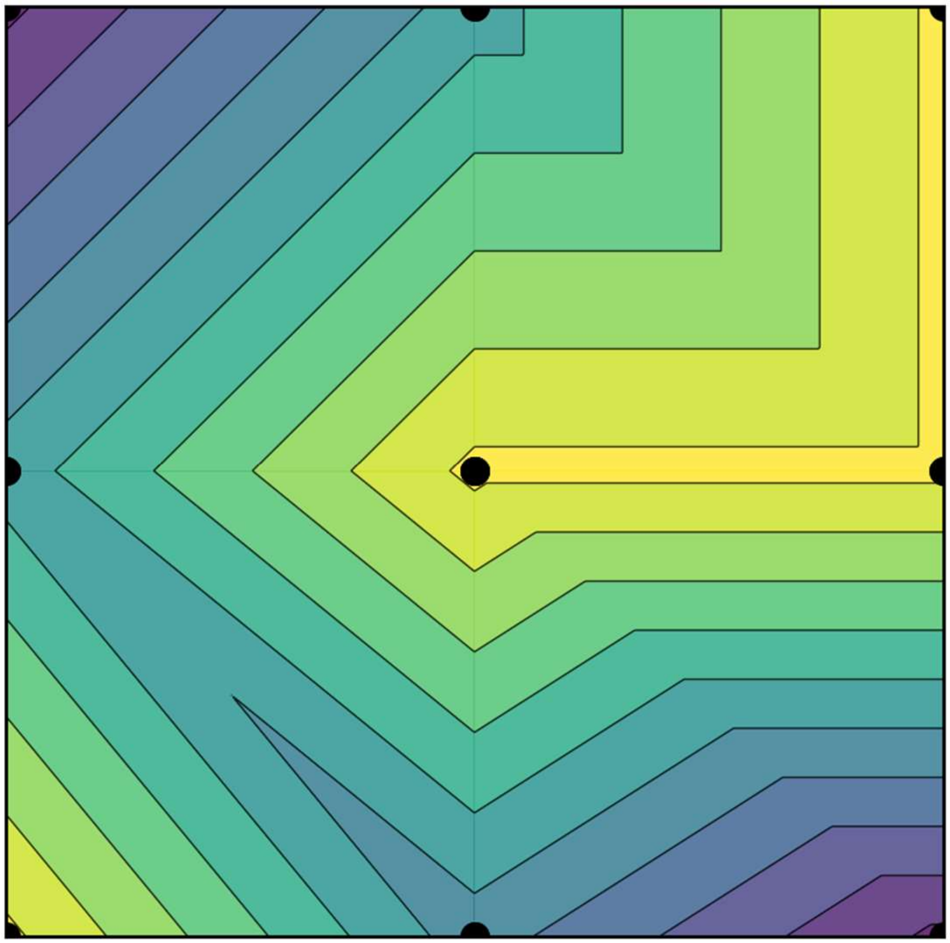
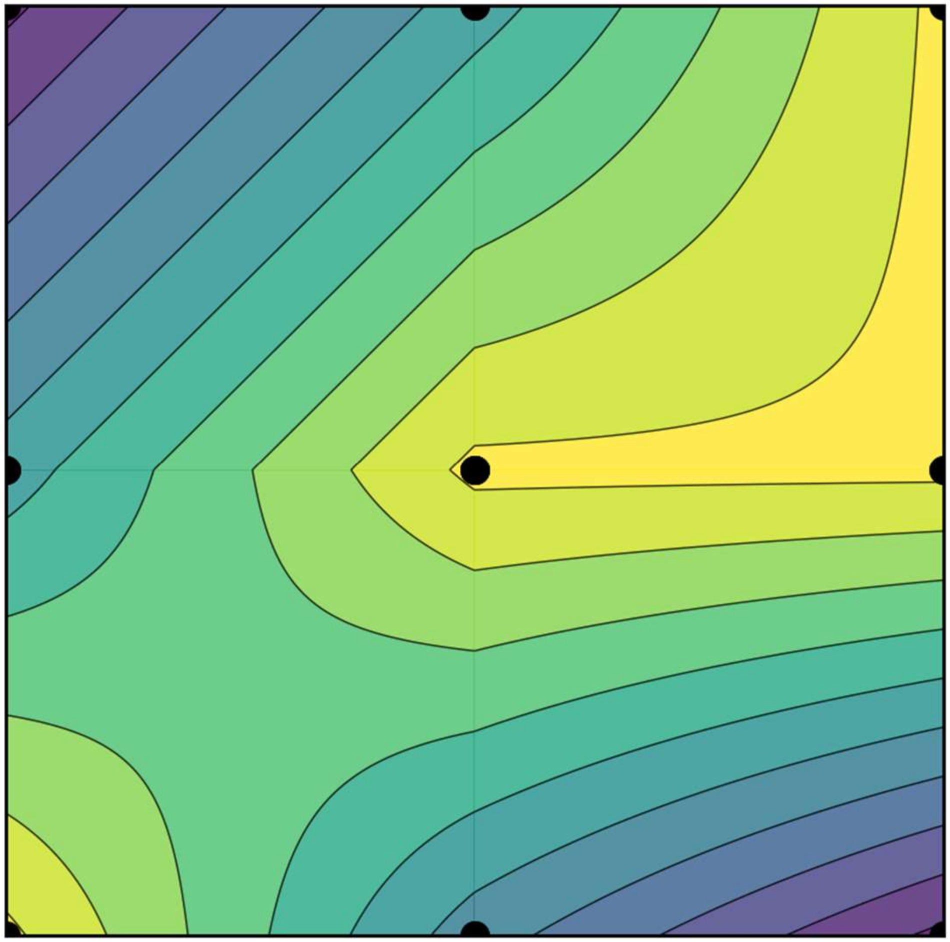


Bi-Linear Interpolation: Comparisons

bi-linear



Thank you.

Thanks for material

- Helwig Hauser
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